Transient Analysis of K-node Tandem Forked Queuing Model with Bulk Arrivals Having Load Dependent Service Rates

M. Sita Rama Murthy, K.Srinivasa Rao, V.Ravindranath, P.Srinivasa Rao

Abstract: ***In this paper a K-node series and parallel queuing model with load dependent service rates is introduced and analysed. It is assumed that the customers arrive to the initial queue in batches and wait for service. After completing the service at first service station they may join any one of the (K-1) parallel queues which are connected to first queue in series and exit from the system after getting service. Here it is assumed that the arrival and service completions follow Poisson processes and service rates depend on number of customers in the queue connected to it. Using difference-differential equations the joint probability function of number of customers in each queue is derived. The system performance measures such as average number of customers, waiting time of customer, variation of number of customers in each queue, throughput of each service station, utilization of each server are derived explicitly. The sensitivity of the model is analysed through numerical illustration and observed that the performance measures are significantly influenced by state dependent service rates. This model also includes the earlier models as particular cases for specific values of the parameters. This model is useful in analysing the practical situations such as communication networks, production process and cargo handling.***

Index Terms: Bulk arrivals, forked queuing model, Load dependent service rates, Performance of system, Poisson Process, Tandem queue.

I. INTRODUCTION

Queuing reflects organized behaviour of large numbers whether it involves persons , items machines thoughts, services etc., For homogeneous services with small numbers simple queues will solve the problem. However when the situation involves large groups bulk arrivals seeking various services the complexity increases. Handling such situations needs mathematical modelling and analysis because mathematical modelling gives a picture of all theoretical possibilities and combinations before working out real time situations. The advantage of forming and analysis of mathematical model is that they are time and cost effective ,always give scope for comparing with practical situation and modify the theoretical model approximately, can be tested with generic data and go back to real time situation for implementation. Various forked queuing models with load dependent service rates have been developed to analyse mathematical situations allowing bulk arrivals. The present study is motivated by Srinivasa Rao et al., [1][2][3][4], S.P.Niranjan.et al.,[5], Raja Sekhar Reddy et al.,[6]. Several queuing models have been developed and analysed in order to evaluate the performance of several systems for control and monitoring. Queueing models formulate a prerequisite for design and and development of several systems arising at places like Communication networks, ATM scheduling, Transportation systems, Production processes etc.,(Boxima O.J., [7] , Bunday.B.D., [8] , Srinivasa Rao et al., [1][2][3][4] , Charan Jeet Singh et al., [9] ,Kin L.Leung et al[10]). Recently much work has been reported regarding Tandem Queueing models. In Tandem Queueing models the output of one Queue formulate the input of the other. Jackson Paul [11] , Srinivasa Rao et al., [1][2][3][4] , Armuganathan et al [12] , Raghavendran et al[13]and others have developed various tandem Queueing models with the assumption that arrivals and services are independent. But in practical situations the service time is to adjust depending upon the number of customers in the Queue. This type of Queueing models are called load dependent Queueing models. Srinivasa Rao et al., [1][2][3][4],Suresh Varma et al., [14], Tirupathi Rao et al., [15], Nageswara Rao et al., [16], Subhashini et al., [17][18] [19], Anyue Chen et al., [20] have developed Queueing models with the assumption that the service rates are dependent on the number of customers in the Queue. In all these papers they assumed that the nodes are connected in tandem and single. But in several practical situations after getting service from the first Queue the customer may join one of the several Queues connected to it for service .For example in Communication networks after getting service from the first transmitter the data/voice packets are to be routed to one of the several buffers connected in parallel for forward transmission. This type of scenario is also visible in Production processes such as Glass manufacturing, where the raw material is converted as liquid glass. It is then transferred to several production lines which are parallel for making different types of glass ware. This type of Queueing models may be called as 2-node series and K parallel Queueing systems, referred as forked Queueing models.

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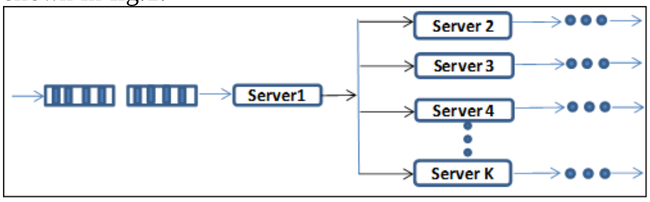
Sita Rama Murthy. M et al., [21] have developed a K-node Tandem Queueing model in which customers or units arrive individually at the first server and after getting service there they may join in any one of (K-1) parallel queues connected in series. In this paper they assumed that the customers arrive single and wait for service. But in other practical situations like store and forward communications network the arrival messages are converted in to random number of packets depending upon size of the messages. Also in railway yards and ports also the cargo handling is done in batches of random size. For analysing such situations one has to consider queuing models with bulk arrivals. The bulk arrival queuing models were initiated by Erlang solution of $M/E_k/1$ model [22] ,Brockmeyer et.al., [23] and O.Brain[24].Later several authors developed queuing models with bulk arrivals. Subhashini A.V.S et.al., [17][18],[19]developed tandem queuing model of 2 servers with non homogeneous Poisson bulk arrivals. Sadu Atchuta Rao et.al., [25] have developed a tandem queuing model of 3 servers with non homogeneous Poisson arrivals having state dependent service rates. Very little work has been reported in literature regarding K-node tandem queuing model with load dependent service having bulk arrivals. Hence in this paper we developed a K-node tandem queuing model with K servers having state dependent service rates.

Here it is assumed that the arrival processes and service processes follow Poisson processes. It is further assumed that the service rate of each service station depends on the number of customers in the Queue connected to it. Using the difference–differential equations the joint probability generating function of the number of customers in each Queue is derived. The performance of the model is analysed by deriving explicit expressions for the system characteristics such as average number of customers in the Queue, Probability of idleness of each service station, Throughput of the nodes, Average waiting time customers in each Queue, Utilisation of each server etc., The sensitivity analysis of the model is carried with a numerical illustration.

II. QUEUEING MODEL

In this section a queuing model with K buffers $B_1, B_2, \dots B_k$ of infinite capacity and K servers $S_1, S_2, \dots S_k$ connected as forked network is considered. It is assumed that the customers arrive in bathes to the first queue and after getting service at first server they may join any of the (K-1) queues connected to the servers $S_2, S_3, \dots S_k$ which are parallel and connected to first server in tandem , with some probability i.e., the customers after getting service at S_1 in batches may join second buffer with probability θ_1 or third buffer with probability θ_2 or K th buffer with probability θ_{k-1}.Let us assume that the actual number of customers in any arriving module is a random variable X with probability C(X).Let $\lambda_x$be the arrival rate of batches of size x and λ is the composite arrival rate. Then $\lambda = \sum \lambda_x$.Therefore the arrival process follows a compound Poisson process with arrival rate $\lambda . E(x)$ .Further it is assumed that the service completion in each service station is random and follows a Poisson process with service rates $\mu_1, \mu_2, \mu_3 \dots \mu_k$ respectively. Here we assume that service rate in each server is linearly dependent on the content of buffer connected to it and queue discipline is first come first serve(FCFS).

The schematic diagram representing the queuing model is shown in fig.1.



Let $P(n_1, n_2, \dots, n_k; t)$ be the probability that there are $n_1$costumers in first queue, n_2 customers in second queue and n_k customers in k^{th} th queue at time t. The customers arrive in batches of size X .The probability generating function of X is $C(Z) = \sum_{m=1}^{\infty} C_m z^m$.

Then difference differential equations governing the system are

$$\frac{\partial P}{\partial t}(n_1, n_2, \dots n_k; t) = -\left[\lambda + \sum_{i=1}^{k} n_i \mu_i\right] P(n_1, n_2, \dots, n_k; t) + (n_1+1)\mu_1[\theta_1 P(n_1+1, n_2-1, n_3, \dots, n_k; t) + \theta_2 P(n_1+1, n_2, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(n_1+1, n_2, \dots, n_k-1; t)] + (n_2+1)\mu_2 P(n_1, n_2+1, n_3, \dots, n_k; t) + (n_3+1)\mu_3 P(n_1, n_2, n_3+1, \dots, n_k; t) + \dots + (n_k+1)\mu_k P(n_1, n_2, \dots, n_k+1; t) + \lambda \sum_{m=1}^{n_1} C_m P(n_1-m, n_2, \dots, n_k, t) \quad (1)$$

$$\frac{\partial P}{\partial t}(0, n_2, \dots n_k; t) = -\left[\lambda + \sum_{i=2}^{k} n_i \mu_i\right] P(0, n_2, \dots, n_k, t) + \mu_1[\theta_1 P(1, n_2-1, n_3, \dots, n_k; t) + \theta_2 P(1, n_2, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(1, n_2, n_3, \dots, n_k-1; t)] (n_2+1)\mu_2 P(0, n_2+1, n_3, \dots, n_k; t) + (n_3+1)\mu_3 P(0, n_2, n_3+1, \dots, n_k; t) \dots + (n_k+1)\mu_k P(0, n_2, n_3, \dots, n_k+1; t) \quad (2)$$

$$\frac{\partial P}{\partial t}(n_1, 0, \dots n_k; t) = -\left[\lambda + \sum_{i=1, i\neq 2}^{k} n_i \mu_i\right] P(n_1, 0, \dots, n_k, t) + (n_1+1)\mu_1[\theta_2 P(n_1+1, 0, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(n_1+1, n_2, \dots, n_k-1; t)] + \mu_3 P(n_1, 0, n_3+1, \dots, n_k; t) + \dots + (n_k+1)\mu_k P(n_1, 0, \dots, n_k+1; t) + \lambda \sum_{m=1}^{n_1} C_m P(n_1-m, 0, \dots, n_k, t) \quad (3)$$

$$\frac{\partial P}{\partial t}(n_1, n_2, \dots, 0; t) = -\left[\lambda + \sum_{i=1}^{k-1} n_i \mu_i\right] P(n_1, n_2, \dots, 0, t) + (n_1+1)\mu_1[\theta_1 P(n_1+1, n_2-1, n_3, \dots, 0; t) + \theta_2 P(n_1+1, n_2, n_3-1, \dots, 0; t) + \dots + \theta_{k-2} P(n_1+1, n_2, \dots, n_{k-1}-1, 0; t)] + (n_2+1)\mu_2 P(n_1, n_2+1, n_3, \dots, 0; t) + (n_3+1)\mu_3 P(n_1, n_2, n_3+1, \dots, 0; t) (n_k+1)\mu_k P(n_1, n_2, \dots, 1; t) + \lambda \sum_{m=1}^{n_1} C_m P(n_1-m, n_2, \dots, n_{k-1}, 0; t) \quad (4)$$

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$\frac{\partial P}{\partial t}(0,0,..n_k;t) = -[\lambda + \sum_{i=3}^{k} n_i\mu_i]P(0,0..,n_k,t) + \mu_1[\theta_2 P(1,0,\ n_3-1,\ldots,n_k;t) + \cdots + \theta_{k-1}P(1,0,\ldots,n_k-1;t)] + \mu_2 P(0,1,n_3,..,n_k;t) + (n_3+1)\mu_3 P(0,0,n_3+1,..,n_k;t) \ldots + (n_k+1)\mu_k P(0,0,n_3 \ldots,n_k+1;t)$ (5)

$\frac{\partial P}{\partial t}(0,n_2,\ldots n_{k-1},0;t) = -[\lambda + \sum_{i=2}^{k-1} n_i\mu_i]P(0,n_2,.,n_{k-1},0;t) + \mu_1[\theta_1 P(1,n_2-1,n_3,..,n_{k-1},0;t) + P(1,n_2+1,n_3,\ldots,n_{k-1},0;t) + \cdots + \theta_{k-2}P(1,n_2,\ldots,n_{k-1}-1,0;t)] + (n_2+1)\mu_2 P(n_1,n_2+1,n_3,..,0;t) + (n_3+1)\mu_3 P(0,n_2,n_3+1,..,0;t) + \ldots + (n_k+1)\mu_k P(0,n_2 \ldots,1;t)$ (6)

$\frac{\partial P}{\partial t}(0,0,0,n_4,\ldots n_k;t) = -[\lambda + \sum_{i=4}^{k} n_i\mu_i]P(0,0,0,n_4,\ldots n_k;t) + \mu_1\theta_3 P(1,0,0,n_4-1,..,n_k;t) + .. + \mu_k\theta_{k-1}P(1,0,0,n_4,..,n_k-1;t)] + \mu_2 P(0,1,0,n_4..,n_k;t) + \mu_3 P(0,0,1,n_4,..,n_k;t) + .. + (n_k+1)\mu_k P(0,0,0,n_4,\ldots,n_k+1;t)$ (7)

$\frac{\partial P}{\partial t}(0,0,n_3,\ldots n_{k-1},0;t) = -[\lambda + \sum_{i=3}^{k-1} n_i\mu_i]P(0,0,n_3..,n_{k-1},0;t) + \mu_1[\theta_2 P(1,0,n_3-1,\ldots,0;t) + \cdots + \theta_{k-2}P(1,0,,n_3,..,n_{k-1}-1,0;t)] + \mu_2 P(0,1,n_3,\ldots,n_{k-1},0;t) + (n_3+1)\mu_3 P(0,0,n_3+1..,n_{k-1},0;t) + .. + \mu_k P(0,1,0,n_3,\ldots,1;t)$ (8)

$\frac{\partial P}{\partial t}(n_1,0,0,n_4,.,0;t) = -[\lambda + \sum_{i=1\neq 2,3}^{k-1} n_i\mu_i]P(n_1,0,0,n_4,.,0;t) + (n_1+)\mu_1[\theta_3 P(n_1+1,0,0,n_4-1,.,0;t) + \ldots + \theta_{k-2}P(n_1+1,0,0,\ldots,n_{k-1}-1,0;t)] + \mu_2 P(n_1,1,0,n_4,.,0;t) + \mu_3 P(n_1,0,1,n_4 \ldots,0;t) + .. + \mu_k P(n_1,0,0,n_4 \ldots,1;t) + \lambda\sum_{m=0}^{n_1} C_m P(n_1-m,0,0,n_4,..,t)$ (9)

$\frac{\partial P}{\partial t}(0,0,.,0;t) = -\lambda P(0,0,\ldots,0;t) + \mu_2 P(0,1,0..,0;t) + \mu_3 P(0,0,1,\ldots,0;t) + .. + \mu_k P(0,0..,0,1;t)$ (10)

Let $P(Z_1,Z_2,..,Z_k;t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_k=0}^{\infty} p(n_1,n_2,\ldots n_k;t)\, z_1^{n_1} z_2^{n_2} .. z_k^{n_k}$ be probability generating function of $p(n_1,n_2,\ldots n_k;t)$.

