SSP Structure on the Cartesian Product S_{m+1} x P_n

R. Mary Jeya Jothi, R, Revathi

Abstract: ***In G, if there exists a minimal dominating set in every induced subgraph H which meets all the cliques (maximal), then G is known as Super Strongly Perfect (SSP). This paper investigates the construction of Cartesian product of SSP graphs like star, path graphs. With this investigation, it is analysed the structural properties of stacked book graph with respect to its diameter and domination numbers.***

Keywords: Cartesian product, minimal dominating set, SSP and stacked book graphs.

I. INTRODUCTION

Multilevel Cartesian products of graphs are denoted by the notation G × H [1]. Here, it is considered the Cartesian product between star and path graphs. All graphs of this paper are considered as connected, finite, simple and undirected. A complete bipartite graph $K_{1,t}$ is called a star where $t \geq 2$. The Cartesian product $G_1 x G_2$ of two graphs $G_1 = (U_1, E_1)$ and $G_2 = (U_2, E_2)$ (where $U_1 \cap U_2 = \varphi$) has U $(G_1 x G_2) = U_1$ x U_2 and $u = (a_1, b_1)$ and $v = (a_2, b_2)$ are connected if $a_1 = a_2$ and b_1 is adjacent to b_2 in G_2 or a_1 is adjacent to a_2 in G_1 and $b_1 = b_2$. $G_1 x G_2$ has $|U_1||U_2|$ vertices and $|U_1||E(G_2)| + |U_2||E(G_1)|$ edges. A subset of V(G) is called a clique if all of its vertices are mutually connected. If every vertex of V(G) is either in D (subset of V(G)) or connected to a vertex in D, then D is called a dominating set. If a dominating set M of V is not a proper subset of any other dominating set of V(G), then it is called a Minimal Dominating Set. The number of vertices of a minimum dominating set is called the domination number and it is denoted by γ(G). $\gamma(\bar{G})$ is the domination number of the complement $\bar{G}$ of the graph G. The length of the longest shortest path of a graph G is called a diameter of G which is denoted by diameter (G) or diam (G). Some classes of SSP graphs have been characterized already [2, 3]. Along this line of thought, Cartesian product of some SSP graphs together with their domination parameters are analysed in this paper.

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R. Mary Jeya Jothi, Department of Mathematics, Sathyabama Institute of Science and Technology, Chennai, Tamilnadu, India

R. Revathi, Department of Mathematics, Sathyabama Institute of Science and Technology, Chennai, Tamilnadu, India

II. SUPER STRONGLY PERFECT GRAPH

In G, if there exists a minimal dominating set in every induced subgraph H which intersects all cliques (maximal), then the graph G is called SSP. SSP and non-SSP graphs are given below in figure 1, 2.

Illustration 1

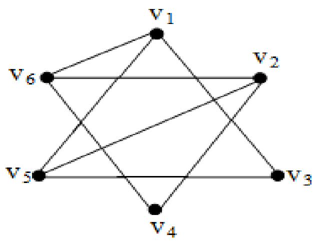

Fig. 1 SSP graph

Illustration 2

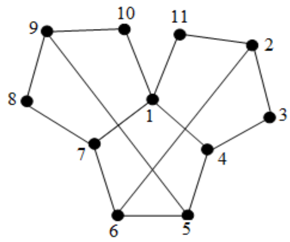

Fig. 2 Non-SSP graph

III. STACKED BOOK GRAPH

A Stacked Book graph $B_{m,n}$ of order (m, n) is defined as the Cartesian product $S_{m+1} \times P_n$. From the construction of the stacked book graph, it is clear that the graph is 2 colourable [4]. Hence every stacked book graph is bipartite. The upcoming Figure 3 illustrates the structure of stacked book graph $B_{3,3}$.

Illustration 3

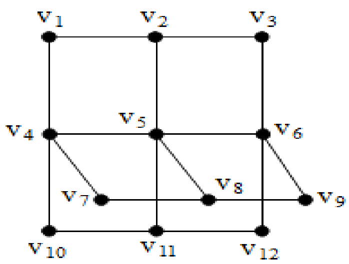

Fig. 3 $B_{3,3}$

Theorem

Every stacked book graph is SSP.

