Basic Study with Support and Support value of Connected Network Graph Support study for Special Graph

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***Abstract:** In this paper we observed the definition of Midvertex graph and Middle graph approach for the connected network topology, and derived some new results and theorems on Support of the Midvertex graph and Middle graphs. And this study leads to new approach from all the other generation of connected network topology. Also obtain new results for support value of some special graphs of like Ladder graph, Wheel graph, C_n+,C_n+ C_n, ,C_n- C_n.*

***Keywords:** Crown graph, Domination set, Middle graph, Support of Midvertex graph and Middle graph*

I. PRELIMINARY & DEFINITIONS

Definition: 1.1

Neighbors. Two vertices of a graph which are adjacent are said to be a neighbor. The set of all neighbors of a vertex v of G is called neighborhood set and it is denoted by N(v) or N[v] . Where N(v) and N[v]are known as open and closed neighborhood set respectively.

Definition: 1.2

Regular graph. A graph G is said to be regular if all the vertices of G have the same degree. If the degree of each vertex in G is k, then the graph is said to be k- regular graph.

Definition: 1.3

Cartesian product of G_1 and G_2. Let G_1 and G_2be two graphs. Then G_1 X $G_2$is the **Cartesian product** of G_1 and G_2if vertex set $V(G_1)$ X V (G_2) and u = (u_1 ,v_1) and v = (u_2,v_2) are adjacent in G_1 X G_2 whenever $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or $v_1 = v_2$ and u_1 and u_2 are adjacent in G_1.

Definition: 1.4

Ladder graph.For $n \geq 2$, the graph P_2XP_n is called as the **Ladder graph** L_n.

Definition: 1.5

Wheel graph. The **Wheel** W_n is defined to be the join C_{n-1} + $K_1(n > 3)$. The vertex corresponding to $K_1$is known as apex vertex and the vertices corresponding to Cycle are known as rim vertices.

Definition: 1.6

Fangraph. A F**angraph**is defined to be a graph which is got by joining every vertex of $P_{n-1}(n \geq 3)$,with a new vertex and is denoted by F_n.

Definition: 1.7

Cut vertex. Let G be a graph. Then a vertex ‘v’ is said to be **cut point** or **cut vertex** of a graph G if removal of vertex ‘v’ from G increases the number of components.

Definition: 1.8

Cut edge. Let G be a graph. Then an edge ‘e’ is said to be **bridge** or **cut edge** of a graph G if removal of an edge ‘e’ from G increases the number of components.

Definition: 1.9

Midvertex of an edge. A Midvertex of an edge uv of a graph G is obtained by introducing a new vertex w**,** joining the edges uw and vw (uv, uw, and vw form C_3).The graph obtained from G by each edge of G exactly once is called the Midvertex graph of G and it is denoted by Mid (G).

Definition: 1.10

Support of a vertex. Let G = (V, E) be a simple graph. The **Support** of a vertex in a graph is defined as the sum of the degrees of its neighbors. (ie)supp (v) = $\sum_{u \in N(v)} \deg(u)$

Definition: 1.11

Support of Graph. LetG = (V, E) be a simple graph. Then **Supp (G)** = $\sum_{v \in V(G)}$ supp(v)

Definition: 1.12

Support regular. A graph G is said to be **Support regular**, if supp (v) is constant for all v $\in$V (G).

Definition: 1.13

Middle graph.Let G be a graph. Then the **Middle graph (**M (G)) of G is said to be a graph whose vertex set is V (G)∪E (G), and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is an edge incident with it.

Definition: 1.14

Domination vertex. Let G be a graph. Then vertex ‘v’ in a graph G is said to be dominate itself and each of its neighbors. In other Words, v is the dominating vertex of its closed neighborhood set (N[v]).

Definition: 1.15

Dominating set. A Set D of a graph G = (V, E) is called a **Dominating set** if every vertex in V - D is adjacent to some vertex in D. The **Domination number** of G is the minimum cardinality taken over all dominating set of G**.**

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II. OPEN SUPPORT OF A GRAPH&CLOSED SUPPORT OF A GRAPH

Open Support of v = $\sum_{u\in N(v)} \deg(u)$

Closed Support of v = $\sum_{u\in N[v]} \deg(u)$

Supp(G) = $\sum_{v\in V(G)}$ supp(v)

Theorem:1.1 For any connected Crown graph C_n+ (n ≥ 3), then Supp (C_n+) = 10n.