Multiplying equations (1) to (10)with probability generating function and summing over $n_1,n_2,\ldots n_k$ from 0 to ∞ we get the Joint Probability generating function of number of customers in first, second,….k[th] queues respectively at any time t as

$$P(Z_1,Z_2,\ldots Z_k;t) = exp\Big[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\cdots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}..\binom{r_{k-1}}{r_k}C_m \left\{(z_1-1) + \frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1} + \frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1} + .+ \frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}..\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\left\{\frac{\theta_{k-2}\mu_1(z_{k-1}-1)}{\mu_{k-1}-\mu_1}\right\}^{r_{k-1}-r_k}\left\{\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_k}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\Big] \quad (11)$$

III. CHARACTERISTICS OF THE MODEL

Putting $z_1 = 0, z_2 = 0, \ldots .z_k = 0$ in (11) and expanding we get the probability that the k-server system is empty at any time t.as

$$P(0,0,..0;t) = exp\Big[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\cdots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}..\binom{r_{k-1}}{r_k}C_m \left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\cdots\left(\frac{\theta_{k-2}\mu_1}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_k}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\Big] \quad (12)$$

III. A. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, \ldots .z_k = 1$ in (11) we get probability generating function of first queue size as

$$P(Z_1;t) = exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (13)$$

Mean number of customers in first buffer is $E(N_1) = L_1(t) = \frac{\lambda}{\mu_1}(1-e^{-\mu_1 t})E(X)$ (14)

Where $E(X)$ is the mean of batch size arrivals to first queue and is given by $E(X) = \sum_{m=1}^{\infty} m.C_m$

Putting $Z_1 = 0$ in (13) we get the probability that the first queue is empty as

$$P(0,\ldots,.;t) = exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (15)$$

Utilization of first server is $U_1(t) = 1 - P(0,\ldots,.;t)$

$$= 1 - exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (16)$$

Throughput of first server is $\text{Thp1}(t) = \mu_1.U_1(t)$

$$= \mu_1\left\{1 - exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\} \quad (17)$$

Average waiting time of a customer in first queue is

$$W_1(t) = \frac{L_1(t)}{Thp_1(t)} = \frac{\left(\frac{\lambda}{\mu_1^2}\right)(1-e^{-\mu_1 t})E(X)}{\left\{1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}} \quad (18)$$

Variation of the number of customers in first queue is

$$V(Z_1) = V_1(t) = \lambda\sum_{m=2}^{\infty}\left[\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \quad (19)$$

Coefficient of variation of the number of customers in first queue is

$$CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)}x100 = \frac{\sqrt{\lambda\sum_{m=2}^{\infty}\left[\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\frac{\lambda}{\mu_1}(1-e^{-\mu_1 t})E(X)}x100 \quad (20)$$

III.B. PERFORMANCE ANALYSIS OF i[th] QUEUE FOR i =2,3,..k

Putting $z_1 = 1, z_2 = 1, z_3 = 1, .., z_{k-1} = 1$ we get probability generating function of i[th] buffer size distribution as

$$P(z_i;t) = exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i}\binom{m}{r_1}\binom{r_1}{r_i}C_m\left\{\frac{\theta_{i-1}\mu_1(z_i-1)}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (21)$$

Mean number of customers in i[th] queue is $E(N_i) = L_i(t)$

$$= \left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_{i}-\mu_1}\right\}\right]E(X) \quad (22)$$

Where E(X) is the mean of batch size arrivals to first queue and is given by $E(X) = \sum_{m=1}^{\infty} m.C_m$

Putting $z_i = 0$ in (21)we get the probability that the i[th] queue is empty as

$$P(.,.,0,.;t) = exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (23)$$

Utilization of i[th] server is $U_i(t) = 1-P(.,.,0,.;t) =$

$$exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (24)$$

Throughput of i[th] server is $Thp_i(t) = \mu_i.U_i(t)$

$$= \mu_i.\left[1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right] \quad (25)$$

Average waiting time of a customer in i[th] queue is

$$W_i(t) = \frac{L_i(t)}{Thp_i(t)} = \frac{\left(\frac{\lambda\theta_{i-1}}{\mu_i^2}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]E(X)}{\left[1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right]} \quad (26)$$

Variance of the number of customers in i[th] queue is

$$V(z_i) = V_i(t) = \lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right)+\left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\} + \left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}E(X)\right] \quad (27)$$

Coefficient of variation of the number of customers in i[th] queue is $CV_i(t)$

$$= \frac{\sqrt{V_i(t)}}{L_i(t)}x100 = \frac{\sqrt{\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right)+\left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\}+\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}E(X)\right]}}{\left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]E(X)}x100 \quad (28)$$

III.C. PERFORMANCE MEASURES OF THE MODEL WHEN BATCH SIZE DISTRIBUTION IS UNIFORM

The performance of the model is influenced by the batch size arrival distribution. It is assumed that the number of customers in any arriving model is random and follows uniform distribution with parameters a and b. The probability mass function of uniform batch size distribution is $C_m = \frac{1}{(b-a+1)}$ for m = a,a+1,..,b.The mean number of customers in a batch is $E(x) = \frac{(a+b)}{2}$ and variance of batch size is $V(x) = \frac{1}{12}[(b+a-1)^2-1]$.

The Joint Probability generating function of number of customers in first, second,….k[th] queues respectively at any time t is

$$P(Z_1,Z_2,...Z_k;t) = exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}....\sum_{r_k=0}^{r_{k-1}}(-1)^{r_k}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}..\binom{r_{k-1}}{r_k}\frac{1}{(b-a+1)}\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_1-r_2}\left(\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right)^{r_3-r_4}...\left(\frac{\theta_{k-2}\mu_1(z_{k-1}-1)}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right)^{r_k}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\right] \quad (29)$$

III. D. CHARACTERISTICS OF THE MODEL

Putting $z_1 = 0, z_2 = 0, ...z_k = 0$ in (29) and expanding we get the probability that the k-server system is empty at any time t is

$$P(0,0,..0;t) = exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}....\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}..\binom{r_{k-1}}{r_k}\frac{1}{(b-a+1)}\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}...\left(\frac{\theta_{k-2}\mu_1}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_k}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\right] \quad (30)$$

III.E.PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, ...z_k = 1$ in (29) we get probability generating function of first queue size as

$$P(Z_1;t) = exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\frac{1}{(b-a+1)}\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (31)$$

Mean number of customers in first buffer is

$$E(N_1) = L_1(t) = \left[\frac{\lambda(a+b)}{2\mu_1}\right](1-e^{-\mu_1 t}) \quad (32)$$

Where E(X) is the mean of batch size arrivals to first queue and is given by

$$E(X) = \frac{(a+b)}{2}$$

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Putting $Z_1 = 0$ in (31) we get the probability that the first queue is empty as

$$P(0,\ldots,\ldots;t) = exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} (-1)^{r_2} \frac{1}{(b-a+1)} \binom{m}{r_1} (-1)^{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (33)$$

Utilization of first server is $U_1(t) = 1 - P(0,\ldots,\ldots;t)$

$$= 1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \frac{1}{(b-a+1)} \binom{m}{r_1} (-1)^{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (34)$$

Throughput of first server is $Thp1(t) = \mu_1 . U_1(t)$

$$= \mu_1 \left\{1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \frac{1}{(b-a+1)} \binom{m}{r_1} (-1)^{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\} \quad (35)$$

Average waiting time of a customer in first queue is

$$W_1(t) = \frac{L_1(t)}{Thp_1(t)} = \frac{\left[\frac{\lambda(a+b)}{\mu_1^2}\right](1-e^{-\mu_1 t})}{\left\{1-exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} (-1)^{r_2} \frac{1}{(b-a+1)} \binom{m}{r_1} (-1)^{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}} \quad (36)$$

Variance of the number of customers in first queue is $V(Z_1) = V_1(t) =$

$$\lambda \sum_{m=a}^{b} \left[\binom{m}{2} \frac{1}{(b-a+1)} \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \quad (37)$$

Coefficient of variation of the number of customers in first queue is $CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)} x100 =$

$$\frac{\sqrt{\lambda \sum_{m=a}^{b} \left[\binom{m}{2} \frac{1}{(b-a+1)} \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\left[\frac{\lambda(a+b)}{2\mu_1}\right](1-e^{-\mu_1 t})} x100 \quad (38)$$

III.F. PERFORMANCE ANALYSIS OF i^{th} QUEUE FOR i =2,3,..k

Putting $z_1 = 1, z_2 = 1, z_3 = 1, \ldots z_{i-1} = 1$ in (29) we get probability generating function of i^{th} queue size distribution as $P(Z_i;t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_i=0}^{r_1} (-1)^{r_i} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_i} \left\{\frac{\theta_{i-1}\mu_1(z_i-1)}{\mu_i-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (39)$$

Mean number of customers in i^{th} queue is

$$E(N_i) = L_i(t) = \left[\frac{\lambda\theta_{i-1}(a+b)}{2\mu_i}\left\{1 - \frac{(\mu_i e^{-\mu_1 t} - \mu_1 e^{-\mu_i t})}{(\mu_i - \mu_1)}\right\}\right] \quad (40)$$

Probability that the i^{th} queue is empty is $P(,\ldots 0,.;t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_i=0}^{r_1} (-1)^{r_i+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_i} \left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (41)$$

Utilization of i^{th} server is $U_i(t) = 1 - P(,\ldots,\ldots,0;t) =$

$$1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_i=0}^{r_1} (-1)^{r_i+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_i} \left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (42)$$

Throughput of i^{th} server is $Thp_i(t) = \mu_i . U_i(t)$

$$= \mu_i \left[1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_i=0}^{r_1} (-1)^{r_i+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_i} \left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right] \quad (43)$$

Average waiting time of a customer in i^{th} queue is

$$W_i(t) = \frac{L_i(t)}{Thp_i(t)} = \frac{\left[\frac{\lambda\theta_{i-1}(a+b)}{2\mu_i}\left\{1 - \frac{(\mu_i e^{-\mu_1 t} - \mu_1 e^{-\mu_i t})}{(\mu_i - \mu_1)}\right\}\right]}{\left[1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_i=0}^{r_1} (-1)^{r_i+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_i} \left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right]} \quad (44)$$

Variance of the number of customers in i^{th} queue is $V(z_i) = V_i(t)$

$$= \lambda \left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i - \mu_1}\right)^2 \sum_{m=a}^{b} \binom{m}{2} \frac{1}{(b-a+1)} . \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right) + \left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\} + \left(\frac{\theta_{i-1}\mu_1}{\mu_i - \mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) - \left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}\right] \quad (45)$$

Coefficient of variation of the number of customers in i^{th} queue is $CV_i(t) = \frac{\sqrt{V_i(t)}}{L_i(t)} x100 =$

$$= \frac{\sqrt{\lambda \left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i - \mu_1}\right)^2 \sum_{m=a}^{b} \binom{m}{2} \frac{1}{(b-a+1)} . \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right) + \left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\} + \left(\frac{\theta_{i-1}\mu_1}{\mu_i - \mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) - \left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}\right]}}{\left[\frac{\lambda\theta_{i-1}(a+b)}{2\mu_i}\left\{1 - \frac{(\mu_i e^{-\mu_1 t} - \mu_1 e^{-\mu_i t})}{(\mu_i - \mu_1)}\right\}\right]} x100 \quad (46)$$

IV. NUMERICAL ILLUSTRATION

For numerical illustration we take k=4 and calculate equations and performance measures

The Joint Probability generating function of number of customers in first, second, third and fourth queues respectively at any time t is $P(Z_1, Z_2, Z_3, Z_4; t) =$

$$exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} \sum_{r_3=0}^{r_2} \sum_{r_4=0}^{r_3} (-1)^{r_2} \binom{m}{r_1} \binom{r_1}{r_2} \binom{r_2}{r_3} \binom{r_3}{r_4} C_m \left\{(z_1 - 1) + \frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1} + \frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1} + \frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1-r_2} \left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3} \left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4} \left\{\frac{\theta_3\mu_4(z_4-1)}{\mu_4-\mu_1}\right\}^{r_4} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \quad (47)$$

IV. A CHARACTERISTICS OF THE MODEL

Putting $z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 0$ in (47) and expanding we get the probability that the 4-server system is empty at any time t as $\boldsymbol{P(0,0,0.0;t)} =$

$$exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} \sum_{r_3=0}^{r_2} \sum_{r_4=0}^{r_3} (-1)^{r_2+r_1} \binom{m}{r_1} \binom{r_1}{r_2} \binom{r_2}{r_3} \binom{r_3}{r_4} C_m \left(1 + \frac{\theta_1\mu_1}{\mu_2-\mu_1} + \frac{\theta_2\mu_1}{\mu_3-\mu_1} + .. + \frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2} \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3} \left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4} \left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_4} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \quad (48)$$

IV . B. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, z_4 = 1$ in(48) we get probability generating function of first queue size as

$$P(Z_1;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} C_m \binom{m}{r_1} (z_1 - 1)^{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (49)$$

Mean number of customers in first queue is $E(N_1) = L_1(t) = \left(\frac{\lambda}{\mu_1}\right)(1 - e^{-\mu_1 t})E(X)$ (50)

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Where $E(X)$ is the mean of batch size arrivals to first queue and is given by $E(X) = \sum_{m=1}^{\infty} m.C_m$

Putting $Z_1 = 0$ we get the probability that the first queue is empty as $P(0,\dots,\dots;t) =$

$$exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (51)$$

Utilization of first server is $U_1(t) = 1 - P(0,\dots,\dots;t) = 1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]$ (52)

Throughput of first server is $\text{Thp1}(t) = \mu_1 . U_1(t) = \mu_1\left\{1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}$

(53)

Average waiting time of a customer in first queue is

$$W_1(t) = \frac{L_1(t)}{Thp_1(t)} = \frac{\left(\frac{\lambda}{\mu_1{}^2}\right)(1-e^{-\mu_1 t})E(X)}{\left\{1-exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1}\left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}}$$

(54)

Variance of the number of customers in first queue is $V(Z_1) = V_1(t) =$

$$\lambda \sum_{m=2}^{\infty} \left[\binom{m}{2} . C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \quad (55)$$

Coefficient of variation in number of customers in first queue is

$$CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)} x100 = \frac{\sqrt{\lambda \sum_{m=2}^{\infty}\left[\binom{m}{2} . C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\left(\frac{\lambda}{\mu_1}\right)(1-e^{-\mu_1 t})E(X)} x100$$

(56)

IV.C. PERFORMANCE ANALYSIS OF SECOND QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$ in (48)we get probability generating function of SECOND queue size distribution as

$$P(Z_2;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{\theta_1 \mu_1 (z_2 - 1)}{\mu_2 - \mu_1}\right\}^{r_1} \left\{\frac{1 - e^{(-\mu_1(r_1 - r_2) + \mu_2 r_2)t}}{\mu_1(r_1 - r_2) + \mu_2 r_2}\right\}\right] \quad (57)$$

Mean number of customers in second queue is $E(N_2) =$

$$\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{1 - \frac{(\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t})}{(\mu_2 - \mu_1)}\right\}\right] E(X) \quad (58)$$

Where $E(X)$ is the mean of batch size arrivals at second queue and $E(X) = \sum_m m.C_m$

Putting $z_2 = 0$ we get the probability that the second queue is empty as $P(,,0,,,;t) =$

$$exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]$$

(59)

Utilization of second server is $U_i(t) = 1 - P(,0,,,;t) =$

$$1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (60)$$

Throughput of second server is$Thp_2(t) = \mu_2 . U_2(t) =$

$$\mu_2\left[1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right] \quad (61)$$

Average waiting time of a customers in the second queue is

$$W_2(t) = \frac{L_2(t)}{Thp_2(t)} = \frac{\left[\left(\frac{\lambda\theta_1}{\mu_2{}^2}\right)\left\{1 - \left(\frac{\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t}}{(\mu_2 - \mu_1)}\right)\right\}\right] E(X)}{\left[1 - exp\left[\lambda \sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}(-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]}$$

(62)

Variation of the number of customers in second queue is $V(z_2) = V_2(t)$

$$\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right) + \left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\} \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\} E(X)\right] \quad (63)$$

Coefficient of variation of the number of customers in second queue is $CV_2(t) = \frac{\sqrt{V_2(t)}}{L_2(t)} x100$

$$= \frac{\sqrt{\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right) + \left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\} + \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\} E(X)\right]}}{\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{1 - \frac{(\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t})}{(\mu_2-\mu_1)}\right\}\right] E(X)} x100$$

(64)

IV.D.PERFORMANCE ANALYSIS OF THIRD QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$ in (48) we get probability generating function of third queue size distribution as

$$P(Z_3;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3} C_m \binom{m}{r_1}\binom{r_1}{r_3} \left\{\frac{\theta_2 \mu_1 (z_3 - 1)}{\mu_3 - \mu_1}\right\}^{r_1} \left\{\frac{1 - e^{(\mu_1(r_1 - r_3) + \mu_3 r_3)t}}{\mu_1(r_1 - r_3) + \mu_3 r_3}\right\}\right]$$

(65)

Mean number of customers in third queue is $E(N_3) =$

$$L_3(t) = \left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{1 - \left(\frac{\mu_3 e^{-\mu_1 t} - \mu_1 e^{-\mu_3 t}}{\mu_3 - \mu_1}\right)\right\}\right] E(X) \quad (66)$$

