Paired Equitable Domination in Inflated Graphs

A.Meenakshi

***Abstract:** Let G be a connected graph. A equitable dominating set U of a connected graph G is called the paired equitable dominating set if U dominates G and the induced sub graph of U has a perfect matching. The minimum cardinality of paired equitable dominating set is called paired equitable domination number of G and is denoted by $\gamma_{pr}{}^{e}(G)$. The inflation graph G_I is obtained from a graph G by modifying every vertex x of degree d(x) by a clique $K_{d(x)}$. In this paper we study the paired equitable domination number for some inflated graphs and arrive with few general results.*

***Keywords:** Paired equitable domination, inflated graph, dominating set and domination number.*

I. INTRODUCTION

Let G be a simple undirected graph. Set of vertices is denoted by $V(G)$ and set of edges by $E(G)$. Order of G is denoted by $|G|$. The degree of a vertex is denoted by d(v) and the closed neighborhood of a vertex v is $N[v] = N(v) \cup \{v\}$. For further graph theoretic terminology refer [1].

A set S is a subset of V is said to be dominating if for every vertex v of V-S is dominated by at least one vertex of S. The domination number of a graph G is denoted by $\gamma(G)$, which is the minimum cardinality of the set S. A set S is a subset of V is said to be total dominating, if every vertex of V is adjacent to at least one vertex of S. The total domination number of a graph G is denoted by $\gamma_t(G)$, which is the minimum cardinality of the set S. A set S is a subset of V is said to be paired dominating if the induced sub graph of S has a perfect matching. The induced sub graph of S is denoted by $\langle S \rangle$. The paired domination number of a graph G is denoted by $\gamma_{pr}(G)$, which is the minimum cardinality of the set S. This concept was introduced by Haynes et al. [5].

More practical applications are developed from domination concepts. Among this one of the concept named equitable domination emerges from the practical application. Same status of pupils combines each other easily in their profession. To execute such type of applications, Prof. Sampath kumar initiated the study of equitable domination parameter and further deliberated by V. Swaminathan et al [4].

II. INFLATED GRAPH

We generally follow the inflated graphs terminology from [2]. Let G be a connected graph with no isolated vertices. The inflation or inflated graph G_I of a graph G is obtained as follows; every vertex a_i of degree $d(a_i)$ of G is modified by a clique $A_i \cong K_{d(a_i)}$ and every edge y_iy_j of G is modified by an edge ab in such a way that $a \in A_i, b \in Y_j,$ and two different edges of G are modified by non adjacent edges of G_I.

2.1. Equitable domination number [4]

Let G be a graph. A subset U of V is called an equitable dominating set of a graph G if for every vertex a of V-U, there exists a vertex b of U such that $ab \in E(G)$ and $|\deg(a) - \deg(b)| \le 1$, where deg(a) is the degree of a and deg(b) is the degree of b in G. The equitable domination number of a graph G is denoted by $\gamma^e(G)$, which is the minimum cardinality of the set U. The corresponding set is denoted by $\gamma^e(G)$ -set of G.

2.2 .Paired equitable domination number [3]

Let G be a connected graph. A equitable dominating set U of a connected graph G is called the paired equitable dominating set if the induced sub graph of U has a perfect matching. The paired equitable domination number of a graph G is denoted by $\gamma_{pr}{}^{e}(G)$, which is the minimum cardinality of the set U. The corresponding set is denoted by $\gamma_{pr}{}^{e}$ -set of G.

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Example:

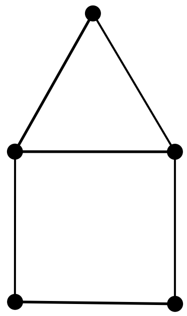

Fig . 1

For the graph G given in fig.1 , $\gamma_{pr}{}^{e}(G) = \gamma^{e}(G) = \gamma(G) = 2.$

2.3. Paired equitable domination number of inflated graph

Aim of this paper is to provide an upper bound of paired equitable domination number of an inflated graph of any connected graph of order n with $\delta \geq 2$ and we study the paired equitable domination number of inflated graph of path, cycle graph, complete graph, complete bipartite graph and bi-regular graph.