Proof:

Let G be a stacked book graph.

⇒ G is the Cartesian product $S_{m+1} \times P_n$

⇒ G is bipartite (2-colourable) with K_2.

⇒G has a partition (V_1, V_2) in which the vertices of V_1 are non-adjacent with the vertices of V_2.

⇒ The non-connected vertices from either V_1 or V_2 intersects every maximal cliques.

⇒ G is SSP

Theorem

Every stacked book graph has $\gamma(\bar{G})$ = 2 if and only if diameter (G) ≥ 3.

Proof:

Consider a stacked book graph G.

Consider $\gamma(\bar{G}) = 2$.

⇒$\bar{G}$ provides a minimum dominating set S = {u_1, u_2} with cardinality 2.

⇒ All the vertices of $\bar{G}$ are either connected to u_1 or u_2.

⇒ G does not have a vertex which is connected to u_1 and u_2.

⇒ diameter (u_1, u_2) ≥ 3.

⇒ diameter (G) ≥ 3.

Now, Consider diameter (G) ≥ 3.

⇒ G has at least two vertices u_1, u_2 for which diameter (u_1, u_2) ≥ 3.

⇒ G does not have a vertex which is connected to u_1 and u_2.

⇒ All the vertices of $\bar{G}$ are either connected to u_1 or u_2.

⇒ $\bar{G}$ has a dominating set {u_1, u_2}.

Also, there is no isolated vertex in $\bar{G}$, $\gamma(\bar{G}) \neq 1$.

⇒ $\gamma(\bar{G}) = 2$.

Theorem

Every stacked book graph has diameter ($\bar{G}$) ≤ 3 if and only if domination number γ(G) >1.

Proof:

Consider a stacked book graph G.

Suppose diameter ($\bar{G}$) > 3.

⇒$\bar{G}$ has atleast two vertices u_1, u_2 for which d (u_1, u_2) > 3.

⇒ $\bar{G}$ does not have a vertex which is connected to u_1 and u_2.

⇒ All vertices are either connected to u_1 or u_2 .

⇒ G has {u_1, u_2} as a dominating set.

⇒γ(G) < 2, a contradiction.

⇒ diameter ($\bar{G}$) ≤ 3.

Now, consider diameter ($\bar{G}$) ≤ 3.

If γ (G) =1.

⇒ G has a vertex u such that {u} is a dominating set.

⇒In $\bar{G}$, u will be an isolated vertex.

⇒ diameter ($\bar{G}$) does not exist, a contradiction.

⇒ γ(G) >1.

Theorem

Every stacked book graph has $\gamma(\bar{G})$ = 2 if and only if domination number γ(G) >1.

Proof:

Consider a stacked book graph G.

Now, γ(G) >1.

⇒ G does not have a vertex u for which all the vertices are connected to u.

As G is bipartite, G has a bipartite set (V_1, V_2) such that the vertices within V_1 and V_2 are not connected.

⇒ In $\bar{G}$, All the vertices of V_1 and V_2 are mutually connected and also $\bar{G}$ has at least one connection between some two vertices of V_1 and V_2 such that $u_1 \in V_1$, $u_2 \in V_2$.

Hence $\gamma(\bar{G}) = 2$.

Now, consider, $\gamma(\bar{G}) = 2$.

⇒ $\bar{G}$ has {u_1, u_2} as a minimum dominating set.

⇒ G does not have a vertex for which both v_1 and v_2 are adjacent to that vertex.

⇒ γ (G) >1.

Proposition

Every stacked book graph has the following structural properties.

1) Number of maximal cliques (K_2) in G is m(2n-1)+n-1.
2) G is 2-colourable.
3) a) If n is even, then in G, there exists a dominating set (minimal) with cardinality $\left\lfloor \frac{n(m+1)}{2} \right\rfloor$ vertices.

b) If n is odd, then then in G, there is a dominating set (minimal) of cardinality $\left\lceil \frac{n}{2} \right\rceil m + \left\lfloor \frac{n}{2} \right\rfloor$ (or) $\left\lfloor \frac{n}{2} \right\rfloor m + \left\lceil \frac{n}{2} \right\rceil$ vertices.