Where $E(X)$ is the mean of batch size arrivals at third queue and $E(X) = \sum_m m.C_m$

Putting $z_3 = 0$ we get the probability that the third queue is empty as $P(,,,0,,;t) =$

$$exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]$$

(67)

Utilization of third server is $U(t) = 1 - P(,,,0,,;t) =$

$$1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]$$

(68)

Throughput of third server is $Thp_3(t) = \mu_3 . U_3(t) =$

$$\mu_3\left[1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]$$

(69)

Average waiting time of a customer in third queue is

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$$W_3\ (t) = \frac{L_3(t)}{Thp_3(t)} = \frac{\left[\left(\frac{\lambda\theta_2}{\mu_3{}^2}\right)\left\{1-\frac{(\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t})}{(\mu_3-\mu_1)}\right\}\right]E(X)}{\left[1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_3=0}^{r_1}(-1)^{r_3+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]} \quad (70)$$

Variation of the number of customers in third queue is

$$V(z_3) = V_3(t) = \lambda\left[\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right)+\left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\}\right.\right.$$
$$\left.\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}E(X)\right] \quad (71)$$

Coefficient of variation of the number of customers in third queue is $CV_3(t) = \frac{\sqrt{V_3(t)}}{L_3(t)}x100$

$$= \frac{\sqrt{\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right)+\left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\}+\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}E(X)\right]}}{\left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{1-\left(\frac{\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t}}{\mu_3-\mu_1}\right)\right\}\right]E(X)}x100 \quad (72)$$

IV.E. PERFORMANCE ANALYSIS OF FOURTH QUEUE

Putting $z_1 = 1, z_2 = 1, z_3 = 1$ in (49)we get probability generating function of fourth queue size distribution as

$$P(Z_4;t) = exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (73)$$

Mean number of customers in fourth queue is $E(N_4) =$

$$L_4\ (t) = \left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right]E(X) \quad (74)$$

Where $E(X)$ is the mean of batch size arrivals at fourth queue and $E(X) = \sum_m m.C_m$

Putting $z_4 = 0$ we get the probability that the fourth queue is empty as $P(.,.,.,.,0;t) =$

$$exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (75)$$

Utilization of fourth server is $U_4\ (t) = 1 - P(.,.,.,.,0;t)$

$$= 1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (76)$$

Throughput of fourth server is $Thp_4\ (t) = \mu_4.U_4\ (t) =$

$$\mu_4[1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right] \quad (77)$$

Average waiting time of a customer in fourth queue is $W_4\ (t)W_4\ (t) = \frac{L_4(t)}{Thp_4(t)} =$

$$\frac{\left[\left(\frac{\lambda\theta_3}{\mu_4{}^2}\right)\left\{1-\frac{(\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t})}{(\mu_4-\mu_1)}\right\}\right]E(X)}{\left[1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right]} \quad (78)$$

Variance of the number of customers in fourth queue is $V(z_4) = V_4(t)$

$$= \lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\right.$$
$$\left.\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}E(X)\right] \quad (79)$$

Coefficient of variation of the number of customers in fourth queue is $CV_4(t) = \frac{\sqrt{V_4(t)}}{L_4(t)}x100$

$$= \frac{\sqrt{\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}E(X)\right]}}{\left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right]E(X)}x100 \quad (80)$$

IV. F .NUMERICAL ILLUSTARTION WITH UNIFORM DISTRIBUTION

The performance of model is influenced by the batch size arrival distribution .It is assumed that number of customers in any arriving module is random and follows uniform distribution with parameters a and b. The probability mass function of uniform distribution is $C_m = \frac{1}{(b-a+1)}$ for m=a,a+1,..b. Mean number of customers in batch is $E(X) = \frac{(a+b)}{2}$.

The Joint Probability generating function of number of customers in first, second, third and fourth queues respectively at any time t is $P(Z_1, Z_2, Z_3, Z_4; t) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\frac{1}{(b-a+1)}\right.$$
$$\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}$$
$$\left.\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\left\{\frac{\theta_3\mu_4(z_4-1)}{\mu_4-\mu_1}\right\}^{r_4}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \quad (81)$$

Putting $z_1 = 0, z_2 = 0, \ldots. z_k = 0$ (81) and expanding we get the probability that the 4-server system is empty at any time t as

$$P(0,0,0,0;t) =$$
$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\frac{1}{(b-a+1)}\right.$$
$$\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}$$
$$\left.\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^4\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \quad (82)$$

IV.G. PERFORMANCE MEASURES OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, \ldots. z_k = 1$ in (11) we get probability generating function of first queue size as $P(Z_1;t) = exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\frac{1}{(b-a+1)}\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]$ (83)

Mean number of customers in first queue is $E(N_1) =$

$$L_1\ (t) = \left[\frac{\lambda(a+b)}{2\mu_1}\right](1-e^{-\mu_1 t}) \quad (84)$$

Putting $Z_1 = 0$ in (83) we get the probability that the first queue is empty as $P(0,.,.,.,.;t) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}(-1)^{r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (85)$$

Utilization of first server is $U_1\ (t) = 1 - P(0,.,.,.,.;t) =$

$$1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}(-1)^{r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]$$

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(86)

Throughput of first server is $Thp1(t) = \mu_1 . U_1(t) = \mu_1\left\{1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} (-1)^{r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}$

(87)

Average waiting time of a customer in first queue is

$$W_1(t) = \frac{L_1(t)}{Thp_1(t)} = \frac{\left[\frac{\lambda(a+b)}{2\mu_1^2}\right](1-e^{-\mu_1 t})}{\left\{1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} (-1)^{r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \left\{\frac{1-e^{\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}}$$

(88)

Variance of the number of customers in first queue is

$$V(Z_1) = V_1(t) = \lambda \sum_{m=a}^{b} \left[\binom{m}{2} \frac{1}{(b-a+1)} \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]$$

(89)

Coefficient of variation of the number of customers in first queue is $CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)} x100 =$

$$\frac{\sqrt{\lambda \sum_{m=a}^{b} \left[\binom{m}{2} \frac{1}{(b-a+1)} \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\left[\frac{\lambda(a+b)}{2\mu_1}\right](1-e^{-\mu_1 t})} x100$$

(90)

IV. H. PERFORMANCE MEASURES OF SECOND QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$) we get probability generating function of second queue size distribution as $P(Z_2; t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_2} \left\{\frac{\theta_1 \mu_1 (z_i - 1)}{\mu_2 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]$$

(91)

Mean number of customers in second queue is

$E(N_2) = L_2(t)$

$$= \left[\frac{\lambda\theta_1(a+b)}{2\mu_2}\left\{1 - \left(\frac{\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t}}{\mu_2 - \mu_1}\right)\right\}\right] \quad (92)$$

Putting $Z_2 = 0$ in (91) we get the probability that the second queue is empty as $P(.,0,.,.;t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_2} \left\{\frac{\theta_1 \mu_1}{\mu_2 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]$$

(93)

Utilization of second server is $U_2(t) = 1 - P(.,0,.,.;t) =$

$$1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_2} \left\{\frac{\theta_1 \mu_1}{\mu_2 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]$$

(94)

Throughput of second server is $Thp_2(t) = \mu_2 . U_2(t) =$

$$\mu_2 . \left[1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_2} \left\{\frac{\theta_1 \mu_1}{\mu_2 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]$$

(95)

Average waiting time of a customers in second queue is $W_2(t) = \frac{L_2(t)}{Thp_2(t)} =$

$$\frac{\left[\frac{\lambda\theta_1(a+b)}{2\mu_2}\left\{1 - \left(\frac{\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t}}{\mu_2 - \mu_1}\right)\right\}\right]}{\left[1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_2} \left\{\frac{\theta_1 \mu_1}{\mu_2 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]}$$

(96)

Variance of the number of customers in second queue is $V(z_2) = V_2(t) =$

$$\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=a}^{b} \binom{m}{2} \frac{1}{(b-a+1)} . \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right) + \left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\} + \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) - \left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\}\right]$$

(97)

Coefficient of variation of the number of customers in second queue is $CV_2(t) = \frac{\sqrt{V_2(t)}}{L_2(t)} x100$

$$= \frac{\sqrt{\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=a}^{b} \binom{m}{2} \frac{1}{(b-a+1)} \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right) + \left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\} + \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) - \left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\}\right]}}{\left[\frac{\lambda\theta_1(a+b)}{2\mu_2}\left\{1 - \left(\frac{\mu_2 e^{-\mu_1 t} - \mu_1 e^{-\mu_2 t}}{\mu_2 - \mu_1}\right)\right\}\right]} x100$$

(98)

IV.I. PERFORMANCE MEASURES OF THIRD QUEUE

Putting $z_1 = 1, z_2 = 1, z_4 = 1$in (81) we get probability generating function of third queue size distribution as $P(Z_3; t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_3} \left\{\frac{\theta_2 \mu_1 (z_3 - 1)}{\mu_3 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]$$

(99)

Mean number of customers in third queue is $E(N_3) =$

$$L_3(t) = \left[\frac{\lambda\theta_2(a+b)}{2\mu_3}\left\{1 - \left(\frac{\mu_3 e^{-\mu_1 t} - \mu_1 e^{-\mu_3 t}}{\mu_3 - \mu_1}\right)\right\}\right] \quad (100)$$

Probability that the third queue is empty is $P(.,.,0,.;t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_3} \left\{\frac{\theta_2 \mu_1}{\mu_3 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]$$

(101)

Utilization of third server is $U_3(t) = 1 - P(.,.,0,.;t) =$

$$1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_3} \left\{\frac{\theta_2 \mu_1}{\mu_3 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]$$

(102)

$Throughput\ of$ third $server\ is\ Thp_3(t) = \mu_3 . U_3(t) =$

$$\mu_3 . \left[1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_3} \left\{\frac{\theta_2 \mu_1}{\mu_3 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]$$

(103)

Average waiting time of a customer in third queue is $W_3(t) = \frac{L_3(t)}{Thp_3(t)} =$

$$\frac{\left[\frac{\lambda\theta_2(a+b)}{2\mu_3^2}\left\{1 - \left(\frac{\mu_3 e^{-\mu_1 t} - \mu_1 e^{-\mu_3 t}}{\mu_3 - \mu_1}\right)\right\}\right]}{\left[1 - exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_3} \left\{\frac{\theta_2 \mu_1}{\mu_3 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]}$$

(104)

Variance of the number of customers in third queue is $V(z_3) = V_3(t) =$

$$\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=a}^{b} \binom{m}{2} \frac{1}{(b-a+1)} . \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right) + \left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\} + \left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) - \left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}\right]$$

(105)

Coefficient of variation of the number of customers in third queue is $CV_3(t) = \frac{\sqrt{V_3(t)}}{L_3(t)} x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=a}^{b} \binom{m}{2} \frac{1}{(b-a+1)} . \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right) + \left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\} + \left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right) - \left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}\right]}}{\left[\frac{\lambda\theta_2(a+b)}{2\mu_3}\left\{1 - \left(\frac{\mu_3 e^{-\mu_1 t} - \mu_1 e^{-\mu_3 t}}{\mu_3 - \mu_1}\right)\right\}\right]} x100$$

(106)

IV. J. PERFORMANCE MEASURES OF FOURTH QUEUE

Putting $z_1 = 1, z_2 = 1, z_3 = 1$ we get probability generating function of fourth queue size distribution as $P(Z_4; t) =$

$$exp\left[\lambda \sum_{m=a}^{b} \sum_{r_1=1}^{m} \sum_{r_4=0}^{r_1} (-1)^{r_4} \frac{1}{(b-a+1)} \binom{m}{r_1} \binom{r_1}{r_4} \left\{\frac{\theta_3 \mu_1 (z_4 - 1)}{\mu_4 - \mu_1}\right\}^{r_1} \left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]$$

(107)

Mean number of customers in fourth queue is $E(N_4) = L_4(t)$

$$= \left[\frac{\lambda\theta_3(a+b)}{2\mu_4}\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right] \quad (108)$$

Putting $z_4 = 0$ in (107) Probability that the fourth queue is empty is $P(.,.,.,.,0:t) = 0$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (109)$$

Utilization of fourth server is $U_4(t) = 1 - P(.,.,.,.,0:t) =$

$$1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (110)$$

Throughput of fourth server is $Thp_4(t) = \mu_4\, U_4(t) =$

$$\mu_4\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right] \quad (111)$$

Average waiting time of a customer in fourth queue is

$$W_4(t) = \frac{L_4(t)}{Thp_4(t)} = \frac{\left[\frac{\lambda\theta_3(a+b)}{2\mu_4{}^2}\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right]}{\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right]} \quad (112)$$

Variance of the number of customers in fourth queue is $V(z_4) = V_4(t) =$

$$\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=a}^{b}\binom{m}{2}\frac{1}{(b-a+1)}.\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}\right.$$
$$\left.\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}\right] \quad (113)$$

Coefficient of variation of the number of customers in fourth queue is $CV_4(t) = \frac{\sqrt{V_4(t)}}{L_4(t)}x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=a}^{b}\binom{m}{2}\frac{1}{(b-a+1)}.\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left(\frac{a+b}{2}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}\right]}}{\left[\frac{\lambda\theta_3(a+b)}{2\mu_4}\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right]}x100$$

(114)

V.NUMERICAL ILLUSTRATION

The transient behaviour of the model is studied by considering uniform batch size distribution and the performance measures are calculated by varying system parameters as

$t = 0.1, 0.2, 0.3, 0.4, 0.5;\ \lambda = 10, 11, 12, 13, 14;\ \mu_i = 10, 11, 12, 13, 14;\ i = 1, 2, 3, 4;\ \theta_j = 0.1, 0.2, 0.3, 0.4, 0.5;\ j = 1, 2,\ a = 1, 2, 3, 4, 5\ and\ b = 10, 15, 20, 25, 30.$

The mean number of customers in each buffer L_1, L_2, L_3, L_4 is calculated along with mean number of customers $L(t)$ in the entire system by varying the parameters $t, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, \theta_1, \theta_2, \theta_3$ one at atime keeping all other fixed and the calculations are recorded in Table1.The probability of emptiness of each server and also the utilization of servers are calculated correspondingly for each value of parameters as above and the values are tabulated in Table2.The throughputs of four servers $Thp_1, Thp_2, Thp_3, Thp_4$ along with average waiting times of customers in four buffers W_1, W_2, W_3, W_4 are also computed and tabulated in Table3.The Variance of the number of customers V_1, V_2, V_3, V_4 along with coefficient of variation of the number of customers in each queue are calculated and the values are tabulated in Table 4.

From Table1. It is observed that as time t increases from 0.1 to 0.5 the mean number of customers is also increasing in each buffer. The same phenomenon is reflected in mean number of customers in the entire system. Also if service rate μ_1 increases from 10 to 14 keeping μ_2, μ_3, μ_4 unchanged the mean number of customers in first server $L_1(t)$ gets decrease, the mean number of customers in the remaining queues are increasing and number of customers in the entire system $L(t)$ is decreased.. Similarly when μ_2 increases L_2 (t) decreases, μ_3 increases L_3 (t) decreases and no change in the other queues.The same phenomenon is observed with the fourth queue. Thus the improvement in performance of first server improves the performance of entire system. In the same pattern when the probability $\theta_1 (or\ \theta_2)$ that the customers from first server join second (or third server) increases the buffer at second server $L_2(t)(or$ at third server $L_3(t)$) is increasing correspondingly. As the batch size distribution parameters 'a' is increasing then $L_1(t), L_2(t), L_3(t)\ and\ L_4(t)$ are increasing .The same phenomenon is observed with the parameter 'b'.