Theorem 2.4[4]

(i) $\gamma^{e}(P_n) = \left\lceil \frac{n}{3} \right\rceil$, for the path P_n

(ii) $\gamma^{e}(C_n) = \left\lceil \frac{n}{3} \right\rceil$, for the cycle C_n

(iii) $\gamma^{e}(K_n) = 1$, for the complete graph K_n

(iv) $\gamma^{e}(K_{m,n}) = 2 \text{ if } |m-n| \leq 1$ and $\gamma^{e}(K_{m,n}) = m+n \text{ if } |m-n| \geq 2, m, n \geq 2,$ for the complete bipartite graph $K_{m,n}$

Theorem 2:5[3]

(i)For the path graph P_n, $\gamma_{pr}{}^{e}(P_n) = 2\left\lceil \frac{n}{4} \right\rceil \text{ if } n \geq 2$

(ii)For the cycle graph C_n , $\gamma_{pr}{}^{e}(C_n) = 2\left\lceil \frac{n}{4} \right\rceil$

(iii)For the complete graph K_n , $\gamma_{pr}{}^{e}(K_n) = 2$

(iv)For the complete bipartite graph $K_{m,n}$,

$$\gamma_{pr}{}^{e}(K_{m,n}) = \begin{cases} 2 , \text{ if } |m-n| \leq 1 \\ \text{not defined, otherwise} \end{cases}$$

Theorem 2:6

(i)For the path G = P_n on n vertices, $\gamma_{pr}^{e}(G_l) = 2\left\lceil \frac{2n-2}{4} \right\rceil$

(ii) For the cycle G = C_n on n vertices, $\gamma_{pr}^{e}(G_l) = 2\left\lceil \frac{n}{2} \right\rceil$

(iii)For the complete graph G = K_n on n vertices, $\gamma_{pr}^{e}(G_l) \leq n+1$

(iv)For the complete bipartite graph G=$K_{m,n}$, $\gamma_{pr}^{e}(G_l) \leq m+n+1 \text{ if } |m-n| \leq 1$

Proof (i):

Let G = P_n , there are n vertices in G, but in G_l , there are $\sum_{i=1}^{n} \deg(v_i)$ number of vertices, that is 2n-2 vertices in G_l.

By theorem 2.5(i) we have, $\gamma_{pr}^{e}(G_l) = 2\left\lceil \frac{2n-2}{4} \right\rceil$.

(ii)Let G = C_n , there are n vertices in G, but in G_l , there are $\sum_{i=1}^{n} \deg(v_i)$ number of vertices, that is 2n vertices in G_l. By theorem 2.5(i) we have, $\gamma^{e}(G_l) = 2\left\lceil \frac{n}{2} \right\rceil$.

(iii) Let G = K_n and V(G)={u_1, u_2, u_3,…,u_n}. Since G is complete, $\sum_{i=1}^{n} \deg(v_i) = n(n-1)$ and hence $|V(G_l)| = n(n-1).$ Arbitrarily choose a vertex say u_i in V(G) which creates a clique K_{n-1} in G_l whose vertices are $u_i u_{i-1}, u_i u_{i-2}, \ldots, u_i u_{i+1}, u_i u_{i+2,\ldots,} u_i u_{n^2-n-1,} u_i u_{n^2-n}$,similarly each vertex in V(G) creates a clique K_{n-1} in G_l. Choose a pair of vertex (u_iu_j ,u_ju_i) , $u_iu_ju_ju_i \in E(G_l)$ dominates exactly two cliques say X_r and Y_r such that $u_iu_j \in V(X_r)$ $u_j u_i \in V(Y_r)$.

Set S = {(u_1u_n , $u_n u_1$), (u_2u_{n-1} , $u_{n-1} u_2$), (u_3u_{n-2} , $u_{n-2} u_3$),…, ($u_{(n/2)-1}u_{(n/2)+2}$, $u_{(n/2)+2} u_{(n/2)-1}$), ($u_{n/2}u_{(n/2)+1}$, $u_{(n/2)+1} \underline{u}_{n/2}$) if n is even} and

set S = {(u_1u_n , $u_n u_1$), (u_2u_{n-1} , $u_{n-1} u_2$), (u_3u_{n-2} , $u_{n-2} u_3$),…, ($u_{(n-3)/2}u_{(n+1)/2+2}$, $u_{(n+1)/2+2} u_{(n-3)/2}$),

