Coefficient Inequalities of a Subclass of Analytic Functions Defined Using q-Differential Operator

K . R. Karthikeyan, M. Musthafa Ibrahim, K. Srinivasan

Abstract: ***By making use of a q-analogue of the Sălăgean differential operator, we define a new subclass of analytic functions. We obtain Fekete-Szegö inequality and coefficient inequality of certain class satisfying bi-convex criteria. Fekete-Szegö inequality of several well-known classes are obtained as special cases from our results. Applications of the result are also obtained on the class defined by convolution.***

Keywords: Fekete-Szegö inequalities and p-valent Functions. [2010]30C45.

I. INTRODUCTION, DEFINITIONS AND PRELIMINARIES

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad (1.1)$$

analytic in the open unit disk $\mathcal{U}$. Also we let $\mathcal{S}$ denote the class of all function in $\mathcal{A}$ which are univalent in $\mathcal{U}$. The well known example in this class is the Koebe function $k(z)$, defined by

$$k(z) = \frac{z}{(z-1)^2} = z + \sum_{n=2}^{\infty} n\, z^n.$$

The Bieberbach conjecture about the coefficient of the univalent functions in the unit disk was formulated by Bieberbach [2] in the year 1916. The conjecture states that for every function $f \in S$ given by (1.1), we have $|a_n| \leq n$, for every n. Strictly inequality holds for all n unless f is the Koebe function or one of its rotation. For many years, this conjecture remained as a challenge to mathematicians. After the proof of $|a_3| \leq 3$ by Lowner in 1923, Fekete-Szegö [7] surprised the mathematicians with the complicated inequality

$$|a_3 - \mu\, a_2^2| \leq 1 + 2\exp\left(\frac{-2\mu}{1-\mu}\right),$$

which holds good for all values $0 \leq \mu \leq 1$. Note that this inequality region was thoroughly investigated by Schaefer and Spencer [16].

Revised Manuscript Received on May 07, 2019.

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For a class functions in $\mathcal{A}$ and a real (or more generally complex) number μ, the Fekete-Szegö problem is all about finding the best possible constant $C(\mu)$ so that $|a_3 - \mu\, a_2^2| \leq C(\mu)$ for every function in $\mathcal{A}$. Many papers have been devoted to this problem see [3, 4, 5, 8, 11, 10]. In this paper, we obtain the estimates of a_2 , a_3 and also the Fekete-Szegö inequality for a subclass of functions of complex order defined using subordination.

It is well known that every function $f \in S$ has a function f^{-1}, defined by

$$f^{-1}[f(z)] = z\,;\,(z \in \mathcal{U})$$

and $f[f^{-1}(w)] = w\,; \quad (\,|w| < r_0(f)\,;\; r_0 f \geq \frac{1}{4}).$

In fact, the inverse function f^{-1} is given by

$$f^{-1}(w) = w - a_2 w^2 + (2a_2 w^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots. \qquad (1.2)$$

A function $f \in \mathcal{A}$ is said to be biunivalent in $\mathcal{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathcal{U}$. Let f and g be analytic in the open unit desk $\mathcal{U}$. The function f is subordinate to g written as $f \prec g$ in $\mathcal{U}$, if there exist a function w analytic in $\mathcal{U}$ with $w(0) = 0$ and $|w(z)| < 1$; $(z \in \mathcal{U})$ such that $f(z) = g(w(z))$, $(z \in \mathcal{U})$.

Let $\mathcal{S}^*(\alpha)$ and $\mathcal{C}(\alpha)$ denote the well known subclasses of the univalent function class $\mathcal{S}$ which are respectively defined as follows.

$$S^*(\alpha) = \left\{f \in \mathcal{A} :\; Re\left(\frac{z\,f'(z)}{f(z)}\right) > \alpha\,;\;\; 0 \leq \alpha < 1\right\}$$

and

$$C(\alpha) = \left\{f \in \mathcal{A} :\; Re\left(1 + \frac{z\,f''(z)}{f(z)}\right) > \alpha\,;\;\; 0 \leq \alpha < 1\right\}.$$

Using Alexander transform, it follows that $f(z) \in C(\alpha)$ if and only if $zf'(z) \in \mathcal{S}^*(\alpha)$.

q-calculus has been studied by various authors due to the fact that applications of basic Gaussian hypergeometric function to the diverse subjects of combinatorics, quantum theory, number theory, statistical mechanics, are constantly being uncovered.