Table 2. indicates that the probability of emptiness has shown decrease with respect to increase in time. In particular it has sudden decrease when t=0.1 and decreasing normally thereafter when t=0.2,0.3,0.4,0.5. Similarly with increase in mean arrival rate λ the probability of emptiness at each server is decreasing while the utilizations of servers U_1, U_2, U_3, U_4 are increasing. This clearly indicates that the system performs in accordance with time. As the service rate μ_1 increases from $10\ to\ 14$ the probability of emptiness in each service station increases while utilization of each server decreases. The probability of emptiness decreases as the probability of customers joining a particular server increases while its utilization gets increased. Thus as θ_1 increases from 0.1 to 0.5 system emptiness decreases marginally from 0.2254 to 0.2252 and probability of emptiness of second server decreases from 0.8595 to 0.5573.This has an impact on the fourth server since the joining probability of fourth queue is directly dependent on $\theta_1\ and\ \theta_2$ ($\theta_3 = 1 - \theta_1 - \theta_2$) .Therefore the probability of emptiness at fourth server increases from 0.4944 to 0.6846 and it's utilization decreases from 0.5056 to 0.3154.Similarly with increase of θ_2 from 0.1 to 0.5 the probability of emptiness of third server decreases from 0.8631 to 0.5639 and it's utilization increases from 0.1369 to 0.4361. It is observed that as the batch size distribution parameter 'a' is increasing the probability of emptiness of the system as well as servers decrease. The same phenomenon is observed with respect to batch size distribution parameter 'b'.From table.3 it is observed that the throughputs $Thp_1, Thp_2, Thp_3, Thp_4$ and mean waiting times of each of the queues W_1, W_2, W_3, W_4 have shown increase with increase in time. Similarly an increase in λ led to an increase in throughputs as well as mean waiting times. Further we can observe that

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the increase in service rate at second, third and fourth servers lead to increase in throughputs and waiting times except at first server where increase in $\mu_1$leads to increase in Thp_1 and decrease in W_1.As the probability of joining second queue increases from θ_1=0.1 to 0.5 the throughput Thp_2 increases correspondingly from 0.9835 to 3.0989 ,this in turn increase the waiting time W_2 from 0.1651 to 0.2621.As this influences on θ_3 which decreases from 0.7 to 0.3 ,the throughput Thp_4 decreases from 4.8078 to 3.4083 while mean waiting time W_4 decreases from 0.2543 to 0.1793. Similar phenomenon is observed with variation in θ_2, the probability of joining third queue after being served at first queue .From Table.4.it is observed that the variance of the number of customers in first, second and third queues increases with increase in time. On the other hand the coefficient of variation of the number of customers in first, second and third queues get decrease as time increases. But the variance and coefficient of variation of the number of customers at fourth queue increases with increase in time. Similarly increase in λ leads to increase in variance in each of the four queues. Further we observe that the increase in service rates μ_2,μ_3,μ_4 led to decrease in variance of each queue .But the increase in μ_1 leads to decrease in $V_1$and increase in V_2, V_3, V_4.The probability of joining the second or third queue, θ_1 or θ_2 increases the variances in second or third queues increases where as the variance in fourth queue decreases. It is also observed that the increase in batch size parameter ‘a‘ increases the variances V_1, V_2, V_3 and V_4 .The same phenomenon is observed with other batch size parameter ‘ b ‘.

VI . SENSITIVITY ANALYSIS

In this section the sensitivity analysis of model with the values of parameters as t=0.1, λ=15, μ_1 =12, μ_2 =14, μ_3 =11, μ_4 =13, θ_1 =0.3 , θ_2 =0.2, $a = 5$ and $b = 20$ is considered. The effect of varying the parameters on performance measures $L_1, L_2, L_3, L_4, L, W_1, W_2, W_3$ and W_4 with change of $\mp$15%,$\mp$10% and $\mp$5% was computed and are presented in Table 5.

From Table 5. we observe that as time t increases the values of $L_1, L_2, L_3, L_4, L, W_1, W_2, W_3$ and W_4 are increasing.. The same phenomenon is observed with variation in arrival rate λ .It is also observed that as μ_1 increases L_1, L, W_1 decrease whereas L_2, L_3, L_4, W_2, W_3 and W_4 increase. When μ_2 increases L_2, L and W_2 decrease whereas L_3, L_4, W_3 and W_4 remain constants. When μ_3 increases L_3, L, W_3 decrease whereas $L_1, L_2, L_4, W_1, W_2$and W_4 remain constant. When μ_3 increases L_3,L, W_3 decrease whereas $L_1, L_2, L_4, W_1, W_2$and W_4 remain constant Similar phenomenon is observed with μ_4 .We also observed that with increase in θ_1 the performance measures L_2, L, W_2 are increasing , L_4, W_4 are decreasing whereas $L_1, L_3, W_1$and W_3 remain constant .Similarly with increase in θ_2 the performance measures L_3, L, W_3 are increasing , L_4 and W_4 are decreasing whereas L_1, L_2, W_1 and W_2 remain constant. We also observed that with increase in batch size distribution parameters ‘ a ‘and ‘ b’ the performance measures $L_1, L_2, L_3, L_4, L, W_1, W_2, W_3, W_4$ increase.

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Table 1.Values of Expected Number (Mean) of Customers in the Queue in Transient State.

t	a	b	λ	μ_1	μ_2	μ_3	μ_4	θ_1	θ_2	θ_3	$L_1(t)$	$L_2(t)$	$L_3(t)$	$L_4(t)$	$L(t)$
0.1	1	10	15	6	7	8	9	0.1	0.2	0.7	6.2038	0.1624	0.3150	1.0697	7.7509
0.2	1	10	15	6	7	8	9	0.1	0.2	0.7	9.6086	0.4375	0.8269	2.7400	13.613
0.3	1	10	15	6	7	8	9	0.1	0.2	0.7	11.4771	0.6808	1.2601	4.0971	17.5151
0.4	1	10	15	6	7	8	9	0.1	0.2	0.7	12.5026	0.8602	1.5663	5.021	19.9501
0.5	1	10	15	6	7	8	9	0.1	0.2	0.7	13.0654	0.9814	1.7651	5.6008	21.4127
0.1	1	10	**10**	6	7	8	9	0.1	0.2	0.7	4.1359	0.1083	0.2100	0.7131	5.1673
0.1	1	10	**11**	6	7	8	9	0.1	0.2	0.7	4.5495	0.1191	0.2310	0.7844	5.6840
0.1	1	10	**12**	6	7	8	9	0.1	0.2	0.7	4.9631	0.1300	0.2520	0.8557	6.2008
0.1	1	10	**13**	6	7	8	9	0.1	0.2	0.7	5.3767	0.1408	0.2730	0.9271	6.7176
0.1	1	10	**14**	6	7	8	9	0.1	0.2	0.7	5.7903	0.1516	0.2940	0.9984	7.2343
0.1	1	10	15	**10**	7	8	9	0.1	0.2	0.7	5.2150	0.2394	0.4638	1.5735	7.4917
0.1	1	10	15	**11**	7	8	9	0.1	0.2	0.7	5.0035	0.2556	0.4952	1.6798	7.4341
0.1	1	10	15	**12**	7	8	9	0.1	0.2	0.7	4.8043	0.2709	0.5247	1.7794	7.3793
0.1	1	10	15	**13**	7	8	9	0.1	0.2	0.7	4.6166	0.2852	0.5523	1.8727	7.3268
0.1	1	10	15	**14**	7	8	9	0.1	0.2	0.7	4.4397	0.2987	0.5782	1.9602	7.2768
0.1	1	10	15	6	**10**	8	9	0.1	0.2	0.7	6.2038	0.1483	0.3150	1.0697	7.7368
0.1	1	10	15	6	**11**	8	9	0.1	0.2	0.7	6.2038	0.1440	0.3150	1.0697	7.7325
0.1	1	10	15	6	**12**	8	9	0.1	0.2	0.7	6.2038	0.1400	0.3150	1.0697	7.7285
0.1	1	10	15	6	**13**	8	9	0.1	0.2	0.7	6.2038	0.1360	0.3150	1.0697	7.7245
0.1	1	10	15	6	**14**	8	9	0.1	0.2	0.7	6.2038	0.1323	0.3150	1.0697	7.7208
0.1	1	10	15	6	7	**10**	9	0.1	0.2	0.7	6.2038	0.1624	0.2967	1.0697	7.7326
0.1	1	10	15	6	7	**11**	9	0.1	0.2	0.7	6.2038	0.1624	0.2881	1.0697	7.7240
0.1	1	10	15	6	7	**12**	9	0.1	0.2	0.7	6.2038	0.1624	0.2799	1.0697	7.7158
0.1	1	10	15	6	7	**13**	9	0.1	0.2	0.7	6.2038	0.1624	0.2721	1.0697	7.7080
0.1	1	10	15	6	7	**14**	9	0.1	0.2	0.7	6.2038	0.1624	0.2641	1.0697	7.7000
0.1	1	10	15	6	7	8	**10**	0.1	0.2	0.7	6.2038	0.1624	0.3150	1.0383	7.7195
0.1	1	10	15	6	7	8	**11**	0.1	0.2	0.7	6.2038	0.1624	0.3150	1.0083	7.6895
0.1	1	10	15	6	7	8	**12**	0.1	0.2	0.7	6.2038	0.1624	0.3150	0.9797	7.6609
0.1	1	10	15	6	7	8	**13**	0.1	0.2	0.7	6.2038	0.1624	0.3150	0.9523	7.6335
0.1	1	10	15	6	7	8	**14**	0.1	0.2	0.7	6.2038	0.1624	0.3150	0.9262	7.6074
0.1	1	10	15	6	7	8	9	**0.1**	0.2	0.7	6.2038	0.1624	0.3150	1.0697	7.7509
0.1	1	10	15	6	7	8	9	**0.2**	0.2	0.6	6.2038	0.3249	0.3150	0.9169	7.7606
0.1	1	10	15	6	7	8	9	**0.3**	0.2	0.5	6.2038	0.4873	0.3150	0.7641	7.7702
0.1	1	10	15	6	7	8	9	**0.4**	0.2	0.4	6.2038	0.6498	0.3150	0.6112	7.7798
0.1	1	10	15	6	7	8	9	**0.5**	0.2	0.3	6.2038	0.8122	0.3150	0.4584	7.7894
0.1	1	10	15	6	7	8	9	0.1	**0.1**	0.8	6.2038	0.1624	0.1575	1.2225	7.7462
0.1	1	10	15	6	7	8	9	0.1	**0.2**	0.7	6.2038	0.1624	0.3150	1.0697	7.7509
0.1	1	10	15	6	7	8	9	0.1	**0.3**	0.6	6.2038	0.1624	0.4725	0.9169	7.7556
0.1	1	10	15	6	7	8	9	0.1	**0.4**	0.5	6.2038	0.1624	0.6301	0.7641	7.7604
0.1	1	10	15	6	7	8	9	0.1	**0.5**	0.4	6.2038	0.1624	0.7876	0.6112	7.7650
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	6.2038	0.1624	0.3150	1.0697	7.7509
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	6.2038	0.1624	0.3150	1.0697	7.7509
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	6.2038	0.1624	0.3150	1.0697	7.7509
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	6.2038	0.1624	0.3150	1.0697	7.7509
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	6.2038	0.1624	0.3150	1.0697	7.7509

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0.1	**1**	10	15	6	7	8	9	0.1	0.2	0.7	6.2038	0.1624	0.315	1.0697	7.7509
0.1	**2**	10	15	6	7	8	9	0.1	0.2	0.7	6.7678	0.1772	0.3437	1.1169	8.4556
0.1	**3**	10	15	6	7	8	9	0.1	0.2	0.7	7.3318	0.192	0.3723	1.2642	9.1603
0.1	**4**	10	15	6	7	8	9	0.1	0.2	0.7	7.8958	0.2067	0.4009	1.3614	9.8648
0.1	**5**	10	15	6	7	8	9	0.1	0.2	0.7	8.4598	0.2215	0.4296	1.4587	10.5696
0.1	1	**10**	15	6	7	8	9	0.1	0.2	0.7	6.2038	0.1624	0.315	1.0697	7.7509
0.1	1	**15**	15	6	7	8	9	0.1	0.2	0.7	9.0238	0.2363	0.4582	1.5559	11.2742
0.1	1	**20**	15	6	7	8	9	0.1	0.2	0.7	11.8437	0.3101	0.6014	2.0421	14.7973
0.1	1	**25**	15	6	7	8	9	0.1	0.2	0.7	14.6636	0.384	0.7446	2.5284	18.3206
0.1	1	**30**	15	6	7	8	9	0.1	0.2	0.7	17.4835	0.4578	0.8878	3.0146	21.8437

Table 2.Probability of Emptiness and Utilization of Servers and System in Transient State.

