CMI Dual Algorithm for the Solution of Linear Programming Problems

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Abstract: In this paper, a new approach CMI Dual algorithm is introduced. This method is very easy to solve linear programming problem. Sometimes it involves less number of iterations than the Existing dual simplex method. This method is very strong with effortless iterations and also it saves the Precious time by avoiding the calculations of net evaluations.

Keywords: CMI Dual Method, LPP, Optimal Solution

I. INTRODUCTION

LPP has its own significant in attaining the solution of a problem. It concerned with the optimal allocation of limited resources to meet certain desired objective.
To Maximize the Objective function F= RX
Subject to PX=Q, X≥0
Where X =n x1 column matrix
P=m x n coefficient matrix
Q = m x 1 column matrix
R = 1 X n row matrix. There are handful methods to attain the solution of the above problem. In that methods simplex method is the most general and powerful.

II. PROPOSED CMI DUAL ALGORITHM

To find the solution of any LPP by CMI Dual method, algorithm is given as follows,

Step 1: The objective function of LPP must be Maximize. If it is Minimize then convert it into a Maximize by using the result Min Z = **-** Max(-Z)

Step 2: Convert all ≥ type constraints in to ≤ by multiplying both sides by **-**1. Also convert the inequality constraints to equality by adding of slack variables and attain an initial basic solution .Indicate the above details in the form of table called as CMI Dual table.

Step 3: To find the Pivot Column

(i) In Starting iteration, find the Pivot column by choosing the large coefficient in the objective function C_j.

(ii) In successive iterations, identify the Maximum positive value of the variables (both basic and non basic).

(iii) If the Maximum value attains 1, it should be chosen from body matrix only.

Step 4: To find the Pivot Row

(i) First identify the Minimum value of the variables (both basic and non basic) except 0 and 1.

(ii) If the Pivot element attains 0 ,skip that corresponding row and consider the next minimum value

(iii) If the Minimum value occurs in tie then consider the corresponding row of largest element in that pivot column.

Step 5: Skip the Pivot row and introduce the Pivot column in the basis and convert the Pivot element to unity and all other elements in its column to zero by using usual simplex method and go to next step.

Step 6: Ignore the corresponding row and column. Proceed to step 3 and 4 for remaining elements and repeat the same procedure until an optimum solution is obtained or there is an indication for an unbounded solution.

Step 7: If all Pivot rows and Pivot columns are ignored and all values of x_B are positive then the current solution is an optimum solution.

In Case of Failure

(i) While finding the Pivot column (step-3), Once the variable leaves, it should not enter again in the basis in the successive iterations. If it happens, leave that corresponding maximum value and consider the next maximum one from the other variables.

(ii) While finding the Pivot row (Step-4), Once the variable enters, it should not leave from the basis in the successive iterations . If it happens, leave that corresponding minimum value and consider the next minimum one from the other variables.

III. NUMERICAL EXAMPLES

Problem -I

$$\text{Minimize } Z = 2x_1 + x_2$$
Subject to the constraints
$$3x_1 + x_2 \geq 3,$$
$$4x_1 + 3x_2 \geq 6,$$
$$x_1 + 2x_2 \geq 3, \text{where } x_1, x_2 \geq 0$$

Solution:

By using step 1 and 2, the given problem can be modified in to the system of equations
The modified objective function is
$$\text{Maximize } Z = -2x_1 - x_2 + 0P_1 + 0P_2 + 0P_3$$
Subject to the constraints
$$-3x_1 - x_2 + P_1 = -3,$$
$$-4x_1 - 3x_2 + P_2 = -6,$$
$$-x_1 - 2x_2 + P_3 = -3,$$

where $x_1, x_2, P_1, P_2, P_3 \geq 0$

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Initial iteration

z			-2	-1	0	0	0
C_B	BASIS	x_B	x_1	x_2	P_1	P_2	P_3
0	P_1	-3	-3	-1	1	0	0
0	P_2 →	-6	-4	**-3**	0	1	0
0	P_3	-3	-1	-2	0	0	1
				↑			

In initial iteration, the variable $x_2$column is having greatest coefficient in objective function. So choose that column as an pivot column .The minimum value is -4which is inP_2row.Hence that corresponding row is a pivot row. The Pivotal element is**(-3).**

First iteration:

By using Step 5, the initial iteration becomes

O.F			-2	-1	0	0	0
C_B	BASIS	x_B	x_1	x_2	P_1	P_2	P_3
0	P_1 →	-1	**-5/3**	0	1	-1/3	0
-1	x_2	2	4/3	1	0	1/3	0
0	P_3	1	5/3	0	0	-2/3	1
			↑				

