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Toroidial Properties of Polychromatic Quasi Crystals

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***Abstract:** A group theoretic method of obtaining the maximum number of non-vanishing independent constants required to describe a chosen toroidial property of the 54 polychromatic classes is presented. The toroidial coefficients obtained for all the 54 polychromatic classes are tabulated and the results obtained are briefly discussed.*

***Keywords:** Toroidial properties, polychromatic quasi crystals, independent constants.*

I. INTRODUCTION

Quasi crystals are fascinating materials: crystal structures, with their 5-fold symmetry, are unconventional and properties are surprising and could be remarkably useful. The root construction of macroscopic models of quasi crystal structures is Penrose tiling, which played a very important role in the theory of quasi crystals. Janssen T. (1992) gave a clear theoretical explanation for quasi periodic structures which may have either crystallographic (or) non-crystallographic point group symmetry. Belov and Tarkhova (1956) gave the interpretation of anti-symmetry as two colour symmetry and also gave the concept of polychromatic symmetry. The notation adopted for the polychromatic classes is due to Indenbom, Belov & Neronova (1960) and the nomenclature for the point groups is that of Hermann-Mauguin (International). The polychromatic structure, the polychromatic point groups and space groups are described by Naish (1963).These groups plays a powerful role in physical applications such as the derivation and description of similarity symmetry point groups and space groups, in description of stem and layer symmetry groups in higher dimensional space. Primary ferroic crystals are also the crystals with the domain states which are differentiated by properties like spontaneous polarization, magnetization (or) strain. Secondary ferroic crystals are the crystals with the domain states that are distinguished by the piezoelectric tensor and they are named as Ferromagnetotoroidic (ev2), Ferromagnetoelastic (aev [v2]) crystals respectively. Here "V" represents a polar vector and "e" and "a" denotes zero rank tensors that change under spatial inversion and time inversion respectively.

Ferrotoroidicity is the property of certain materials that exhibit Spontaneous toroidial moment. A spontaneous toroidial moment is an independent term in the multipole expansion of electromagnetic fields besides magnetic and electric multipoles. This paper accounts the concept of 18 polychromatic point groups with the help of 10 crystallographic point groups not containing one dimensional complex irreducible representation (Indenbom et al). It was extended to the quasi crystals with five, eight, ten and twelve fold symmetries. The 54 polychromatic classes with the help of 11 non-crystallographic point groups not containing one dimensional complex irreducible representation are formulated. By using Bhagavantham's formula, the maximum number of non-vanishing independent constants of the 54 polychromatic classes is calculated and tabulated which helps in choosing toroidial property.

Classification of Toroidial Properties:

S. No.	Toroidial Property	Character $\chi'_j(R)$
1.	Ferromagnetic	aev
2.	Ferromagnetoelastic	aev $[v^2]$
3.	Ferromagnetoelectric	aev^2
4.	Ferromagnetotoroidic	ev^2
5.	Ferroelectrotoroidic	av^2
6.	Ferroelastotoroidic	$av[v^2]$

Polychromatic Classes of Quasi Crystals:

The polychromatic point group C, is given by

$$C = \alpha_1 H + \alpha_2 R_p H + \alpha_3 R_p^{\,2} H + ------ + \alpha_p R_p^{\,p-1} H$$

Where H is a subgroup of some point group G and is of order (1/p) × the order of G and $\alpha_1, \alpha_2 ----\alpha_p$ are the left coset representatives in the expansion of G. $G = \alpha_1 H + \alpha_2 H + \alpha_3 H + ------ + \alpha_p H$

If p = 3,4 or 6 we can derive p-coloured point groups. Bradley and cracknel gave the 58 black and white point groups are derived by studying the real one dimensional representation of the ordinary point groups and multiplying an element whose character is -1 by the colour changing operation. Similarly to derive the 3-coloured,4-coloured and 6-coloured point groups(Indenbom et al)

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complex one dimensional representation of the ordinary point groups are useful. The complex numbers $w=e^{\frac{2\pi i}{3}}$, $i=e^{\frac{2\pi i}{4}}$ *and* $w^{*}=-e^{\frac{2\pi i}{6}}$ can be associated with colour changing operations for 3-coloured,4-coloured and 6-coloured point groups respectively. The two members of a pair of complex conjugate representations of an ordinary point group both lead to the same polychromatic point group. In the same way 5-coloured, 8-coloured 10-coloured and 12-coloured point groups are defined by taking the complex one dimensional representation of the quasi crystals. The complex numbers $\rho=e^{\frac{2\pi i}{5}}$, $\rho=e^{\frac{2\pi i}{8}}$ *and* $\rho=e^{\frac{2\pi i}{10}}$ *and* $\rho=e^{\frac{2\pi i}{12}}$ can associated with colour changing operations for 5-coloured,8coloured 10-coloured and 12-coloured point groups respectively. The results obtained of all the 54 polychromatic classes of quasi crystals are tabulated in the following table.