t	a	b	λ	μ_1	μ_2	μ_3	μ_4	θ_1	θ_2	θ_3	$P_{0000}t$	$P_{0..}t$	$P_{.0.}t$	$P_{.0.}t$	$P_{..0}t$	$U_1(t)$	$U_2(t)$	$U_3(t)$	$U_4(t)$
0.1	1	10	15	6	7	8	9	0.1	0.2	0.7	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.2	1	10	15	6	7	8	9	0.1	0.2	0.7	0.0536	0.0657	0.6698	0.4951	0.1873	0.9343	0.3302	0.5049	0.8127
0.3	1	10	15	6	7	8	9	0.1	0.2	0.7	0.0142	0.0238	0.536	0.3415	0.0791	0.9762	0.4641	0.6585	0.9209
0.4	1	10	15	6	7	8	9	0.1	0.2	0.7	0.0045	0.0116	0.4531	0.2596	0.0404	0.9884	0.5460	0.7404	0.9596
0.5	1	10	15	6	7	8	9	0.1	0.2	0.7	0.0018	0.0073	0.4035	0.2158	0.0252	0.9927	0.5965	0.7842	0.9748
0.1	1	10	**10**	6	7	8	9	0.1	0.2	0.7	0.3704	0.3818	0.904	0.8321	0.6253	0.6182	0.096	0.1679	0.3747
0.1	1	10	**11**	6	7	8	9	0.1	0.2	0.7	0.3354	0.3467	0.8949	0.817	0.5966	0.6533	0.1051	0.183	0.4034
0.1	1	10	**12**	6	7	8	9	0.1	0.2	0.7	0.3036	0.3149	0.8859	0.8021	0.5692	0.6851	0.1141	0.1979	0.4308
0.1	1	10	**13**	6	7	8	9	0.1	0.2	0.7	0.2749	0.2860	0.8770	0.7875	0.5431	0.714	0.1230	0.2125	0.4569
0.1	1	10	**14**	6	7	8	9	0.1	0.2	0.7	0.2489	0.2597	0.8682	0.7731	0.5182	0.7403	0.1318	0.2269	0.4818
0.1	1	10	15	**10**	7	8	9	0.1	0.2	0.7	0.2268	0.2484	0.8053	0.6813	0.4124	0.7516	0.1947	0.3187	0.5876
0.1	1	10	15	**11**	7	8	9	0.1	0.2	0.7	0.2271	0.2523	0.7947	0.6668	0.3995	0.7477	0.2053	0.3332	0.6005
0.1	1	10	15	**12**	7	8	9	0.1	0.2	0.7	0.2275	0.2564	0.7849	0.6538	0.3885	0.7436	0.2151	0.3462	0.6115
0.1	1	10	15	**13**	7	8	9	0.1	0.2	0.7	0.2278	0.2608	0.7760	0.6421	0.3790	0.7392	0.2240	0.3579	0.6210
0.1	1	10	15	**14**	7	8	9	0.1	0.2	0.7	0.2281	0.2655	0.7677	0.6315	0.3707	0.7345	0.2323	0.3685	0.6293
0.1	1	10	15	6	**10**	8	9	0.1	0.2	0.7	0.2255	0.2359	0.8700	0.7591	0.4944	0.7641	0.13	0.2409	0.5056
0.1	1	10	15	6	**11**	8	9	0.1	0.2	0.7	0.2255	0.2359	0.8733	0.7591	0.4944	0.7641	0.1267	0.2409	0.5056
0.1	1	10	15	6	**12**	8	9	0.1	0.2	0.7	0.2255	0.2359	0.8764	0.7591	0.4944	0.7641	0.1236	0.2409	0.5056
0.1	1	10	15	6	**13**	8	9	0.1	0.2	0.7	0.2255	0.2359	0.8794	0.7591	0.4944	0.7641	0.1206	0.2409	0.5056
0.1	1	10	15	6	**14**	8	9	0.1	0.2	0.7	0.2256	0.2359	0.8823	0.7591	0.4944	0.7641	0.1177	0.2409	0.5056
0.1	1	10	15	6	7	**1**	9	0.1	0.2	0.7	0.2255	0.2359	0.8595	0.7696	0.4944	0.7641	0.1405	0.2304	0.5056
0.1	1	10	15	6	7	**0**	9	0.1	0.2	0.7	0.2255	0.2359	0.8595	0.7746	0.4944	0.7641	0.1405	0.2254	0.5056
0.1	1	10	15	6	7	**1**	9	0.1	0.2	0.7	0.2256	0.2359	0.8595	0.7795	0.4944	0.7641	0.1405	0.2205	0.5056
0.1	1	10	15	6	7	**1**	9	0.1	0.2	0.7	0.2256	0.2359	0.8595	0.7842	0.4944	0.7641	0.1405	0.2158	0.5056
0.1	1	10	15	6	7	**1**	9	0.1	0.2	0.7	0.2257	0.2359	0.8595	0.7888	0.4944	0.7641	0.1405	0.2112	0.5056
0.1	1	10	15	6	7	8	**1**	0.1	0.2	0.7	0.2256	0.2359	0.8595	0.7591	0.5003	0.7641	0.1405	0.2409	0.4997
0.1	1	10	15	6	7	8	**0**	0.1	0.2	0.7	0.2257	0.2359	0.8595	0.7591	0.5062	0.7641	0.1405	0.2409	0.4938
0.1	1	10	15	6	7	8	**1**	0.1	0.2	0.7	0.2259	0.2359	0.8595	0.7591	0.5121	0.7641	0.1405	0.2409	0.4879
0.1	1	10	15	6	7	8	**1**	0.1	0.2	0.7	0.2260	0.2359	0.8595	0.7591	0.5180	0.7641	0.1405	0.2409	0.482
0.1	1	10	15	6	7	8	**1**	0.1	0.2	0.7	0.2262	0.2359	0.8595	0.7591	0.5238	0.7641	0.1405	0.2409	0.4762
0.1	1	10	15	6	7	8	9	**0.1**	0.2	0.7	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	10	15	6	7	8	9	**0.2**	0.2	0.7	0.2253	0.2359	0.7536	0.7591	0.5287	0.7641	0.2464	0.2409	0.4713
0.1	1	10	15	6	7	8	9	**0.3**	0.2	0.7	0.2253	0.2359	0.6721	0.7591	0.5703	0.7641	0.3279	0.2409	0.4297
0.1	1	10	15	6	7	8	9	**0.4**	0.2	0.7	0.2253	0.2359	0.6082	0.7591	0.6213	0.7641	0.3918	0.2409	0.3787
0.1	1	10	15	6	7	8	9	**0.5**	0.2	0.7	0.2252	0.2359	0.5573	0.7591	0.6846	0.7641	0.4427	0.2409	0.3154
0.1	1	10	15	6	7	8	9	0.1	**0.1**	0.8	0.2254	0.2359	0.8595	0.8631	0.4658	0.7641	0.1405	0.1369	0.5342
0.1	1	10	15	6	7	8	9	0.1	**0.2**	0.7	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	10	15	6	7	8	9	0.1	**0.3**	0.6	0.2254	0.2359	0.8595	0.6784	0.5287	0.7641	0.1405	0.3216	0.4713
0.1	1	10	15	6	7	8	9	0.1	**0.4**	0.5	0.2254	0.2359	0.8595	0.6148	0.5703	0.7641	0.1405	0.3852	0.4297
0.1	1	10	15	6	7	8	9	0.1	**0.5**	0.4	0.2253	0.2359	0.8595	0.5639	0.6212	0.7641	0.1405	0.4361	0.3788

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0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	**1**	10	15	6	7	8	9	0.1	0.2	0.7	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	**2**	10	15	6	7	8	9	0.1	0.2	0.7	0.2234	0.2277	0.8479	0.7409	0.4672	0.7726	0.1521	0.2591	0.5328
0.1	**3**	10	15	6	7	8	9	0.1	0.2	0.7	0.2232	0.225	0.8367	0.7237	0.4443	0.775	0.1633	0.2763	0.5557
0.1	**4**	10	15	6	7	8	9	0.1	0.2	0.7	0.2231	0.2239	0.8258	0.7074	0.4249	0.7761	0.1742	0.2626	0.5751
0.1	**5**	10	15	6	7	8	9	0.1	0.2	0.7	0.2231	0.2235	0.8152	0.6919	0.4082	0.7765	0.1848	0.3081	0.5918
0.1	1	**10**	15	6	7	8	9	0.1	0.2	0.7	0.2254	0.2359	0.8595	0.7591	0.4944	0.7641	0.1405	0.2409	0.5056
0.1	1	**15**	15	6	7	8	9	0.1	0.2	0.7	0.2246	0.2316	0.8089	0.6879	0.4268	0.7682	0.1911	0.3121	0.5732
0.1	1	**20**	15	6	7	8	9	0.1	0.2	0.7	0.2186	0.2294	0.7647	0.6315	0.3857	0.7706	0.2353	0.3685	0.6143
0.1	1	**25**	15	6	7	8	9	0.1	0.2	0.7	0.0003	0.2281	0.7258	0.5861	0.3586	0.7719	0.2742	0.4139	0.6414
0.1	1	**30**	15	6	7	8	9	0.1	0.2	0.7	0.0001	0.2273	0.6914	0.5490	0.3407	0.7727	0.3086	0.4510	0.6593

Table 3.Values of Throughput and Waiting Time of Customers in Queues in Transient State.

t	a	b	λ	μ_1	μ_2	μ_3	μ_4	θ_1	θ_2	θ_3	$Thp_1(t)$	$Thp_2(t)$	$Thp_3(t)$	$Thp_4(t$	$W_1(t)$	$W_2(t)$	$W_3(t)$	$W_4(t)$
0.1	1	10	15	6	7	8	9	0.1	0.2	0.7	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.2	1	10	15	6	7	8	9	0.1	0.2	0.7	5.6058	2.3314	4.0392	7.3143	1.7140	0.1893	0.2047	0.3746
0.3	1	10	15	6	7	8	9	0.1	0.2	0.7	5.8572	3.2480	5.268	8.2881	1.9595	0.2096	0.2392	0.4943
0.4	1	10	15	6	7	8	9	0.1	0.2	0.7	5.9304	3.8283	2.9232	8.6364	2.1082	0.2247	0.2644	0.5814
0.5	1	10	15	6	7	8	9	0.1	0.2	0.7	5.9562	4.1755	6.2736	8.7732	2.1936	0.2350	0.2814	0.6384
0.1	1	10	**10**	6	7	8	9	0.1	0.2	0.7	3.7092	0.6720	1.3432	3.3723	1.1150	0.1612	0.1563	0.2115
0.1	1	10	**11**	6	7	8	9	0.1	0.2	0.7	3.9198	0.7357	1.4640	3.6306	1.1606	0.1619	0.1578	0.2161
0.1	1	10	**12**	6	7	8	9	0.1	0.2	0.7	4.1106	0.7987	1.5832	3.8772	1.2074	0.1628	0.1592	0.2207
0.1	1	10	**13**	6	7	8	9	0.1	0.2	0.7	4.2840	0.8610	1.7001	4.1121	1.2551	0.1635	0.1606	0.2255
0.1	1	10	**14**	6	7	8	9	0.1	0.2	0.7	4.4418	0.9226	1.8152	4.3362	1.3036	0.1643	0.1620	0.2302
0.1	1	10	15	**10**	7	8	9	0.1	0.2	0.7	7.516	1.3629	2.5496	5.2884	0.6939	0.1757	0.1819	0.2975
0.1	1	10	15	**11**	7	8	9	0.1	0.2	0.7	8.2247	1.4371	2.6656	5.4045	0.6084	0.1779	0.1858	0.3108
0.1	1	10	15	**12**	7	8	9	0.1	0.2	0.7	8.9232	1.5057	2.7696	5.5035	0.5384	0.1799	0.1894	0.3233.
0.1	1	10	15	**13**	7	8	9	0.1	0.2	0.7	9.6096	1.568	2.8632	5.589	0.4804	0.1819	0.1929	0.3351
0.1	1	10	15	**14**	7	8	9	0.1	0.2	0.7	10.283	1.6261	2.9480	5.6637	0.4318	0.1837	0.1961	0.3461
0.1	1	10	15	6	**10**	8	9	0.1	0.2	0.7	4.5846	1.3001	1.9272	4.5504	1.3532	0.1141	0.1634	0.2351
0.1	1	10	15	6	**11**	8	9	0.1	0.2	0.7	4.5846	1.3937	1.9272	4.5504	1.3532	0.1033	0.1634	0.2351
0.1	1	10	15	6	**12**	8	9	0.1	0.2	0.7	4.5846	1.4832	1.9272	4.5504	1.3532	0.0944	0.1634	0.2351
0.1	1	10	15	6	**13**	8	9	0.1	0.2	0.7	4.5846	1.5678	1.9272	4.5504	1.3532	0.0867	0.1634	0.2351
0.1	1	10	15	6	**14**	8	9	0.1	0.2	0.7	4.5846	1.6478	1.9272	4.5504	1.3532	0.0803	0.1634	0.2351
0.1	1	10	15	6	7	**10**	9	0.1	0.2	0.7	4.5846	0.9835	2.304	4.5504	1.3532	0.1651	0.1288	0.2351
0.1	1	10	15	6	7	**11**	9	0.1	0.2	0.7	4.5846	0.9835	2.4794	4.5504	1.3532	0.1651	0.1162	0.2351
0.1	1	10	15	6	7	**12**	9	0.1	0.2	0.7	4.5846	0.9835	2.646	4.5504	1.3532	0.1651	0.1058	0.2351
0.1	1	10	15	6	7	**13**	9	0.1	0.2	0.7	4.5846	0.9835	2.8054	4.5504	1.3532	0.1651	0.0970	0.2351
0.1	1	10	15	6	7	**14**	9	0.1	0.2	0.7	4.5846	0.9835	2.9568	4.5504	1.3532	0.1651	0.0893	0.2351
0.1	1	10	15	6	7	8	**10**	0.1	0.2	0.7	4.5846	0.9835	1.9272	4.9970	1.3532	0.1651	0.1634	0.2078
0.1	1	10	15	6	7	8	**11**	0.1	0.2	0.7	4.5846	0.9835	1.9272	5.4318	1.3532	0.1651	0.1634	0.1856
0.1	1	10	15	6	7	8	**12**	0.1	0.2	0.7	4.5846	0.9835	1.9272	5.8548	1.3532	0.1651	0.1634	0.1673
0.1	1	10	15	6	7	8	**13**	0.1	0.2	0.7	4.5846	0.9835	1.9272	6.2660	1.3532	0.1651	0.1634	0.1520
0.1	1	10	15	6	7	8	**14**	0.1	0.2	0.7	4.5846	0.9835	1.9272	6.6668	1.3532	0.1651	0.1634	0.1389
0.1	1	10	15	6	7	8	9	**0.1**	0.2	0.7	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	10	15	6	7	8	9	**0.2**	0.2	0.6	4.5846	1.7248	1.9272	4.2417	1.3532	0.1884	0.1634	0.2162
0.1	1	10	15	6	7	8	9	**0.3**	0.2	0.5	4.5846	2.2953	1.9272	3.8673	1.3532	0.2123	0.1634	0.1976
0.1	1	10	15	6	7	8	9	**0.4**	0.2	0.4	4.5846	2.7426	1.9272	3.4083	1.3532	0.2369	0.1634	0.1793
0.1	1	10	15	6	7	8	9	**0.5**	0.2	0.3	4.5846	3.0989	1.9272	2.8386	1.3532	0.2621	0.1634	0.1615

0.1	1	10	15	6	7	8	9	0.1	**0.1**	0.8	4.5846	0.9835	1.0952	4.8078	1.3532	0.1651	0.1438	0.2543
0.1	1	10	15	6	7	8	9	0.1	**0.2**	0.7	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	10	15	6	7	8	9	0.1	**0.3**	0.6	4.5846	0.9835	2.5728	4.2417	1.3532	0.1651	0.1837	0.2162
0.1	1	10	15	6	7	8	9	0.1	**0.4**	0.5	4.5846	0.9835	3.0816	3.8673	1.3532	0.1651	0.2045	0.1976
0.1	1	10	15	6	7	8	9	0.1	**0.5**	0.4	4.5846	0.9835	3.4888	3.4083	1.3532	0.1651	0.2258	0.1793
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	**1**	10	15	6	7	8	9	0.1	0.2	0.7	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	**2**	10	15	6	7	8	9	0.1	0.2	0.7	4.6338	1.0647	2.0728	4.7952	1.4605	0.1664	0.1658	0.2433
0.1	**3**	10	15	6	7	8	9	0.1	0.2	0.7	4.6500	1.1431	2.2104	5.0013	1.5767	0.1680	0.1684	0.2528
0.1	**4**	10	15	6	7	8	9	0.1	0.2	0.7	4.6556	1.2194	2.3408	5.1759	1.6956	0.1695	0.1713	0.2630
0.1	**5**	10	15	6	7	8	9	0.1	0.2	0.7	4.6590	1.2936	2.4648	5.3262	1.8158	0.1712	0.1743	0.2739
0.1	1	**10**	15	6	7	8	9	0.1	0.2	0.7	4.5846	0.9835	1.9272	4.5504	1.3532	0.1651	0.1634	0.2351
0.1	1	**15**	15	6	7	8	9	0.1	0.2	0.7	4.6104	1.3377	2.4968	5.1588	1.9573	0.1766	0.1835	0.3016
0.1	1	**20**	15	6	7	8	9	0.1	0.2	0.7	4.6236	1.6471	2.948	5.5287	2.5616	0.1833	0.2040	0.3694
0.1	1	**25**	15	6	7	8	9	0.1	0.2	0.7	4.6314	1.9194	3.3112	5.7726	3.1661	0.2001	0.2249	0.4380
0.1	1	**30**	15	6	7	8	9	0.1	0.2	0.7	4.6362	2.1602	3.6080	5.9337	3.7711	0.2119	0.2461	0.5080