The q-difference operator denoted as $D_q f(z)$ is defined by

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$D_q f(z) = \frac{f(z)-f(qz)}{z(1-q)}, \quad (f \in \mathcal{A}, z \in \mathcal{U} - \{0\}),$

and $D_q f(0) = f'(0)$, where $q \in (0,1)$. It can be easily seen that $D_q f(z) \to f'(z)$ as $q \to 1^-$. If $f(z)$ is of the form (1.1), a simple computation yields

$$D_q f(z) = 1 + \sum_{n=2}^{\infty} \frac{1-q^n}{1-q} a_n z^n, \quad (z \in \mathcal{U}). \qquad (1.3)$$

The inverse function of (1.3) is given by

$$D_q g(w) = 1 - (1+q)\, a_2 w - (1+q+q^2)\, a_3 w^2 - \cdots.$$

The q-analogue of Sălăgean differential operator (see [15]) $R_q^m f(z) : \mathcal{A} \to \mathcal{A}$ for $m \in N$, is formed as follows.

$$R_q^0 f(z) = f(z)$$

$$R_q^1 f(z) = z(D_q f(z))$$

$$\vdots$$

$$R_q^m f(z) = R_q^1\left(R_q^{m-1} f(z)\right).$$

Motivated by [6], we define the following.

Definition 1.1 *Let $\phi(z)$ be analytic in $\mathcal{U}$ with $\phi(0) = 1$. A function $f(z) \in \mathcal{A}$ is said to be in the class $C_q^m(\lambda;\phi)$ satisfies the differential inequality if and only if*

$$\left[\frac{(1-\lambda)\, z\, (R_q^m f(z))^{(q)} + \lambda\, z (R_q^{m+k} f(z))^{(q)}}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)}\right] \prec \phi(z) \qquad (1.4)$$

$$(z \in \mathcal{U}; m, k \in M_0;\ \lambda \geq 0)$$

where (q) denotes the q-derivative of f as defined in (1.3).

Remark 1.1 *We note that by specializing m, λ, q, k in the function class $C_q^m(\lambda;\phi)$, we obtain several well-known and new subclasses of analytic functions. Here we list a few of them:*

1. If we let $q \to 1$ then the $C_q^m(\lambda;\phi)$ reduces to the class $G_{\lambda,n,m}(\phi)$, studied by Darwish, Lashin and Alnayyef [6].
2. If we let $q \to 1$ and $k = 1$ then the $C_q^m(\lambda;\phi)$ reduces to the class $M_{\alpha,n}(\phi)$, studied by Orhan and Gunes [14].

Lemma 1.1 *[12] Let the function $\phi(z)$ given by $\phi(z) = \sum_{n=1}^{\infty} \mathcal{B}_n z^n$ be convex in $\mathcal{U}$. If $h(z) \prec \phi(z)$, $(z \in U)$, then $|h_n| \leq |\mathcal{B}_1|, n \in \mathcal{N} = \{1,2,3,\ldots\}$.*

Lemma 1.2 *[10]If $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$ is a function with positive real part in $\mathcal{U}$ and μ is a complex number, then*

$$|c_2 - \mu c_1^2| \leq 2\max\{1; |2\mu - 1|\}.$$

The result is sharp for the functions given by

$$p(z) = \frac{1+z^2}{1-z^2} \text{ and } p(z) = \frac{1+z}{1-z}.$$

II. FEKETE-SZEGÖ INEQUALITY

We begin with the following result.

Theorem 2.1

Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots (B_1 \neq 0)$.