Proof:

1) Consider a stacked book graph G.

⇒G is the product (Cartesian) $S_{m+1} \times P_n$

⇒G has (m+1)(n-1)+mn = mn+mn -m-1+n

=2mn-1-m+n

= n-1+ m(2n-1) edges.

⇒ Number of maximal cliques (K_2) in G is n-1+ m(2n-1).

2) Consider a stacked book graph G.

⇒ By the construction itself, G is 2-colourable (as G is bipartite).

3) Consider a stacked book graph.

⇒G is constructed from Cartesian product $S_{m+1} \times P_n$

⇒ By the second part, G is 2-colourable.

⇒ G has a bipartition {V_1, V_2} for which all the vertex of V_1 and V_2 are coloured by colours 1 and 2.

⇒In G, there is a dominating set (minimal) with cardinality $|V_1|$ or $|V_2|$.

⇒If n is even, then $|V_1|$ has $\left\lfloor \frac{n}{2} \right\rfloor (m+1)$ vertices and $|V_2|$ has $\left\lfloor \frac{n}{2} \right\rfloor (m+1)$ vertices.

⇒ G has a dominating set (minimal) of $\left\lfloor \frac{n}{2} \right\rfloor (m+1)$ vertices.

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If n is odd, then $|V_1|$ has $\left\lceil \frac{n}{2} \right\rceil m + \left\lfloor \frac{n}{2} \right\rfloor$ and $|V_2|$ has $\left\lfloor \frac{n}{2} \right\rfloor m + \left\lceil \frac{n}{2} \right\rceil$ vertices.

$\Rightarrow$ G has a dominating set (minimal) of $\left\lceil \frac{n}{2} \right\rceil m + \left\lfloor \frac{n}{2} \right\rfloor$ (or) $\left\lfloor \frac{n}{2} \right\rfloor m + \left\lceil \frac{n}{2} \right\rceil$ vertices.

Figure 4 gives the illustration of proposition 4.5.

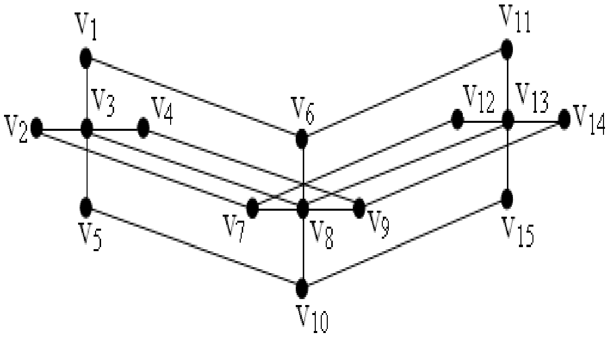

Fig. 4 $B_{4,3}$

Here,

1) $B_{4,3}$ has 22 maximal cliques, each of which is a K_2.
2) $B_{4,3}$ has a dominating set (minimal) with cardinality 6 (or) 9 vertices.

IV. CONCLUSION

It is analysed the Cartesian product of some SSP graphs like star, path graphs. It is given the structural construction of SSP graph on stacked book graphs. From this analysation, one can investigate any arbitrary graph.

REFERENCES

1. Aurenhammer, F.; Hagauer, J.; Imrich, W., “Cartesian graph factorization at logarithmic cost per edge”. Comput. Complexity 1992, vol. 2. no.4: 331-349.
2. R. Mary Jeya Jothi, “A discussion on SSP Structure of Pan, Helm and Crown Graphs”, ARPN Journal of Engineering and Applied Sciences, 2015, vol. 10, no. 9, pp. 4115 - 4121.
3. R. Mary Jeya Jothi, etal, “Super Strongly Perfect Graphs in the Selection of Cluster Heads in Wireless Sensor Networks”, Applied Mechanics and Materials, 2014, vol. 591, pp. 206-210.
4. R. Revathi and S.Ganesh, “Modular Multiplicative Divisor Labeling of k-Multilevel Corona Related Graphs”, Journal of Computational and Theoretical Nanoscience, 2016, vol. 13, pp.7634-7639.

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