Proof: let $v_1,v_2,v_3,\ldots,v_n,u_1,u_2,u_3,\ldots,u_n$ be the vertices of C_n+ with deg (v_i) =3 and deg (u_i) = 1,for i = 1,2,…,n .It consists of 2n vertices.

Define Supp: V (C_n+) ⟶ {3, 7} by

Supp (v_i) = 7, for i =1, 2,... , n,and

Supp (u_i) = 3, for i =1, 2,..., n.

Supp (C_n+) = $\sum_{v\in V(C_n+)} \deg(v)$

= $\sum_{i=1}^{n}$ supp (v_i) + $\sum_{i=1}^{n}$ supp (u_i)

= $\sum_{i=1}^{n}(7)$ + $\sum_{i=1}^{n}(3)$ = 7n +3n = 10n

Supp (C_n+) = 10n.

Observation of Supp (C_n+)

1. Support value of connected domination set is 7n
2. Minimum Support value of independent domination set is 3n.
3. Support value of neither connected norindependent domination set is lies between 3n and 7n.
4. Supp (G) = Support value of connected domination set + Minimum Support value of independent domination set.

Theorem:1.2 For anyconnoted graph G =M (C_n+), (n ≥ 3), then Supp [M (C_n+)] = 62n.

Proof: Letthe vertices M(C_n+) be $v_1,v_2,v_3,\ldots,v_n,u_1,u_2,u_3,\ldots u_n$, $x_1,x_2,x_3,\ldots,x_n,y_1,y_2,y_3,\ldots,y_n$, where deg ($v_i$) =3, for i= 1,2,…,n, deg(u_i) = 1, for i = 1,2,…,n, deg (x_i) = 6, for i= 1,2,…,n, and deg (y_i) = 4, for i= 1,2,…,n, The number of vertices of M(Cn+) are 4n.

Define Supp: V [M(C_n+)]⟶ {4, 16, 26} by

$$\text{Supp}(V) = \begin{cases} \text{supp } (v_i) = 16 & \text{for i} = 1,2,\ldots,n \\ \text{supp } (u_i) = 4 & \text{for i} = 1,2,\ldots,n \\ \text{supp } (x_i) = 26 & \text{for i} = 1,2,\ldots,n \\ \text{supp } (y_i) = 16 & \text{for i} = 1,2,\ldots,n \end{cases}$$

NowSupp(G) = $\sum_{v\in V(G)}$ supp(v)= $\sum_{i=1}^{n}$ supp (v_i) + $\sum_{i=1}^{n}$ supp (u_i)+ $\sum_{i=1}^{n}$ supp (x_i) + $\sum_{i=1}^{n}$ supp (y_i)

= $\sum_{i=1}^{n}(16)+\sum_{i=1}^{n}(4)+\sum_{i=1}^{n}(26)+\sum_{i=1}^{n}(16)$

= 16n +4n +26n +16n = 62n

Supp[M (C_n+)] = 62n.

Theorem:1.3 For any graph G =Mid (C_n+), (n ≥ 3), then Supp[Mid (C_n+)] = 48n.

Observation of Supp (C_n+), Supp[Mid (C_n+)],&Supp M (C_n+).

1. Supp [M (C_n+)] = 6 Supp (C_n+) + 2n.
2. Supp [Mid (C_n+)] = 5 Supp(C_n+) - 2n.
3. Supp [M (C_n+)] = Supp [Mid (C_n+)] + Supp (C_n+) + 4n

Theorem:1.4 For anyconnected graph G = C_n + C_n (n ≥ 3) thenSupp (C_n + C_n) = 18n.

Proof. Let G = C_n + C_n be a 3- regular graph having 2n vertices with deg (v_i) =deg (u_i) =3, for all u_i,v_i∈V (G).

For any regular graph G, supp (v) = $[\deg (v)]^2$for all v∈ V(G).

Here C_n + C_n is a regular graph, with deg (v_i) = 3 for all$v_1,v_2,v_3,\ldots,v_n,u_1,u_2,u_3,\ldots,u_n$.