In the above iteration the maximum value is 5/3 which occurs in x_1 column, therefore it is a pivot column. The minimum value is -5/3 which is inP_1row.Hence that corresponding row is a pivot row. The Pivotal element is **(-5/3).**

Second Iteration:

By using Step 5, the first iteration becomes

O.F			-2	-1	0	0	0
C_B	BASIS	x_B	x_1	x_2	P_1	P_2	P_3
-2	x_1	3/5	1	0	-3/5	1/5	0
-1	x_2	6/5	0	1	4/5	-3/5	0
0	P_3	0	0	0	1	-1	1

Since all the rows and columns are ignored and all values of $\boldsymbol{x_B}$ are positive , hence current solution is an optimal

Therefore the optimum solution is

$\boldsymbol{x_1 = 3/5,\ x_2 = 6/5}$ **, & Max Z = - 12/5**

But Min Z = - Max (-Z) = - (-12/5) = 12/5

Problem -2:

$$\boldsymbol{Maximize\ Z = -2x_1 - 2x_2 - 4x_3}$$

Subject to the constraints

$$\boldsymbol{2x_1 + 3x_2 + 5x_3 \geq 2,}$$
$$\boldsymbol{3x_1 + x_2 + 7x_3 \leq 3,}$$
$$\boldsymbol{x_1 + 4x_2 + 6x_3 \leq 5} \textbf{where}$$
$$\boldsymbol{x_1, x_2, x_3 \geq 0}$$

Solution:

By using step 2, the given problem can be modified in to the system of equations

The modified objective function is

$$\boldsymbol{Maximize\ Z = -2x_1 - 2x_2 - 4x_3 + 0P_1 + 0P_2 + 0P_3}$$

Subject to the constraints

$$\boldsymbol{x_1 - 3x_2 - 5x_3 + P_1 = -2,}$$
$$\boldsymbol{3x_1 + x_2 + 7x_3 + P_2 = 3,}$$
$$\boldsymbol{x_1 + 4x_2 + 6x_3 + P_3 = 5} \textbf{where}$$
$$\boldsymbol{x_1, x_2, x_3 \geq 0}$$

Initial iteration

O.F			-2	-2	-4	0	0	0
C_B	BASIS	x_B	x_1	x_2	x_3	P_1	P_2	P_3
0	P_1 →	-2	-2	**-3**	-5	1	0	0
0	P_2	3	3	1	7	0	1	0
0	P_3	5	1	4	6	0	0	1
				↑				

In initial iteration, the variable $x_1$and$x_2$columns are having same greatest coefficient in objective function. So choose any one as arbitrarily. selecting x_2 column as a pivot column .The minimum value is -5 which is in P_1row.Hence that corresponding row is a pivot row. The Pivotal element is **(-3).**

First iteration:

By using Step 5, the initial iteration becomes

O.F			-2	-2	-4	0	0	0
C_B	BASIS	x_B	x_1	x_2	x_3	P_1	P_2	P_3
-2	x_2	2/3	2/3	1	5/3	-1/3	0	0
0	P_2	7/3	7/3	0	16/5	1/3	1	0
0	P_3	7/3	-5/3	0	-2/3	4/3	0	1

In the above iteration all x_B values are positive. Hence the current solution is an optimal solution.

Therefore the optimal solution is

$\boldsymbol{x_1 = 0}$ **,** $\boldsymbol{x_2 = 2/3}$ **,** $\boldsymbol{x_3 = 0}$ **&Max Z = - 4/3**

Failure Case:

Problem -3:

$$\boldsymbol{Minimize\ Z = x_1 + x_2}$$

Subject to the constraints

$$\boldsymbol{2x_1 + x_2 \geq 2,}$$
$$\boldsymbol{-x_1 - x_2 \geq 1,}\ \textbf{where}\ \boldsymbol{x_1, x_2 \geq 0}$$

Solution:

By using step 1 and 2, the given problem can be modified in to the system of equations

The modified objective function is

$$Maximize\ Z = -x_1 - x_2 + 0P_1 + 0P_2$$

Subject to the constraints

$$-2x_1 - x_2 + P_1 = -2,$$
$$x_1 + x_2 + P_2 = -1,$$
$$\text{where } x_1, x_2, P_1, P_2, \geq 0$$

Initial iteration

O.F			-1	-1	0	0
C_B	BASIS	x_B	x_1	x_2	P_1	P_2
0	P_1 →	-2	**-2**	-1	1	0
0	P_2	-1	1	1	0	1

↑

In initial iteration, the variable x_1 and x_2 columns are having the same greatest coefficient in objective function. So choose arbitrarily x_1 column as a pivot column .The minimum value is -2 which is in P_1 row. Hence that corresponding row is a pivot row. The Pivotal element is (-2).