Table.4.Values of Variances and coefficients of Variation of Customers in Queues in Transient State

t	a	b	λ	μ_1	μ_2	μ_3	μ_4	θ_1	θ_2	θ_3	$V_1(t)$	$V_2(t)$	$V_3(t)$	$V_4(t)$	$CV_1(t)$	$CV_2(t)$	$CV_3(t)$	$CV_4(t)$
0.1	1	10	15	6	7	8	9	0.1	0.2	0.7	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.20
0.2	1	10	15	6	7	8	9	0.1	0.2	0.7	66.333	1.6749	3.0166	318.35	84.760	295.81	210.04	651.19
0.3	1	10	15	6	7	8	9	0.1	0.2	0.7	74.554	2.2307	5.3377	4777.2	75.220	219.38	183.35	1687.0
0.4	1	10	15	6	7	8	9	0.1	0.2	0.7	78.418	2.7328	7.7684	22607	70.830	192.18	177.95	2994.6
0.5	1	10	15	6	7	8	9	0.1	0.2	0.7	80.344	3.1185	9.7582	51809	68.610	179.94	176.98	4064.0
0.1	1	10	**10**	6	7	8	9	0.1	0.2	0.7	31.624	1.1317	1.3071	3.9555	135.97	982.28	544.42	278.90
0.1	1	10	**11**	6	7	8	9	0.1	0.2	0.7	34.787	1.1458	1.3425	4.5385	129.64	898.76	501.59	271.59
0.1	1	10	**12**	6	7	8	9	0.1	0.2	0.7	37.949	1.1601	1.3790	5.2076	124.12	828.53	466.00	266.68
0.1	1	10	**13**	6	7	8	9	0.1	0.2	0.7	41.112	1.1746	1.4164	5.9752	119.25	796.74	435.94	263.66
0.1	1	10	**14**	6	7	8	9	0.1	0.2	0.7	44.274	1.1892	1.4549	6.8561	114.92	719.33	410.27	262.26
0.1	1	10	15	**10**	7	8	9	0.1	0.2	0.7	37.045	1.3344	1.5146	39.243	116.71	482.52	265.35	398.12
0.1	1	10	15	**11**	7	8	9	0.1	0.2	0.7	35.017	1.3653	1.5768	57.739	118.29	457.14	253.58	452.35
0.1	1	10	15	**12**	7	8	9	0.1	0.2	0.7	33.166	1.3953	1.6394	84.076	119.87	436.04	244.02	515.31
0.1	1	10	15	**13**	7	8	9	0.1	0.2	0.7	31.474	1.4246	1.7022	121.02	132.60	418.50	236.23	587.45
0.1	1	10	15	**14**	7	8	9	0.1	0.2	0.7	29.922	1.4529	1.7649	172.09	123.21	403.54	229.76	669.24
0.1	1	10	15	6	**10**	8	9	0.1	0.2	0.7	47.437	1.1821	1.4943	7.8667	111.02	733.14	388.07	262.50
0.1	1	10	15	6	**11**	8	9	0.1	0.2	0.7	47.437	1.1757	1.4943	7.8667	111.02	752.98	388.07	262.50
0.1	1	10	15	6	**12**	8	9	0.1	0.2	0.7	47.437	1.1696	1.4943	7.8667	111.02	772.49	388.07	262.50
0.1	1	10	15	6	**13**	8	9	0.1	0.2	0.7	47.437	1.1639	1.4943	7.8667	111.02	793.27	388.07	262.50
0.1	1	10	15	6	**14**	8	9	0.1	0.2	0.7	47.437	1.1585	1.4943	7.8667	111.02	813.56	388.07	262.50
0.1	1	10	15	6	7	**10**	9	0.1	0.2	0.7	47.437	1.2040	1.4515	7.8667	111.02	675.66	406.06	262.50
0.1	1	10	15	6	7	**11**	9	0.1	0.2	0.7	47.437	1.2040	1.4323	7.8667	111.02	675.66	415.41	262.50
0.1	1	10	15	6	7	**12**	9	0.1	0.2	0.7	47.437	1.2040	1.4145	7.8667	111.02	675.66	424.91	262.50
0.1	1	10	15	6	7	**13**	9	0.1	0.2	0.7	47.437	1.2040	1.3978	7.8667	111.02	675.66	434.50	262.50
0.1	1	10	15	6	7	**14**	9	0.1	0.2	0.7	47.437	1.2040	1.3823	7.8667	111.02	675.66	444.34	262.50

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0.1	1	10	15	6	7	8	**10**	0.1	0.2	0.7	47.437	1.2040	1.4943	7.1584	111.02	675.66	388.07	257.68
0.1	1	10	15	6	7	8	**11**	0.1	0.2	0.7	47.437	1.2040	1.4943	6.5564	111.02	675.66	388.07	253.95
0.1	1	10	15	6	7	8	**12**	0.1	0.2	0.7	47.437	1.2040	1.4943	6.0408	111.02	675.66	388.07	250.87
0.1	1	10	15	6	7	8	**13**	0.1	0.2	0.7	47.437	1.2040	1.4943	5.5962	111.02	675.66	388.07	248.41
0.1	1	10	15	6	7	8	**14**	0.1	0.2	0.7	47.437	1.2040	1.4943	5.2102	111.02	675.66	388.07	246.45
0.1	1	10	15	6	7	8	9	**0.1**	0.2	0.7	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.20
0.1	1	10	15	6	7	8	9	**0.2**	0.2	0.6	47.437	1.5184	1.4943	5.1882	111.02	379.27	388.07	212.94
0.1	1	10	15	6	7	8	9	**0.3**	0.2	0.5	47.437	2.0058	1.4943	3.5632	111.02	290.64	388.07	176.45
0.1	1	10	15	6	7	8	9	**0.4**	0.2	0.4	47.437	2.7754	1.4943	2.5484	111.02	256.38	388.07	149.24
0.1	1	10	15	6	7	8	9	**0.5**	0.2	0.3	47.437	4.0227	1.4943	1.8980	111.02	246.94	388.07	128.79
0.1	1	10	15	6	7	8	9	0.1	**0.1**	0.8	47.437	1.2040	1.1962	12.421	111.02	675.66	694.42	329.48
0.1	1	10	15	6	7	8	9	0.1	**0.2**	0.7	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.20
0.1	1	10	15	6	7	8	9	0.1	**0.3**	0.6	47.437	1.2040	1.9494	5.1882	111.02	675.66	295.49	212.94
0.1	1	10	15	6	7	8	9	0.1	**0.4**	0.5	47.437	1.2040	2.6556	3.5633	111.02	675.66	258.63	176.47
0.1	1	10	15	6	7	8	9	0.1	**0.5**	0.4	47.437	1.2040	3.7779	2.5484	111.02	675.66	246.79	149.24
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.50
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.50
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.50
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.50
0.1	1	10	15	6	7	8	9	0.1	0.2	**0.7**	47.437	1.2040	1.4943	7.8667	111.02	675.66	388.07	262.50
0.1	**1**	10	15	6	7	8	9	0.1	0.2	0.7	47.437	1.2040	1.4943	2.5484	111.02	675.66	388.07	149.24
0.1	**2**	10	15	6	7	8	9	0.1	0.2	0.7	52.708	1.2071	1.5088	2.6419	107.27	620.02	357.39	139.29
0.1	**3**	10	15	6	7	8	9	0.1	0.2	0.7	58.937	1.2108	1.5261	2.7568	104.71	573.11	331.82	131.34
0.1	**4**	10	15	6	7	8	9	0.1	0.2	0.7	66.124	1.2150	1.5462	2.8956	102.99	533.27	310.17	124.99
0.1	**5**	10	15	6	7	8	9	0.1	0.2	0.7	74.270	1.2199	1.5694	3.0614	101.87	498.64	291.61	119.95
0.1	1	**10**	15	6	7	8	9	0.1	0.2	0.7	47.437	1.2040	1.4943	3.8220	111.02	679.66	388.07	182.76
0.1	1	**15**	15	6	7	8	9	0.1	0.2	0.7	107.33	1.2398	1.6671	9.6011	114.81	471.21	281.79	199.15
0.1	1	**20**	15	6	7	8	9	0.1	0.2	0.7	191.18	1.2916	1.9431	34.863	116.75	366.49	231.78	289.14
0.1	1	**25**	15	6	7	8	9	0.1	0.2	0.7	298.99	1.3615	2.3660	182.98	117.92	303.86	206.58	535.02
0.1	1	**30**	15	6	7	8	9	0.1	0.2	0.7	430.76	1.4521	3.0100	1368.3	118.71	263.22	195.42	1227.1

Table 5.Values of L_1 , L_2 , L_3 , L_4 , L, W_1 , W_2 , W_3 *and* W_4 for different Values of t, λ, μ_1 , μ_2, μ_3, μ_4, θ_1, θ_2 *and* θ_3. (SENSITIVITY ANALYSIS)

Variation Parameter	Performance Measure	Percentage Change in Parameter						
		-15%	-10%	-5%	0	5%	10%	15%
t = 0.1	$L_1(t)$	5.4932	5.7372	5.9740	6.2038	6.4269	6.6433	6.8533
	$L_2(t)$	0.1248	0.1371	0.1496	0.1624	0.1755	0.1887	0.2021
	$L_3(t)$	0.2431	0.2666	0.2906	0.3150	0.3398	0.3649	0.3903
	$L_4(t)$	0.8269	0.9078	0.9881	1.0697	1.1523	1.2358	1.3201
	$L(t)$	6.6880	7.0487	7.4023	7.7509	8.0945	8.4327	8.7658
	$W_1(t)$	1.2904	1.3115	1.3324	1.3532	1.3736	1.3940	1.4140
	$W_2(t)$	0.1615	0.1627	0.1639	0.1651	0.1664	0.1676	0.1688
	$W_3(t)$	0.1571	0.1592	0.1613	0.1634	0.1655	0.1676	0.1698
	$W_4(t)$	0.2138	0.2212	0.2281	0.2351	0.2421	0.2491	0.2562
λ=15	$L_1(t)$	5.2733	5.5835	5.8936	6..2038	6.5140	6.8242	7.1344
	$L_2(t)$	0.1381	0.1462	0.1543	0.1624	0.1706	0.1787	0.1868
	$L_3(t)$	0.2678	0.2835	0.2993	0.3150	0.3308	0.3465	0.3623
	$L_4(t)$	0.9092	0.9627	1.0162	1.0697	1.1232	1.1767	1.2301
	$L(t)$	6.5884	6.9759	7.3634	7.7509	8.1386	8.5261	8.9136
	$W_1(t)$	1.2429	1.2792	1.3158	1.3532	1.3908	1.4292	1.4678
	$W_2(t)$	0.1633	0.1639	0.1645	0.1651	0.1658	0.1664	0.1670
	$W_3(t)$	0.1602	0.1613	0.1624	0.1634	0.1645	0.1656	0.1667

	$W_4(t)$	0.2242	0.2278	0.2315	0.2351	0.2369	0.2425	0.2462
μ_1=12	$L_1(t)$	5.1717	5.0448	4.9224	4.8043	4.6904	4.5804	4.4793
	$L_2(t)$	0.2427	0.2525	0.2619	0.2709	0.2796	0.2880	0.2961
	$L_3(t)$	0.4703	0.4891	0.5073	0.5247	0.5415	0.5576	0.5732
	$L_4(t)$	1.5953	1.6591	1.7204	1.7794	1.8361	1.8906	1.9431
	$L(t)$	7.4800	7.4455	7.4120	7.3793	7.3476	7.3166	7.2867
	$W_1(t)$	0.5740	0.5617	0.5498	0.5384	0.5275	0.5170	0.5069
	$W_2(t)$	0.1761	0.1774	0.1778	0.1799	0.1811	0.1823	0.1834
	$W_3(t)$	0.1827	0.1850	0.1873	0.1894	0.1915	0.1936	0.1955
	$W_4(t)$	0.3002	0.3082	0.3159	0.3233	0.3305	0.3373	0.3440
μ_2=14	$L_1(t)$	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**
	$L_2(t)$	6.2038	6.2038	6.2038	6.2038	6.2038	6.2038	6.2038
	$L_3(t)$	0.2172	0.2172	0.2172	0.2172	0.2172	0.2172	0.2172
	$L_4(t)$	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**
	$L(t)$	**1.0697**	**1.0697**	**1.0697**	**1.0697**	**1.0697**	**1.0697**	**1.0697**
	$W_1(t)$	7.8057	7.8057	7.8057	7.8057	7.8057	7.8057	7.8057
	$W_2(t)$	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**
	$W_3(t)$	0.1252	0.1252	0.1252	0.1252	0.1252	0.1252	0.1252
	$W_4(t)$	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**
μ_3=11	$L_1(t)$	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**
	$L_2(t)$	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**
	$L_3(t)$	0.3024	0.3024	0.3024	0.3024	0.3024	0.3024	0.3024
	$L_4(t)$	**1.0697**	**1.0697**	**1.0697**	**1.0697**	**1.0697**	**1.0697**	**1.0697**
	$L(t)$	7.7383	7.7383	7.7383	7.7383	7.7383	7.7383	7.7383
	$W_1(t)$	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**
	$W_2(t)$	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**
	$W_3(t)$	0.1176	0.1176	0.1176	0.1176	0.1176	0.1176	0.1176
	$W_4(t)$	**0.2351**	**0.2351**	**0.2351**	**0.2351**	**0.2351**	**0.2351**	**0.2351**
μ_4=13	$L_1(t)$	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**
	$L_2(t)$	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**
	$L_3(t)$	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**
	$L_4(t)$	1.0069	1.0069	1.0069	1.0069	1.0069	1.0069	1.0069
	$L(t)$	7.6881	7.6881	7.6881	7.6881	7.6881	7.6881	7.6881
	$W_1(t)$	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**
	$W_2(t)$	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**
	$W_3(t)$	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**
	$W_4(t)$	0.1569	0.1569	0.1569	0.1569	0.1569	0.1569	0.1569
θ_1=0.3	$L_1(t)$	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**
	$L_2(t)$	0.4142	0.4142	0.4142	0.4142	0.4142	0.4142	0.4142
	$L_3(t)$	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**	**0.3150**
	$L_4(t)$	0.8328	0.8328	0.8328	0.8328	0.8328	0.8328	0.8328
	$L(t)$	7.7658	7.7658	7.7658	7.7658	7.7658	7.7658	7.7658
	$W_1(t)$	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**
	$W_2(t)$	0.2014	0.2014	0.2014	0.2014	0.2014	0.2014	0.2014
	$W_3(t)$	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**	**0.1634**
	$W_4(t)$	0.2059	0.2059	0.2059	0.2059	0.2059	0.2059	0.2059
θ_2=0.2	$L_1(t)$	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**	**6.2038**
	$L_2(t)$	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**	**0.1624**
	$L_3(t)$	0.2678	0.2678	0.2678	0.2678	0.2678	0.2678	0.2678
	$L_4(t)$	1.1155	1.1155	1.1155	1.1155	1.1155	1.1155	1.1155
	$L(t)$	7.7495	7.7495	7.7495	7.7495	7.7495	7.7495	7.7495
	$W_1(t)$	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**	**1.3532**
	$W_2(t)$	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**	**0.1651**
	$W_3(t)$	0.1575	0.1575	0.1575	0.1575	0.1575	0.1575	0.1575
	$W_4(t)$	0.2408	0.2408	0.2408	0.2408	0.2408	0.2408	0.2408

a=5	$L_1(t)$	8.0368	8.1778	8.3188	8.4598	8.6008	8.7418	8.8828
	$L_2(t)$	0.2104	0.2141	0.2178	0.2215	0.2252	0.2289	0.2326
	$L_3(t)$	0.4081	0.4153	0.4224	0.4296	0.4367	0.4439	0.4511
	$L_4(t)$	1.3857	1.4100	1.4344	1.4587	1.4830	1.5073	1.5316
	$L(t)$	10.041	10.2172	10.3934	10.5696	10.7457	10.9219	11.0981
	$W_1(t)$	1.8593	1.8406	1.8262	1.8158	1.8086	1.8045	1.8028
	$W_2(t)$	0.2057	0.1924	0.1810	0.1712	0.1627	0.1553	0.1487
	$W_3(t)$	0.2041	0.1926	0.1828	0.1743	0.1670	0.1606	0.1551
	$W_4(t)$	0.3004	0.2900	0.2812	0.2739	0.2677	0.2625	0.2582
b=20	$L_1(t)$	10.1517	10.7157	11.2797	11.8437	12.4077	12.9717	13.5357
	$L_2(t)$	0.2658	0.2806	0.2954	0.3101	0.3249	0.3397	0.3544
	$L_3(t)$	0.5155	0.5441	0.5728	0.6014	0.6301	0.6587	0.6873
	$L_4(t)$	1.7504	1.8476	1.9449	2.0421	2.1394	2.2366	2.3339
	$L(t)$	12.6834	13.3880	14.0928	14.7943	15.5021	16.2067	16.9113
	$W_1(t)$	2.0558	2.2180	2.3866	2.5616	2.7430	2.9307	3.1246
	$W_2(t)$	0.1403	0.1555	0.1715	0.1883	0.2058	0.2242	0.2433
	$W_3(t)$	0.1543	0.1701	0.1867	0.2040	0.2222	0.2411	0.2608
	$W_4(t)$	0.2866	0.3144	0.3413	0.3694	0.3986	0.4289	0.4604