If $f(z) \in C_q^m(\lambda;\phi)$, then

$$\left|a_3 - \mu\, a_2^2\right| \leq \frac{|B_1|}{2q\,(q+1)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)} \times$$

$$\max\left\{1, \left|\frac{B_2}{B_1} + \left(\frac{(1+q)^{2m-1}(1-\lambda+\lambda(1+q)^k)^2}{(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)} - \mu\right)\frac{B_1(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k}{2q(1+q)^{2m-1}(1-\lambda+\lambda(1+q)^k)^2}\right|\right\}.$$

Proof. Let $f(z) \in C_q^m(\gamma;\phi)$, then there exists a Schwarz function $w(z)$ in $\mathcal{U}$ with $w(0) = 0$ and $|w(z)| < 1$ in $\mathcal{U}$ such that

$$\left[\frac{(1-\lambda)\, z\, (R_q^m f(z))^q + \lambda\, z (R_q^{m+k} f(z))^q}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)}\right] = \phi(w(z)). \qquad (2.1)$$

Define the function $f(z)$ by

$$p(z) = \frac{1+w(z))}{1-w(z)} = 1 + c_1 z + c_2 z^2 + \ldots,\ z \in \mathcal{U}. \qquad (2.2)$$

Since $w(z)$ is Schwarz function, it can be easily seen that

$$Re\left(\frac{1+w(z)}{1-w(z)}\right) > 0.$$

Then $Re(p(z)) > 0$ and $p(0) = 1$. Therefore

$$\phi(w(z)) = \phi\left(\frac{p(z)-1}{p(z)+1}\right)$$

$$= \phi\left(\frac{1}{2}\left[c_1 z + \left(c_2 - \frac{c_1^2}{2}\right) z^2 + \left(c_3 - c_1 c_2 \frac{c_1^3}{2}\right) z^3 + \cdots\right]\right)$$

$$= 1 + \frac{1}{2} B_1 c_1 z + \left[\frac{1}{2} B_1 \left(c_2 - \frac{c_1^2}{2}\right) + \frac{1}{4} B_2 c_1^2\right] z^2 + \cdots. \qquad (2.3)$$

Now by substituting (2.1) in (2.3),

$$\left[\frac{(1-\lambda)\, z\, (R_q^m f(z))^q + \lambda\, z (R_q^{m+k} f(z))^q}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)}\right]$$

$$= 1 + \frac{1}{2} B_1 c_1 z + \left[\frac{1}{2} B_1 \left(c_2 - \frac{c_1^2}{2}\right) + \frac{1}{4} B_2 c_1^2\right] z^2 + \cdots.$$

From this equation, we get

$$q(1+q)^m(1-\lambda+\lambda(1+q)^k) a_2 = \frac{1}{2} B_1 c_1$$

and

$$q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k) a_3 - q(1+q)^{2m}(1-\lambda+\lambda(1+q)^k)^2 a_2^2 = \frac{B_1 c_2}{2} - \frac{B_1 c_1^2}{4} + \frac{B_2 c_1^2}{4}.$$

Or, equivalently

$$a_2 = \frac{B_1 c_1}{2q(1+q)^m(1-\lambda+\lambda(1+q)^k)},$$

$$a_3 = \frac{\left(\frac{B_1 c_2}{2} - \frac{B_1 c_1^2}{4} + \frac{B_2 c_1^2}{4}\right) + q(1+q)^{2m}(1-\lambda+\lambda(1+q)^k)^2 a_2^2}{q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)}.$$

On computation, we have

$$a_3 - \mu\, a_2^2 = \frac{\frac{B_1 c_2}{2} - \frac{B_1 c_1^2}{4} + \frac{B_2 c_1^2}{4}}{q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)} + \left[\frac{(1+q)^{2m-1}(1-\lambda+\lambda(1+q)^k)^2}{(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)} - \mu\right] \frac{B_1^2 c_1^2}{4q^2(1+q)^{2m}(1-\lambda+\lambda(1+q)^k)^2}. \qquad (2.4)$$

Therefore,

$$a_3 - \mu\, a_2^2 = \frac{B_1}{2q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)} [c_2 - \delta c_1^2], \qquad (2.5)$$

where

$$\delta = \frac{1}{2} - \frac{B_2}{2B_1} - \left[\frac{(1+q)^{2m-1}(1-\lambda+\lambda(1+q)^k)^2}{(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)} - \mu\right] \times \frac{B_1(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)}{2q(1+q)^{2m-1}(1-\lambda+\lambda(1+q)^k)^2} \quad (2.6)$$

On rearranging the terms and taking modulus on both sides, the result follows on the application of the Lemma 1.2. The result is sharp for the functions

$$\left[\frac{(1-\lambda)\, z\,(R_q^m f(z))^{(q)} + \lambda\, z (R_q^{m+k} f(z))^{(q)}}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)}\right] = \phi(z^2)$$

and

$$\left[\frac{(1-\lambda)\, z\,(R_q^m f(z))^{(q)} + \lambda\, z (R_q^{m+k} f(z))^{(q)}}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)}\right] = \phi(z).$$

This completes the proof of the theorem.