($u_{(n-1)/2}u_{(n+1)/2+1}$, $u_{(n+1)/2+1} u_{(n-1)/2}$), ($u_{(n+1)/2} u_{(n+1)/2+1}$, $u_{(n+1)/2+1} u_{(n+1)/2}$) if n is odd}.Clearly the set S is a dominating set of G_l for both cases n is even and odd and also $|\deg(u_iu_j) - \deg(u_iu_r)| \leq 1$ for every $u_iu_j \in S$ and $u_iu_r \in V-S$ and u_iu_j $u_iu_r \in E(G_l), 1 \leq i, j \leq n.$ Furthermore the induced sub graph of S has a perfect matching in G_l.,

$|S| \leq 2\left(\frac{n}{2}\right) \leq n$ if n is even …………….(i)

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and $\quad |S| \le 2\left(\frac{n+1}{2}\right) \le n+1$ if n is odd ………..(ii)

From (i) and (ii) we get $|S| \le n+1$

Hence $\gamma_{pr}^{e}(G_l) \le n+1$

Fig.2 is G=K_4 complete graph and its inflated graph is given in fig.3, in fig.4 the encircle vertices forms the paired equitable dominating set of G_l

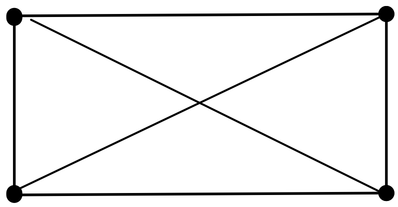

Fig . 2 Graph G=K_4

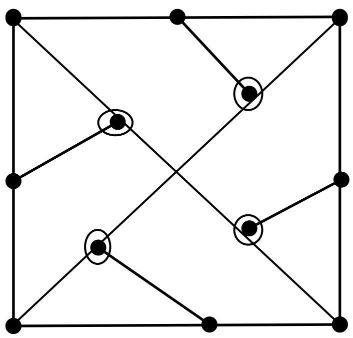

Fig . 3 Inflated Graph G_l

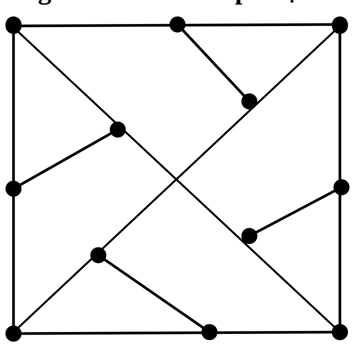

Fig . 4

(iv)Let G = $K_{m,n}$ be a complete bipartite graph. Let $V(G) = X \cup Y$ where X={x_1, x_2 ,x_3,…,x_m}and Y={y_1, y_2 ,y_3,…,y_n}. Construct the graph G_l . Let S be a minimal paired equitable dominating set of G_l. Since each vertex $x_i \in X(G)$ is of degree n and each $y_i \in Y(G)$ is of degree m, the inflated graph G_l consists of m cliques of order n and n cliques of order m. The group of m cliques of order n is represented as CG_{X_m} it consists of $G_{X_1}, G_{X_2},..., G_{X_i},..., G_{X_m}$ cliques and the group of m cliques of order n is represented as CH_{Y_n} it consists of $H_{Y_1}, H_{Y_2},..., H_{Y_j},..., H_{Y_n}$ cliques .Let H_{Y_i} be one of the cliques of order m whose vertices are $y_i x_1$, $y_i x_2$,…,$y_i x_m$. Select a pair of vertices ($x_i y_j$,$y_j x_i$) such that $x_i y_j$ $y_j x_i$ $\in E(G_l)$, $x_i y_j \in V(G_{X\,i})$ and $y_j x_i \in V(H_{Y\,j})$,these two vertices dominate exactly two cliques G_{X_i} and H_{Y_j} . Likewise choose a pair of vertex one vertex from the group CG_{X_m} and one from the group CH_{Y_n} from this form the set S'. Since n=m+1, one of the clique in CH_{Y_n} is not dominated by any vertex of S'. Now form $S = S' \cup \{y_n x_i, y_n x_s\}$, where $y_n x_i y_n x_s \in E(H_{Y_n})$ is a minimal dominating set of G_l. Furthermore $|\deg(x_i y_j) - \deg(x_i y_r)| \le 1$ for every $x_i y_j$ $\in S$ and $x_i y_r \in V - S$ and $x_i y_j x_i y_r \in E(G_l)$, $1 \le i, j \le n.$ Furthermore the induced sub graph of S has a perfect matching in G_l. Hence $|S| = m + (n-1) + 2 = m + n + 1.$

Theorem 2.7: If G is a (k_1,k_2) bi regular graph then $\gamma_{pr}^{e}(G_l) \le m+n+1$ where m is the number of vertices of degree k_1 and n is the number of vertices of degree k_2.