VII.STEADY STATE ANALYSIS

In this section we study the steady-state analysis of queuing model .The Joint Probability generating function of number of customers in first, second,….k[th] queues respectively in steady state is

$$\lim_{t\to\infty} P(Z_1,Z_2,\dots Z_k;t) = P(Z_1,Z_2,\dots Z_k) = \exp\Bigg[\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\dots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\dots\binom{r_{k-1}}{r_k}C_m \left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\dots+\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\dots\left\{\frac{\theta_{k-2}\mu_1(z_{k-1}-1)}{\mu_{k-1}-\mu_1}\right\}^{r_{k-1}-r_k}\left\{\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_k}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\dots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\Bigg] \quad (115)$$

VII.A.CHARACTERISTICS OF THE MODEL UNDER EQUILIBRIUM

Putting $z_1=0, z_2=0,\dots z_k=0$ in (45) and expanding we get the probability that the k-server system is empty in steady state as

$$P(0,0,..0) = \exp\Bigg[\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\dots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\dots\binom{r_{k-1}}{r_k}C_m \left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+\dots+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\dots\left(\frac{\theta_{k-2}\mu_1}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_k}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\dots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\Bigg] \quad (116)$$

VII.B.PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2=1, z_3=1,\dots z_k=1$ in (45) we get probability generating function of first queue size as

$$P(Z_1) = \exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1}{\mu_1 r_1}\right\}\right] \quad (117)$$

Mean number of customers in first queue is

$$E(N_1) = L_1 = \frac{\lambda}{\mu_1}E(X) \quad (118)$$

Where $E(X)$ is the mean of batch size arrivals to first queue and is given by $E(X)=\sum_{m=1}^{\infty} m.C_m$

Putting $Z_1=0$ in (117) we get the probability that the first queue is empty as

$$P(0,.,.,.,.) = \exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right] \quad (119)$$

Utilization of first server is $U_1 = 1-P(0,.,.,.,.) = 1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]$

(120)

Throughput of first server is $Thp1 = \mu_1.U_1 = \mu_1\left\{1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]\right\}$ (121)

Average waiting time customers of a customer in first queue is $W_1 = \frac{L_1}{Thp_1} = \frac{\left(\frac{\lambda}{\mu_1}\right)E(X)}{\left\{1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]\right\}}$

(122)

Variance of the number of customers in first queue is

$$V(Z_1) = V_1 = \left(\frac{2\lambda}{\mu_1}\right)\sum_{m=2}^{\infty} C_m\binom{m}{2} \quad (123)$$

Coefficient of variation of the number of customers in first queue is $CV_1 = \frac{\sqrt{V_1}}{L_1}x100 = \frac{\sqrt{\left(\frac{2\lambda}{\mu_1}\right)\sum_{m=2}^{\infty} C_m\binom{m}{2}}}{\frac{\lambda}{\mu_1}E(X)}x100$ (124)

VII.C. PERFORMANCE ANALYSIS OF i[th] QUEUE FOR i =2,3,..k

Putting $z_1=1, z_2=1, z_3=1,\dots z_{i-1}=1$ in (115) we get probability generating function of i[th] queue size distribution as $P(Z_i) =$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1(z_i-1)}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (125)$$

Mean number of customers in i[th] queue is

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$$E(N_i) = L_i = \left(\frac{\lambda\theta_{i-1}}{\mu_i}\right) E(X) \tag{126}$$

Where $E(X)$ is the mean of batch size arrivals at **i**^th^ queue and $E(X) = \sum_m m.C_m$

Putting $z_i = 0$ in (125) we get the probability that the **i**^th^ buffer is empty as $P(,,.,0,..) =$

$$exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \tag{127}$$

Utilization of **i**^th^ server is $U_i = 1 - P(,,.,.,0,.) =$

$$1 - exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \tag{128}$$

Throughput of **i**^th^ server is $Thp_i = \mu_i.U_i =$

$$\mu_i.\left[1 - exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right] \tag{129}$$

Average waiting time of a customer in **i**^th^ queue is

$$W_i = \frac{L_i}{Thp_i} = \frac{\left(\frac{\lambda\theta_{i-1}}{\mu_i^2}\right)E(X)}{\left[1 - exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right]} \tag{130}$$

Variation of the number of customers in **i**^th^ queue is $V(z_i) = V_i =$

$$\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1}{\mu_1}\right)-\left(\frac{4}{\mu_i+\mu_1}\right)+\left(\frac{1}{\mu_i}\right)\right\}+\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1}{\mu_1}\right)+\left(\frac{1}{\mu_i}\right)\right\}E(X)\right] \tag{131}$$

Coefficient of variation in number of the number of customers in **i**^th^ queue is $CV_i = \frac{\sqrt{V_i}}{L_i}x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}\binom{m}{2}.C_m\left\{\left(\frac{1}{\mu_1}\right)-\left(\frac{4}{\mu_i+\mu_1}\right)+\left(\frac{1}{\mu_i}\right)\right\}+\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1}{\mu_1}\right)+\left(\frac{1}{\mu_i}\right)\right\}E(X)\right]}}{\left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)E(X)}x100 \tag{132}$$

VII. D. PERFORMANCE MEASURES OF THE STAEDY STATE MODEL WHEN BATCH SIZE DISTRIBUTION IS UNIFORM

The Joint Probability generating function of number of customers in first, second, third and fourth queues respectively in steady state is $P(Z_1,Z_2,Z_3,Z_4) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\frac{1}{(b-a+1)}\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\left\{\frac{\theta_3\mu_4(z_4-1)}{\mu_4-\mu_1}\right\}^{r_4}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \tag{133}$$

VII. E. CHARACTERISTICS OF THE MODEL

Putting $z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 0$ in (133) expanding and collecting constant terms we get the probability that the 4-server system is empty in steady state as $P(0,0,0,0) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}C_m\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^4\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \tag{134}$$

VII . F. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, z_4 = 1$ in(133) we get probability generating function of first queue size as $P(Z_1) =$

$$xp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\frac{1}{(b-a+1)}\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1}{\mu_1 r_1}\right\}\right] \tag{135}$$

Mean number of customers in first buffer is

$$E(N_1) = L_1 = \left[\frac{\lambda(a+b)}{2\mu_1}\right] \tag{136}$$

Putting $Z_1 = 0$ in (135) we get the probability that the first queue is empty as $P(0,.,.,.) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}(-1)^{r_2}\frac{1}{(b-a+1)}\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right] \tag{137}$$

Utilization of first server is $U_1 = 1 - P(0,.,.,.) =$

$$1 - exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}(-1)^{r_2}\frac{1}{(b-a+1)}\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right] \tag{138}$$

Throughput of first server is $Thp1 = \mu_1.U_1 =$

$$\mu_1\left\{1 - exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}(-1)^{r_2}\frac{1}{(b-a+1)}\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]\right\} \tag{139}$$

Average waiting time of a customer in first queue is

$$W_1 = \frac{L_1}{Thp_1} = \frac{\left[\frac{\lambda(a+b)}{2\mu_1^2}\right]}{\left\{1 - exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}(-1)^{r_2}\frac{1}{(b-a+1)}\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]\right\}} \tag{140}$$

Variance of number of customers in first queue is

$$V(Z_1) = V_1 = \left(\frac{\lambda}{\mu_1}\right)\left[\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2}+\frac{(a+b)}{2}\right] \tag{141}$$

Coefficient of variation of number of customers in first queue is $CV_1 = \frac{\sqrt{V_1}}{L_1}x100 =$

$$\frac{\sqrt{\left(\frac{\lambda}{\mu_1}\right)\left[\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2}+\frac{(a+b)}{2}\right]}}{\left[\frac{\lambda(a+b)}{2\mu_1}\right]}x100 \tag{142}$$

VII. G. PERFORMANCE ANALYSIS OF SECOND QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$ in (133) we get probability generating function of **second** queue size distribution as $P(Z_2) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}(-1)^{r_2}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \tag{143}$$

Mean number of customers in **second** queue is

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$$E(N_2) = L_2 = \left[\frac{\lambda\theta_1(a+b)}{2\mu_2}\right] \quad (144)$$

Probability that the **second** queue is empty is $P(.,0,.,.) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}(-1)^{r_2+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_2}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (145)$$

Utilization of **second** server is $U_2 = 1 - P(.,0,.,.) =$

$$1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}(-1)^{r_2+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_2}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (146)$$

Throughput of **second** server is $Thp_2 = \mu_2.U_2 =$

$$\mu_2.\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}(-1)^{r_2+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_2}\left\{\frac{\theta_1\mu_2}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right] \quad (147)$$

Average waiting time of customers in **second** queue is

$$W_2 = \frac{L_2}{Thp_2} = \frac{\left[\frac{\lambda\theta_1(a+b)}{2\mu_2^2}\right]}{\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}(-1)^{r_2+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_2}\left(\frac{\theta_1\mu_2}{\mu_2-\mu_1}\right)^{r_1}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]} \quad (148)$$

Variance of the number of customers in **second** queue is

$$V(z_2) = V_2 = \left(\frac{\lambda}{\mu_2}\right)\left[\left(\frac{\mu_1\theta_1^2}{\mu_1+\mu_2}\right)\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2} + \left\{\frac{\theta_1(a+b)}{2\mu_2}\right\}\right] \quad (149)$$

Coefficient of variation of number of customers in **second** queue is $CV_2 = \frac{\sqrt{V_2}}{L_2}x100 =$

$$\frac{\left(\frac{\lambda}{\mu_2}\right)\left[\left(\frac{\mu_1\theta_1^2}{\mu_1+\mu_2}\right)\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2}+\left\{\frac{\theta_1(a+b)}{2\mu_2}\right\}\right]}{\left[\frac{\lambda\theta_1(a+b)}{2\mu_2}\right]}x100 \quad (150)$$

VII. F. PERFORMANCE ANALYSIS OF THIRD QUEUE

Putting $z_1 = 1, z_2 = 1, z_4 = 1$ in (133) we get probability generating function of **third** queue size distribution as

$$P(Z_3) = exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_3=0}^{r_1}(-1)^{r_3}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (151)$$

Mean number of customers in **third** queue is

$$E(N_3) = L_3 = \left[\frac{\lambda\theta_2(a+b)}{2\mu_3}\right] \quad (152)$$

Probability that the **third** queue is empty is $P(.,.,0,.) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_3=0}^{r_1}(-1)^{r_3+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (153)$$

Utilization of **third** server is $U_3 = 1 - P(.,.,0,.) =$

$$1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_3=0}^{r_1}(-1)^{r_3+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (154)$$

Throughput of **third** server is $Thp_3 = \mu_3.U_3 =$

$$\mu_3.\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_3=0}^{r_1}(-1)^{r_3+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right] \quad (155)$$

Average waiting time of a customer in **third** queue is

$$W_3 = \frac{L_3}{Thp_3} = \frac{\left[\frac{\lambda\theta_2(a+b)}{2\mu_3^2}\right]}{\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_3=0}^{r_1}(-1)^{r_3+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]} \quad (156)$$

Variance of the number of customers in **third** queue is

$$V(z_3) = V_3 = \left(\frac{\lambda}{\mu_3}\right)\left[\left(\frac{\mu_1\theta_2^2}{\mu_1+\mu_3}\right)\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2} + \left\{\frac{\theta_2(a+b)}{2\mu_3}\right\}\right] \quad (157)$$

Coefficient of variation of the number of customers in **third** queue is $CV_3(t) = \frac{\sqrt{V_3(t)}}{L_3(t)}x100 =$

$$\frac{\sqrt{\left(\frac{\lambda}{\mu_3}\right)\left[\left(\frac{\mu_1\theta_2^2}{\mu_1+\mu_3}\right)\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2}+\left\{\frac{\theta_2(a+b)}{2\mu_3}\right\}\right]}}{\left[\frac{\lambda\theta_2(a+b)}{2\mu_3}\right]}x100 \quad (158)$$

VII .G. PERFORMANCE ANALYSIS OF FOURTH QUEUE

Putting $z_1 = 1, z_2 = 1, z_3 = 1$ in (133) we get probability generating function of **fourth** queue size distribution as

$P(Z_4) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1(z_i-1)}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (159)$$

Mean number of customers in **fourth** queue is

$$E(N_4) = L_4 = \left[\frac{\lambda\theta_3(a+b)}{2\mu_4}\right] \quad (160)$$

Probability that the **fourth** queue is empty is $P(.,.,.,0) =$

$$exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (161)$$

Utilization of **fourth** server is $U_4 = 1 - P(.,.,.,0) =$

$$1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_1}\left(\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right)\right] \quad (162)$$

Throughput of **fourth** server is $Thp_4 = \mu_4\, U_4 =$

$$\mu_4\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_1}\left(\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right)\right]\right] \quad (163)$$

Average waiting time of customer in **fourth** queue is

$$W_4 = \frac{L_4}{Thp_4} = \frac{\left[\frac{\lambda\theta_3(a+b)}{2\mu_4^2}\right]}{\left[1-exp\left[\lambda\sum_{m=a}^{b}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\frac{1}{(b-a+1)}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_1}\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right]} \quad (164)$$

Variance of the number of customers in **fourth** queue is

$$V(z_4) = V_4 = \left(\frac{\lambda}{\mu_4}\right)\left[\left(\frac{\mu_1\theta_3^2}{\mu_1+\mu_4}\right)\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2} + \left\{\frac{\theta_3(a+b)}{2\mu_4}\right\}\right] \quad (165)$$

Coefficient of variation of the number of customers in **fourth** queue is $CV_4(t) = \frac{\sqrt{V_4(t)}}{L_4(t)}x100 =$

$$\frac{\sqrt{\left(\frac{\lambda}{\mu_4}\right)\left[\left(\frac{\mu_1\theta_3^2}{\mu_1+\mu_4}\right)\sum_{m=a}^{b}\frac{1}{(b-a+1)}\binom{m}{2}+\left\{\frac{\theta_3(a+b)}{2\mu_4}\right\}\right]}}{\left[\frac{\lambda\theta_3(a+b)}{2\mu_4}\right]}x100 \quad (166)$$

VIII.COMPARITIVE STUDY

A comparative study between transient and steady state of the system with values of time t = 0.1, 0.2, 0.3, 0.4,0.5,1,2,3,4 and 5 is carried. The difference and percentage of variation in all performance measures are calculated and given in Table.6.

FromTable.6 it is observed that there is high significant difference between transient behaviour and steady state behaviour of the model. At t=0.1 the variation and percentage of variation in measures is highly significant which can be observed in last two columns of Table.6. As we move towards 0.5 the variation narrows down. At t=1the percentage of variation is reduced further and some of the measures differ very closely. This explains that as t increases the difference between transient and steady state behaviour become negligible and from t=3 onwards we find no difference between them. This indicates that the system attains equilibrium after time t=3 units

.IX.CONCLUSION

This paper addresses the development and analysis of a K-node tandem Queuing model with bulk arrivals and state dependent service rates. It is assumed that the K servers are connected in tandem where the customers arrive to the first Queue in batches and after getting service at first server they may join at any one of the (K-1) Queues which are in parallel with certain probability. The service rate of each service station is dependent on the content of the buffers connected to it. The explicit expressions for system characteristics such as average number of customers in the Queue, probability of idleness of each service station, throughput of the nodes, average waiting time customers in each Queue, utilization of each server are derived. The sensitivity of the model with respect to the changes in parameters is explained. The bulk size distribution parameters have significant influence on system performance measures. The congestion in the queues and the mean waiting time or delay in service can be reduced by regulating the bulk size distribution parameters. A comparative study of the model between transient and steady state revealed that the time t has significant influence on the performance measures. The proposed model is very useful for scheduling the Communication networks at LAN,VAN and MAN. It is possible to obtain the optimal operating policies of the model with suitable cost considerations which will be considered elsewhere. This model also includes some of the earlier models as particular cases for specific values of the parameters.