Now, Supp (G) = $\sum_{v\in V(G)}$ supp(v) = $\sum_{i=1}^{n}$ $[\deg (v_i)]^2$+$\sum_{i=1}^{n}$ $[\deg (u_i)]^2$ = $\sum_{i=1}^{n}(9)+\sum_{i=1}^{n}(9)$ = 9n+9n = 18n.

Theorem:1.5 For anyconnected graph G = C_n- C_n(n ≥ 3) ,thenSupp(C_n - C_n) =2(4n+5)

Proof. Let G = C_n - C_n be a graph of 2n vertices. In a graph G , e = uv be a cut edge with deg (u) = deg (v) = 3, and remaining (2n – 2) vertices are having degree 2.

For any graph G, Supp (G) = $\sum_{v\in V(G)}$ $[\deg(v)]^2$

Now, Supp (G) = $[\deg (u)]^2$ + $[\deg(v)]^2$ + $\sum_{i=1}^{2n-2}$ $[\deg (v_i)]^2$

= 9 + 9 + $\sum_{i=1}^{2n-2}(4)$ = 18 + 4(2n -2) = 18 + 8n - 8 = 8n +10 = 8n + 10 = 2 (4n + 5).

Theorem:1.6 Let G = W_n(n > 3), then Supp (W_n) = (n - 1) (n + 8).

Proof .Let G = W_n be a Wheel graph with n vertices and (2n – 2) edges.

Here (n – 1) vertices are degree 3 and 1 domination vertex (Apex vertex) is degree (n – 1).

Now, Supp (G) = $\sum_{v\in V(G)}$ supp(v) = $\sum_{i=1}^{n}$ $[\deg (v_i)]^2$ = $\sum_{i=1}^{n-1}$ supp (v_i) +supp (Domination vertex)

= $\sum_{i=1}^{n-1}(9)$ +1 $(n – 1)^2$ = (n -1) 9 + $(n – 1)^2$= (n -1) (n -1 + 9) = (n - 1) (n + 8) = Supp (W_n) = (n - 1) (n + 8).

Observation ofW_n

1. W_n (n ≥ 5) has (n – 1) triangles.
2. W_4 has 4 triangles.

Theorem:1.7 For any connected graph G = $F_n$thenSupp (F_n) = (n – 2) (n + 9).

Proof .Let G = F_n ($n \geq 3$) be a Fan graph with 'n' vertices and (2n – 3) edges.
Here 2 vertices (end vertices of path P_{n-1})are degree 2, (n – 3) vertices are degree 3 and 1 domination vertex is degree (n – 1).NowSupp (G) = $\sum_{i=1}^{n}$ $[\deg(v_i)]^2$ = 2(4) + (n - 3) (9) + 1 $(n-1)^2$.

= 8 + 9n - 27 + n^2 - 2n + 1 = n^2 + 7n - 18 = Supp (F_n) = (n - 2) (n + 9).

Observation of F_n

1. F_n ($n \geq 3$) has odd number of edges
2. F_n ($n \geq 3$) has(n – 2) number of triangles.

Theorem:1.8 For any connected graphG = L_n(n > 4)then Supp (L_n) = 2(9n – 10).

Method - I

Proof: Let G = L_n be the Ladder graph with 2n vertices.
Let V (G) = { $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n$}, deg ($v_i$) = deg($u_i$) = 2 for i = 1&n.
and deg (v_i) = deg(u_i) = 3, for all i = 2,3,…,(n-1).

supp (v_1) = $\sum_{v \in N(v_1)} \deg(v)$ = deg (u_1) + deg (v_2) = 2 + 3 =5

Similarly, supp (v_n) = supp (u_1) = supp (u_n) = 5

supp (v_2) = $\sum_{v \in N(v_2)} \deg(v)$ = deg (u_2) + deg (v_1) + deg (v_3) = 3 +2 +3 = 8

Similarly, supp (u_2) = supp (u_{n-1}) = supp (v_{n-1}) = 8

supp (v_i) = $\sum_{v \in N(v_i)} \deg(v)$ = deg (u_i) + deg (v_{i-1}) +deg (v_{i+1}),for i = 3, 4,…,(n-2) = 3 + 3 +3 = 9.

Similarly,supp (u_i) = 9, i = 3, 4, 5,…, (n-2).