First iteration:

By using Step 5, the initial iteration becomes

O.F			-1	-1	0	0
C_B	BASIS	x_B	x_1	x_2	P_1	P_2
-1	x_1	1	1	1/2	-1/2	0
0	P_2 →	-2	0	**1/2**	1/2	1

↑

In the above iteration the maximum value is 1/2 which occurs in both x_2 and P_1 columns. By using failure case (i), choose x_2 is a pivot column. The minimum value is -1/2 which occurs in x_1 row but by using failure case (ii) leave that corresponding row and consider the next minimum value that is ½ in P_2 row. Hence that corresponding row is a pivot row. The Pivotal element is **(1/2).**

Second iteration:

By using Step 5, the first iteration becomes

O.F			-1	-1	0	0
C_B	BASIS	x_B	x_1	x_2	P_1	P_2
-1	x_1	3	1	0	-1	-1
-1	x_2	-4	0	1	1	2

Since all the rows and columns are ignored, but x_B contains –ve value. Therefore it has no feasible solution to the given LPP. Hence it is an unbounded solution.

Problem -4

$$Minimize\ Z = x_1 + 2x_2 + 3x_3$$

Subject to the constraints

$$x_1 - x_2 + x_3 \geq 4,$$
$$x_1 + x_2 + 2x_3 \leq 8,$$
$$x_2 - x_3 \geq 2 \text{ where } x_1, x_2, x_3 \geq 0$$

Solution:

By using step 1 and 2, the given problem can be modified in to the system of equations

The modified objective function is

$$Maximize\ Z = -x_1 - 2x_2 - 3x_3 + 0P_1 + 0P_2 + 0P_3$$

Subject to the constraints

$$-x_1 + x_2 - x_3 + P_1 = -4,$$
$$x_1 + x_2 + 2x_3 + P_2 = 8,$$
$$-x_2 + x_3 + P_3 = -2 \text{ where } x_1, x_2, x_3 \geq 0$$

Initial iteration

O.F			-1	-2	-3	0	0	0
C_B	BASIS	x_B	x_1	x_2	x_3	P_1	P_2	P_3
0	P_1 →	-4	**-1**	1	-1	1	0	0
0	P_2	8	1	1	2	0	1	0
0	P_3	-2	0	-1	1	0	0	1

↑

In initial iteration the variable x_1 column is having greatest coefficient in objective function. So select that column as a pivot column .The minimum value is -1 which occurs in both P_1 and P_3 rows. If we select P_3 row, pivot element attains 0. So by using step-4 condition(ii),select P_1 is a pivot row .Hence that corresponding row is a pivot row. The Pivotal element is **(-1).**

First iteration:

By using Step 5, the initial iteration becomes

O.F			-1	-2	-3	0	0	0
C_B	BASIS	x_B	x_1	x_2	x_3	P_1	P_2	P_3
-1	x_1	4	1	-1	1	-1	0	0
0	P_2	4	0	2	1	1	1	0
0	P_3 →	-2	0	**-1**	1	0	0	1

↑

In the above iteration the maximum value is 2 which occurs in x_2 column, therefore it is a pivot column. The minimum value is -1 which occurs in both x_1 and P_3 but according to condition (ii) in failure case, leave that x_1 row and select P_3 is a pivot row. The Pivotal element is **(-1).**

Second Iteration:

By using Step 5, the first iteration becomes

O.F	-1	-2	-3	0	0	0

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C_B	BASIS	x_B	x_1	x_2	x_3	P_1	P_2	P_3
-1	x_1	6	1	0	0	-1	0	1
0	P_2	0	0	0	3	1	1	2
-2	x_2	2	0	1	-1	0	0	-1

In the above iteration all x_B values are positive. Hence the current solution is an optimal solution.

Therefore the optimal solution is

$$x_1 = 6 \;, x_2 = 2 \,, x_3 = 0 \;\&\text{Max } Z = -10$$

$$\text{But Min } Z = -\text{Max}(-Z) = -(-10) = 10$$

IV. CONCLUSION

CMI Dual method has been evaluated to obtain the solution of linear Programming problem. We observed that the solution obtained in less iteration or at the most equal iterations by our modified technique. It is very easy to apply and reduces our manual work. This method save our precious time as there is no need to calculate the net evaluations.

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