If we let $\lambda = m = 0$ and $q \to 1$ in Theorem 2.1, we have

Corollary 2.2 *(see [13]) Let $f(z) \in \mathcal{A}$ satisfy the inequality*

$$\alpha < Re\left\{\frac{zf'(z)}{f(z)}\right\} < \beta, \quad (2.7)$$

then

$$|a_3 - \mu a_2^2| \le \frac{(\beta-\alpha)}{2\pi} \sin\left[\frac{\pi(1-\alpha)}{(\beta-\alpha)}\right] \times \max\left\{1, \left|\frac{B_2}{B_1} + \frac{B_1}{2}(1-2\mu)\right|\right\},$$

where

$$B_n = \frac{\beta-\alpha}{n\pi}\, i\left[1 - e^{2n\pi i((1-\alpha)/(\beta-\alpha))}\right].$$

Proof. Let

$$\phi(z) = 1 + \frac{\beta-\alpha}{\pi} i \log\left(\frac{1-e^{2\pi i((1-\alpha)/(\beta-\alpha))} z}{1-z}\right).$$

Clearly, it can be seen that $\phi(z)$ maps $\mathcal{U}$ onto a convex domain conformally and is of the form

$h(z) = 1 + \sum_{n=1}^{\infty} B_n z^n$

where $B_n = \frac{\beta-\alpha}{n\pi}\, i\left(1 - e^{2n\pi i((1-\alpha)/(\beta-\alpha))}\right)$. From the equivalent subordination condition proved by Kuroki and Owa in [9], the inequality (2.7) can be rewritten in the form

$$\frac{zf'(z)}{f(z)} \prec \phi(z).$$

Following the steps as in Theorem 2.1, we get the desired result.

Corollary 2.3

(see[6]) Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots (B_1 \ne 0)$. If $f \in \mathcal{A}$ satisfies the analytic criterion,

$$\frac{(1-\lambda)\, z\,(D^m f(z))' + \lambda\, z (D^{m+k} f(z))'}{(1-\lambda) D^m f(z) + \lambda D^{m+k} f(z)} \prec \phi(z),$$

$(z \in \mathcal{U}; m, k \in M_0;\ \lambda \ge 0)$

then

$$|a_3 - \mu\, a_2^2| \le \frac{|B_1|}{4\, 3^m\, (1-\lambda+3^k\lambda)^k} \times \max\left(1, \left|\frac{B_2}{B_1} + \frac{B_1}{2} - \frac{\mu B_1 3^m (1-\lambda+3^k\lambda)}{2^{2m}(1-\lambda+2^k\lambda)^2}\right|\right).$$

The result is sharp by taking $q \to 1$.

Corollary 2.4

(see [14]) Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots (B_1 \ne 0)$. If $f \in \mathcal{A}$ satisfies the analytic criterion,

$$\frac{(1-\lambda)\, z\,(D^m f(z))' + \lambda\, z (D^{m+1} f(z))'}{(1-\lambda) D^m f(z) + \lambda D^{m+1} f(z)} \prec \phi(z),$$

$(z \in \mathcal{U}; m, k \in M_0;\ \lambda \ge 0),$

then

$$|a_3 - \mu\, a_2^2| \le \frac{|B_1|}{4\, 3^m\, (1+2\lambda)} \times \max\left(1, \left|\frac{B_2}{B_1} + \frac{B_1}{2} - \frac{\mu B_1 3^m (1+2\lambda)}{2^{2m}(1+\lambda)^2}\right|\right).$$

The result is sharp by taking $q \to 1$ and $k = 1$.

III. COEFFICIENT INEQUALITIES OF BIUNIVALENT FUNCTIONS

In this section, we obtain coefficient estimates of a biconvex functions.