S. No.	Polychromatic class	No. of independent constants required to describe the ferrotoroidial properties					
		1	2	3	4	5	6
1	$5_1^{(5)}$	1	1	1	2	2	4
2	$5_2^{(5)}$	0	1	0	1	1	3
3	$\bar{5}_1^{(5)}$	1	0	0	0	2	4
4	$\bar{5}_2^{(5)}$	0	0	0	0	1	3
5	$\bar{5}_{1^1}^{(5)}$	0	1	1	2	0	0
6	$\bar{5}_{2^1}^{(5)}$	0	1	0	1	0	0
7	$8_1^{(8)}$	1	1	1	2	2	4
8	$8_2^{(8)}$	0	1	0	1	1	2
9	$8_1^{(4)}$	0	0	0	0	0	1
10	$\bar{8}_1^{(8)}$	0	1	1	2	2	1
11	$\bar{8}_2^{(8)}$	0	1	0	1	1	2
12	$\bar{8}_1^{(4)}$	1	0	0	0	0	4
13	$8/m^{(8)}$	2	0	0	0	4	8
14	$8/m'^{(8)}$	0	0	0	0	2	4
15	$8/m''^{(8)}$	0	0	0	0	0	2
16	$8/m^{(4)}$	0	2	2	4	0	0
17	$8/m'^{(4)}$	0	2	0	2	0	0
18	$8/m''^{(4)}$	0	0	0	0	0	0

19	$10_1^{(10)}$	2	2	2	4	4	8
20	$10_2^{(10)}$	0	2	0	2	2	4
21	$10_1^{(5)}$	0	0	0	0	0	2
22	$10_2^{(5)}$	0	0	0	0	0	0
23	$1\bar{0}_1^{(10)}$	0	1	1	2	0	0
24	$1\bar{0}_2^{(10)}$	0	0	0	0	1	2
25	$1\bar{0}_1^{(5)}$	1	0	0	0	2	4
26	$1\bar{0}_2^{(5)}$	0	1	0	1	0	1
27	$10/m^{(10)}$	2	0	0	0	4	8
28	$10/m'^{(10)}$	0	0	0	0	2	4
29	$10/m''^{(10)}$	0	0	0	0	0	2
30	$10/m'''^{(10)}$	0	0	0	0	0	0
31	$10/m^{(5)}$	0	2	2	4	0	0
32	$10/m'^{(5)}$	0	2	0	2	0	0
33	$10/m''^{(5)}$	0	0	0	0	0	0
34	$10/m'''^{(5)}$	0	0	0	0	0	0
35	$12_1^{(12)}$	2	2	2	4	4	8
36	$12_2^{(12)}$	0	2	0	2	2	4
37	$12_3^{(12)}$	0	0	0	0	0	2
38	$12_1^{(6)}$	0	0	0	0	0	0
39	$12_2^{(6)}$	0	0	0	0	0	0
40	$1\bar{2}_1^{(12)}$	0	1	1	2	0	0
41	$1\bar{2}_2^{(12)}$	0	0	0	0	1	2
42	$1\bar{2}_3^{(12)}$	0	0	0	0	0	1

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43	$12\bar{2}_1{}^{(6)}$	0	1	0	1	0	0
44	$12\bar{2}_2{}^{(6)}$	1	0	0	0	2	4
45	$12/m^{(12)}$	0	2	2	4	0	0
46	$12/m'^{(12)}$	0	2	0	2	0	0
47	$12/m''^{(12)}$	0	0	0	0	0	0
48	$12/m'''^{(12)}$	0	0	0	0	0	0
49	$12/m^{IV\ (12)}$	0	0	0	0	0	0
50	$12/m^{(6)}$	2	0	0	0	4	8
51	$12/m'^{(6)}$	0	0	0	0	2	4
52	$12/m''^{(6)}$	0	0	0	0	0	2
53	$12/m'''^{(6)}$	0	0	0	0	0	0
54	$12/m^{1V\ (6)}$	0	0	0	0	0	0

II. CONCLUSIONS

This paper deals with the group theoretical methods of studying the effect of symmetry on some toroidial properties of polychromatic quasi crystals. The number of independent constants is determined and tabulated which helps in describing the toroidial properties of these crystals.

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