Now,Supp (G) = $\sum_{v \in V(G)}$ supp(v)= supp (v_1) + supp (u_1) + supp (v_n) + supp (u_n) + supp (v_2) + supp (u_2) + supp (v_{n-1}) + supp (u_{n-1}) + $\sum_{i=3}^{n-2}$ supp (v_i) + $\sum_{i=3}^{n-2}$ supp (u_i)= 5 +5+5 + 5 + 8 + 8 + 8 + 8 + (n-4)9+ (n-4)9

= 20 +32 +18(n -4)= 52 +18n –7 = 18n -20 = Supp (G) = 2(9n - 10).

Method - II

Let the vertices of L_n be $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n$ The number of vertices of L_n are 2n. Define Supp: V (Ln) → {5, 8, 9} by

$$\text{Supp}(v_i) = \text{supp}(u_i) = \begin{Bmatrix} 5 & for\, i = 1\&n \\ 8 & for\, i = 2\&(n-1) \\ 9 & for\, i = 3,4,\ldots,(n-2) \end{Bmatrix}$$

Supp (L_n) = $\sum_{v \in V(L_n)} \deg(v)$ = supp (v_1) + supp (u_1) + supp (v_n) + supp (u_n) + supp (v_2) + supp (u_2) + supp (v_{n-1}) + supp(u_{n-1})+ $\sum_{i=3}^{n-2}$ supp (v_i) + $\sum_{i=3}^{n-2}$ supp (u_i)= 5 +5 +5 +5 +8 +8+8+8+ (n -4)9 + (n -4)9.
= 20 +32 +18 (n - 4) = 52 +18n – 72 = 18n – 20 = 2(9n - 10).

Observation of L_n

a) Supp (L_2) = 16. b) Supp (L_3) = 34. c) Supp (L_4) = 52.

Theorem:1.9. For any connected graph G = P_m x P_n (m & n ≥ 3) be a graph with mn vertices and (2mn- m – n) edges, then Supp [P_m x P_n] = 2(8mn -7m - 7n+ 4).

Proof. In a graph G = P_m x $P_n$has4 corner vertices are degree 2, 2(m - 2) + 2(n - 2) boundary vertices (except four corners) are degree 3, and (mn - 2m - 2n + 4) vertices are degree 4.

Now,Supp (G) = $\sum_{v \in V(G)}$ Supp(v) = $\sum_{i=1}^{mn}$ $[\deg(v_i)]^2$=4(4) + (2m + 2n - 8) (9) + (mn - 2m - 2n + 4)(16) = 16 + 18m + 18n - 72 + 16mn - 32m - 32n + 64= 16mn - 14m -14n+8= 2(8mn -7m - 7n+ 4) = Supp [P_m x P_n] = 2(8mn -7m - 7n+ 4).

Observation of Supp [P_m x P_n]

1.If either m = 2 and n ≥ 2 or m ≥ 2 andn = 2, then P_m x P_n= L_n
2. Supp [P_2 x P_n] = 18n – 20 = 2(9n – 10).
3. Supp [P_m x P_2] = 18m – 20 =2(9m – 10).
4. In G = P_m x P_n (m & n ≥ 3) contains (m -1) (n -1) number of C_4 are formed.

Theorem:1.10 For any connected graph G = P_m x C_n (m & n ≥ 3) be a graph with mn vertices and (2mn –n) edges, then Supp [P_mxC_n] = 2n (8m– 7).

Proof. LetG = P_m x $C_n$having 2n vertices (vertices of inner and outer Cycles of C_n) are degree 3, and (mn - 2n) = n (m -2) vertices are degree 4.

Now,Supp (G) = $\sum_{v \in V(G)}$ Supp(v) = $\sum_{i=1}^{mn}$ $[\deg(v_i)]^2$ = 2n (9) + (mn - 2n)(16) = 18n + 16mn - 32n= 16mn -14n
= 2n(8m-7) = Supp [P_mxC_n] = 2n(8m-7).

Observation of G = P_m x C_n.

1. If m = 2 and n ≥3, then P_2 x C_n= C_n + C_n,
2. Supp [P_2 x C_n] = 18n
3. P_m x C_n = C_n+ C_n + C_n+,…m times…+C_n .

III. CONCLUSION

This paper exhibits new approach of connected network topology.

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