Theorem 3.1

Let $f(z)$ be of the form (1.1) and let g denote the inverse function of f given by (1.4) . Suppose that f and g satisfies the following analytic condition

$$\frac{(1-\lambda)\, z\,(R_q^m f(z))^{(q)} + \lambda\, z (R_q^{m+k} f(z))^{(q)}}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)} \prec \phi(z)$$

$$\frac{(1-\lambda)\, w(R_q^m g(z))^{(q)} + \lambda\, w (R_q^{m+k} g(z))^{(q)}}{(1-\lambda) R_q^m g(z) + \lambda R_q^{m+k} g(z)} \prec \phi(w) \quad (3.1)$$

then

$$|a_2| \le \sqrt{\frac{|B_1|}{|2(1+q)^{2m}(2+q)(1-\lambda+\lambda(1+q)^{2k})|}}.$$

and

$$|a_3| \le \sqrt{\frac{|B_1|}{|q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)|}}.$$

Proof. Let g denote the inverse function of f to $\mathcal{U}$. It follows from (3.1) that there exists functions $p(z); q(z) \in \mathcal{P}$ (the class of functions with positive real part), such that

$$\frac{(1-\lambda)\, z\,(R_q^m f(z))^{(q)} + \lambda\, z (R_q^{m+k} f(z))^{(q)}}{(1-\lambda) R_q^m f(z) + \lambda R_q^{m+k} f(z)} \prec \phi(z). \quad (3.2)$$

$$\frac{(1-\lambda)\, w(R_q^m g(z))^{(q)} + \lambda\, w (R_q^{m+k} g(z))^{(q)}}{(1-\lambda) R_q^m g(z) + \lambda R_q^{m+k} g(z)} \prec \phi(w), \quad (3.3)$$

where $p(z) \prec h(z)$ and $q(w) \prec g(w)$ have the forms

$p(z) = 1 + p_1 z + p_2 z^2 + \cdots.$

and

$q(w) = 1 + q_1 w + q_2 w^2 + \cdots.$

respectively. It follows from (3.2) and (3.3), we deduce

$$q(1+q)^m(1-\lambda+\lambda(1+q)^k)a_2 = p_1 \quad (3.4)$$

$$q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)a_3 - q(1+q)^{2m}(1-\lambda+\lambda(1+q)^k)^2 a_2^2 = p_2 \quad (3.5)$$

and

$$q(1+q)^m(1-\lambda+\lambda(1+q)^k)a_2 = q_1 \quad (3.6)$$

$$2(1+q)^{2m} q(1+q)(1-\lambda+\lambda(1+q+q^2))a_2^2 - q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)a_3 + q(1+q)^{2m}(1-\lambda+\lambda(1+q)^k)^2 a_2^2 = q_2 \quad (3.7)$$

From (3.4) and (3.6) we obtain

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$p_1 = -q_1.$

By adding (3.5) and (3.7), we get

$$p_1 + q_1 = 2(1+q)^{2m} q(1+q)(1-\lambda+\lambda(1+q+q^2))a_2^2. \quad (3.8)$$

Since $p, q \in h(\mathcal{U})$, applying Lemma1.1, we have

$$|p_m| = |\frac{p^m(0)}{m!}| \leq |\mathcal{B}_1|, m \in \mathcal{N} \quad (3.9)$$

and

$$|q_m| = |\frac{q^m(0)}{m!}| \leq |\mathcal{B}_1|, m \in \mathcal{N}. \quad (3.10)$$

Applying (3.9), (3.10) and Lemma1.1 for the coefficients p_1, p_2, q_1 and $q_2,$ we readily get

$$|a_2| \leq \sqrt{\frac{|B_1|}{|2(1+q)^{2m}(2+q)(1-\lambda+\lambda(1+q)^{2k})|}}$$

Subtracting (3.5) from (3.7) we have

$$p_2 - q_2 = 2q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)a_3 \quad (3.11)$$

Applying (3.9), (3.10) and Lemma1.1 once again for the coefficients p_1, p_2, q_1 and $q_2,$ we readily get

$$|a_3| \leq \sqrt{\frac{|B_1|}{|q(1+q)(1+q+q^2)^m(1-\lambda+\lambda(1+q+q^2)^k)|}}$$

This completes the proof of Theroem 3.1.

Remark 3.1 We note that Theorem3.1 can be reduced to various other well-known and new results (see Altınkaya and Yalçın[1]).

IV. CONCLUSION

This paper define a new subclass of analytic functions to obtain Fekete-Szego Inequality.

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