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Table 6.Comparative tables of performance measures between Transient and Steady state for values of t=0.1,0.2,0.3,0.4,0.5,1,2,3,4 and 5

$(\lambda = 5, \mu_1 = 6, \mu_2 = 7, \mu_3 = 8, \mu_4 = 9, \theta_1 = 0.1, \theta_2 = 0.2, a = 5, b = 20)$

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=0.1	L_1	6.2038	13.7500	-7.726	-56.191
	L_2	0.1624	1.1786	-1.016	-86.221
	L_3	0.3150	2.0625	-1.748	-84.727
	L_4	1.0697	6.4167	-5.347	-83.329
	L	7.7509	23.4078	-15.837	-67.657
	P_{0000}	0.2254	0.0004	0.225	63680.419
	$P_{0...}$	0.2359	0.0039	0.232	5997.183
	$P_{.0..}$	0.8595	0.3325	0.527	158.496
	$P_{..0.}$	0.7591	0.1616	0.598	369.740
	$P_{...0}$	0.4944	0.0119	0.483	4058.116
	U_1	0.7641	0.9961	-0.232	-23.293
	U_2	0.1405	0.6675	-0.527	-78.951
	U_3	0.2409	0.8384	-0.598	-71.267
	U_4	0.5056	0.9881	-0.483	-48.832
	Thp_1	4.5846	5.9768	-1.392	-23.293
	Thp_2	0.9835	4.6725	-3.689	-78.951
	Thp_3	1.9272	6.7072	-4.780	-71.267
	Thp_4	4.5504	8.8930	-4.343	-48.832
	W_1	1.3139	2.3006	-0.987	-42.887
	W_2	0.1651	0.2522	-0.087	-34.537
	W_3	0.1634	0.3075	-0.144	-46.847
	W_4	0.2351	0.7215	-0.486	-67.420

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=0.2	L_1	9.6086	13.7500	-4.141	-43.101
	L_2	0.4375	1.1786	-0.741	-169.394
	L_3	0.8269	2.0625	-1.236	-149.426

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	L_4	2.7400	6.4167	-3.677	-134.186
	L	13.6130	23.4078	-9.795	-71.952
	P_{0000}	0.0536	0.0004	0.053	99.341
	$P_{0...}$	0.0657	0.0039	0.062	94.113
	$P_{.0..}$	0.6698	0.3325	0.337	50.358
	$P_{..0.}$	0.4951	0.1616	0.334	67.360
	$P_{...0}$	0.1873	0.0119	0.175	93.652
	U_1	0.9343	0.9961	-0.062	-6.620
	U_2	0.3302	0.6675	-0.337	-102.150
	U_3	0.5049	0.8384	-0.334	-66.053
	U_4	0.8127	0.9881	-0.175	-21.584
	Thp_1	5.6057	5.9768	-0.371	-6.620
	Thp_2	2.3114	4.6725	-2.361	-102.150
	Thp_3	4.0392	6.7072	-2.668	-66.053
	Thp_4	7.3143	8.8930	-1.579	-21.584
	W_1	1.7141	2.3006	-0.586	-34.216
	W_2	0.1893	0.2522	-0.063	-33.264
	W_3	0.2047	0.3075	-0.103	-50.209
	W_4	0.3746	0.7215	-0.347	-92.613

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=0.3	L_1	11.4771	13.7500	-2.273	-19.804
	L_2	0.6808	1.1786	-0.498	-73.120
	L_3	1.2601	2.0625	-0.802	-63.677
	L_4	4.0971	6.4167	-2.320	-56.616
	L	17.5151	23.4078	-5.893	-33.644
	P_{0000}	0.0142	0.0004	0.014	97.511
	$P_{0...}$	0.0238	0.0039	0.020	83.709
	$P_{.0..}$	0.5360	0.3325	0.204	37.966
	$P_{..0.}$	0.3415	0.1616	0.180	52.679
	$P_{...0}$	0.0791	0.0119	0.067	84.968
	U_1	0.9762	0.9961	-0.020	-2.036
	U_2	0.4641	0.6675	-0.204	-43.858
	U_3	0.6585	0.8384	-0.180	-27.320
	U_4	0.9209	0.9881	-0.067	-7.298
	Thp_1	5.8572	5.9768	-0.119	-2.036
	Thp_2	3.2480	4.6725	-1.425	-43.858
	Thp_3	5.2680	6.7072	-1.439	-27.320
	Thp_4	8.2881	8.8930	-0.605	-7.298
	W_1	1.9595	2.3006	-0.341	-17.413
	W_2	0.2096	0.2522	-0.043	-20.341
	W_3	0.2392	0.3075	-0.068	-28.556
	W_4	0.4943	0.7215	-0.227	-45.963

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=0.4	L_1	12.5026	13.7500	-1.247	-9.977
	L_2	0.8602	1.1786	-0.318	-37.015
	L_3	1.5663	2.0625	-0.496	-31.680
	L_4	5.0210	6.4167	-1.396	-27.797
	L	19.9501	23.4078	-3.458	-17.332
	P_{0000}	0.0045	0.0004	0.004	92.174
	$P_{0...}$	0.0116	0.0039	0.008	66.589
	$P_{.0..}$	0.4531	0.3325	0.121	26.617

	$P_{..0.}$	0.2596	0.1616	0.098	33.750
	$P_{...0}$	0.0404	0.0119	0.029	70.569
	U_1	0.9884	0.9961	-0.008	-0780
	U_2	0.5469	0.6675	-0.121	-22.052
	U_3	0.7404	0.8384	-0.098	-13.236
	U_4	0.9596	0.9881	-0.029	-2.971
	Thp_1	5.9304	5.9768	-0.046	-0.780
	Thp_2	3.8283	4.6725	-0.844	-22.052
	Thp_3	5.9232	6.7072	-0.784	-13.236
	Thp_4	8.6364	8.8930	-0.257	-2.971
	W_1	2.1082	2.3006	-0.192	-9.126
	W_2	0.2247	0.2522	-0.028	-12.260
	W_3	0.2644	0.3075	-0.043	-16.288
	W_4	0.5814	0.7215	-0.140	-24.110

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=0.5	L_1	13.0654	13.7500	-0.6846	-5.2398
	L_2	0.9814	1.1786	-0.1972	-20.0937
	L_3	1.7651	2.0625	-0.2974	-16.8489
	L_4	5.6008	6.4167	-0.8159	-14.5676
	L	21.4127	23.4078	-1.9951	-9.3174
	P_{0000}	0.0018	0.0004	0.0015	80.8143
	$P_{0...}$	0.0073	0.0039	0.0034	47.1014
	$P_{.0..}$	0.4035	0.3325	0.0710	17.5960
	$P_{..0.}$	0.2158	0.1616	0.0542	25.1158
	$P_{...0}$	0.0252	0.0119	0.0133	52.8175
	U_1	0.9927	0.9961	-0.0034	-0.3470
	U_2	0.5965	0.6675	-0.0710	-11.9028
	U_3	0.7842	0.8384	-0.0542	-6.9115
	U_4	0.9748	0.9881	-0.0133	-1.3654
	Thp_1	5.9562	5.9768	-0.0207	-0.3470
	Thp_2	4.1755	4.6725	-0.4970	-11.9028
	Thp_3	6.2736	6.7072	-0.4336	-6.9115
	Thp_4	8.7732	8.8930	-0.1198	-1.3654
	W_1	2.1936	2.3006	-0.1070	-4.8758
	W_2	0.2350	0.2522	-0.0172	-7.3197
	W_3	0.2814	0.3075	-0.0262	-9.2950
	W_4	0.6384	0.7215	-0.0831	-13.0243

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=1	L_1	13.7159	13.7500	-0.0341	-0.2486
	L_2	1.1766	1.1786	-0.0020	-0.1700
	L_3	2.0441	2.0625	-0.0184	-0.9002
	L_4	6.3705	6.4167	-0.0462	-0.7252
	L	23.3071	23.4078	-0.1007	-0.4321
	P_{0000}	0.0004	0.0004	0.0000	10.5316
	$P_{0...}$	0.0040	0.0039	0.0001	3.2750
	$P_{.0..}$	0.3371	0.3325	0.0046	1.3646
	$P_{..0.}$	0.1646	0.1616	0.0030	1.8226
	$P_{...0}$	0.0125	0.0119	0.0006	4.8800
	U_1	0.9960	0.9961	-0.0001	-0.0132

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	U_2	0.6629	0.6675	-0.0046	-0.6939
	U_3	0.8354	0.8384	-0.0030	-0.3591
	U_4	0.9875	0.9881	-0.0006	-0.0618
	Thp_1	5.9760	5.9768	-0.0008	-0.2354
	Thp_2	4.6403	4.6725	-0.0322	0.5203
	Thp_3	6.6832	6.7072	-0.0240	-0.5391
	Thp_4	8.8875	8.8930	-0.0055	-0.0618
	W_1	2.2952	2.3006	-0.0054	-0.2354
	W_2	0.2536	0.2522	0.0013	0.5203
	W_3	0.3059	0.3075	-0.0016	-0.5391
	W_4	0.7168	0.7215	-0.0048	-0.6630

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=2	L_1	13.7499	13.7500	-0.0001	-0.0007
	L_2	1.1786	1.1786	0.0000	0.0000
	L_3	2.0625	2.0625	0.0000	0.0000
	L_4	6.4165	6.4167	-0.0002	-0.0031
	L	23.4075	23.4078	-0.0003	-0.0013
	P_{0000}	0.0004	0.0004	0.0000	0.0283
	$P_{0...}$	0.0039	0.0039	0.0000	0.0258
	$P_{.0..}$	0.3325	0.3325	0.0000	0.0000
	$P_{..0.}$	0.1617	0.1616	0.0001	0.0618
	$P_{...0}$	0.0119	0.0119	0.0000	0.0000
	U_1	0.9961	0.9961	0.0000	-0.0001
	U_2	0.6675	0.6675	0.0000	0.0000
	U_3	0.8383	0.8384	-0.0001	-0.0119
	U_4	0.9881	0.9881	0.0000	0.0000
	Thp_1	5.9768	5.9768	0.0000	-0.0001
	Thp_2	4.6725	4.6725	0.0000	0.0000
	Thp_3	6.7064	6.7072	-0.0008	-0.0119
	Thp_4	8.8930	8.8930	0.0000	0.0000
	W_1	2.3006	2.3006	0.0000	-0.0006
	W_2	0.2522	0.2522	0.0000	0.0000
	W_3	0.3075	0.3075	0.0000	-0.0119
	W_4	0.7215	0.7215	0.0000	-0.0031

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=3	L_1	13.7500	13.7500	0.0000	0.0000
	L_2	1.1786	1.1786	0.0000	0.0000
	L_3	2.0625	2.0625	0.0000	0.0000
	L_4	6.4167	6.4167	0.0000	0.0000
	L	23.4078	23.4078	0.0000	0.0000
	P_{0000}	0.0004	0.0004	0.0000	0.0000
	$P_{0...}$	0.0039	0.0039	0.0000	0.0000
	$P_{.0..}$	0.3325	0.3325	0.0000	0.0000
	$P_{..0.}$	0.1616	0.1616	0.0000	0.0000
	$P_{...0}$	0.0119	0.0119	0.0000	0.0000
	U_1	0.9961	0.9961	0.0000	0.0000
	U_2	0.6675	0.6675	0.0000	0.0000
	U_3	0.8384	0.8384	0.0000	0.0000
	U_4	0.9881	0.9881	0.0000	0.0000
	Thp_1	5.9768	5.9768	0.0000	0.0000

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	Thp_2	4.6725	4.6725	0.0000	0.0000
	Thp_3	6.7072	6.7072	0.0000	0.0000
	Thp_4	8.8930	8.8930	0.0000	0.0000
	W_1	2.3006	2.3006	0.0000	0.0000
	W_2	0.2522	0.2522	0.0000	0.0000
	W_3	0.3075	0.3075	0.0000	0.0000
	W_4	0.7215	0.7215	0.0000	0.0000

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=4	L_1	13.7500	13.7500	0.0000	0.0000
	L_2	1.1786	1.1786	0.0000	0.0000
	L_3	2.0625	2.0625	0.0000	0.0000
	L_4	6.4167	6.4167	0.0000	0.0000
	L	23.4078	23.4078	0.0000	0.0000
	P_{0000}	0.0004	0.0004	0.0000	0.0000
	$P_{0...}$	0.0039	0.0039	0.0000	0.0000
	$P_{.0..}$	0.3325	0.3325	0.0000	0.0000
	$P_{..0.}$	0.1616	0.1616	0.0000	0.0000
	$P_{...0}$	0.0119	0.0119	0.0000	0.0000
	U_1	0.9961	0.9961	0.0000	0.0000
	U_2	0.6675	0.6675	0.0000	0.0000
	U_3	0.8384	0.8384	0.0000	0.0000
	U_4	0.9881	0.9881	0.0000	0.0000
	Thp_1	5.9768	5.9768	0.0000	0.0000
	Thp_2	4.6725	4.6725	0.0000	0.0000
	Thp_3	6.7072	6.7072	0.0000	0.0000
	Thp_4	8.8930	8.8930	0.0000	0.0000
	W_1	2.3006	2.3006	0.0000	0.0000
	W_2	0.2522	0.2522	0.0000	0.0000
	W_3	0.3075	0.3075	0.0000	0.0000
	W_4	0.7215	0.7215	0.0000	0.0000

Time t	Performance	Transient State	Steady State	Difference between Transient and Steady state	% of variation
t=5	L_1	13.7500	13.7500	0.0000	0.0000
	L_2	1.1786	1.1786	0.0000	0.0000
	L_3	2.0625	2.0625	0.0000	0.0000
	L_4	6.4167	6.4167	0.0000	0.0000
	L	23.4078	23.4078	0.0000	0.0000
	P_{0000}	0.0004	0.0004	0.0000	0.0000
	$P_{0...}$	0.0039	0.0039	0.0000	0.0000
	$P_{.0..}$	0.3325	0.3325	0.0000	0.0000
	$P_{..0.}$	0.1616	0.1616	0.0000	0.0000
	$P_{...0}$	0.0119	0.0119	0.0000	0.0000
	U_1	0.9961	0.9961	0.0000	0.0000
	U_2	0.6675	0.6675	0.0000	0.0000
	U_3	0.8384	0.8384	0.0000	0.0000
	U_4	0.9881	0.9881	0.0000	0.0000
	Thp_1	5.9768	5.9768	0.0000	0.0000
	Thp_2	4.6725	4.6725	0.0000	0.0000
	Thp_3	6.7072	6.7072	0.0000	0.0000
	Thp_4	8.8930	8.8930	0.0000	0.0000

	W_1	2.3006	2.3006	0.0000	0.0000
	W_2	0.2522	0.2522	0.0000	0.0000
	W_3	0.3075	0.3075	0.0000	0.0000
	W_4	0.7215	0.7215	0.0000	0.0000

performance	t=0.1	t=0.2	t=0.3	t=0.4	t=0.5	Steady State
L_1	6.2038	9.6086	11.4771	12.5026	13.0654	13.7500
L_2	0.1624	0.4375	0.6808	0.8602	0.9814	1.1786
L_3	0.3150	0.8269	1.2601	1.5663	1.7651	2.0625
L_4	1.0697	2.7400	4.0971	5.0210	5.6008	6.4167
L	7.7509	13.6130	17.5151	19.9501	21.4127	23.4078
P_{0000}	0.2254	0.0536	0.0142	0.0045	0.0018	0.0004
$P_{0...}$	0.2359	0.0657	0.0238	0.0116	0.0073	0.0039
$P_{.0..}$	0.8595	0.6698	0.5360	0.4531	0.4035	0.3325
$P_{..0.}$	0.7591	0.4951	0.3415	0.2596	0.2158	0.1616
$P_{...0}$	0.4944	0.1873	0.0791	0.0404	0.0252	0.0119
U_1	0.7641	0.9343	0.9762	0.9884	0.9927	0.9961
U_2	0.1405	0.3302	0.4641	0.5469	0.5965	0.6675
U_3	0.2409	0.5049	0.6585	0.7404	0.7842	0.8384
U_4	0.5056	0.8127	0.9209	0.9596	0.9748	0.9881
Thp_1	4.5846	5.6057	5.8572	5.9304	5.9562	5.9768
Thp_2	0.9835	2.3114	3.2480	3.8283	4.1755	4.6725

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