Proof: Let G be a (k_1, k_2) bi regular graph with m number of vertices of degree k_1 and n number of vertices of degree k_2 and let G_l be an inflated graph of G.

Case(i): Suppose G = (k_1, k_1+1) bi regular graph, then by theorem 2.6(iv), we have $\gamma_{pr}^{e}(G_l) \le m+n+1.$

Case(ii): Suppose G = (k_1, k_2) bi regular graph with k_1 =2 and $k_2 \ge 3$ then the inflated graph G_l consists of two partitions of the vertex set say X and Y, where X ={ x_1,x_2,…,x_m} each vertex in X(G_l) is of degree 2 and Y= { y_1,y_2,…,y_n} each vertex in Y(G_l) is of degree ≥3.

By theorem 2.6(ii) and (iii)

$$\gamma_{pr}^{e}(G_l) \le 2\left\lceil \frac{2m-2}{4} \right\rceil + n + 1$$

$$\gamma_{pr}^{e}(G_l) \le m+n+1.$$

Case(iii): Suppose G = (k_1, k_2) bi regular graph with $k_1 \ne k_2$ (where k_1 and k_2>3). In this case by theorem 2.6(iii) $\gamma_{pr}^{e}(G_l) \le m+n+1.$

Theorem 2:8: For any connected graph G of order n with $\delta(G) \ge 2$ then $\gamma_{pr}^{e}(G_l) \le n+1$

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Proof: Let G be any connected graph with $\delta(G) \geq 2$ .Construct the graph G_l. Let S be a minimal paired equitable

$$\in V(G_l)/x_i y_j \in X_r, y_j x_i \in Y_s, x_i y_j y_j x_i \in E(G_l)\, such\ that |r-s| = 0\}$$

$$\in V(G_l)/x_i y_j \in X_r, y_j x_i \in Y_s, x_i y_j y_j x_i \in E(G_l)\, such\ that |r-s| = 1\}$$

$$\in V(G_l)/x_i y_j \in X_r, y_j x_i \in Y_s, x_i y_j y_j x_i \in E(G_l)\, such\ that |r-s| \geq 2\}$$

Choose an arbitrary pair of vertex say ($u_i u_j$,$u_j u_i$) , $u_i u_j u_j u_i \in E(G_l)$ in V(G_l) dominates exactly two cliques say X_r and Y_r such that $u_i u_j \in V(X_r)$ u_j $u_i \in V(Y_r)$.

Case(i): Suppose S contains only the elements of A then we have

If n is even then $|S| \leq 2\left(\frac{n}{2}\right) \leq n$ ....(i)

If n is odd then $|S| \leq 2\left(\frac{n+1}{2}\right) \leq n+1$.........(ii)

From (i) and (ii) we get $|S| \leq n+1$

Hence $\gamma_{pr}^{e}(G_l) \leq n+1$

Case(ii): Suppose S contains only the elements of B . Let m_1 be the number of cliques of degree r and m_2 be the number of cliques of degree r+1 in G_l.

$$|S| \leq m_1 + m_2 + 1$$

$$\leq 2\frac{m_1}{2} + 2\left(\frac{n-m_1}{2}\right) + 1$$

$$|S| \leq n+1$$

Case(iii): Suppose S contains only the elements of C. Let m_i denote the number of cliques of degree i ,i=2 to n-1.

$$|S| \leq 2\frac{m_2}{2} + 2\frac{m_3}{2} + ... + 2\frac{m_{n-1}}{2}$$

Since the sum of the vertices of a graph G is the number of cliques in G_l

$$|S| \leq 2\frac{m_2}{2} + 2\frac{m_3}{2} + ... + 2\frac{m_{n-1}}{2}$$

$$\leq n+1$$

Case(iv): Suppose S contains the elements of A,B and C, $|S| \leq |A| + |B| + |C|$. Since the order of a graph G is n, there are n cliques in G_l. Select a vertex say $x_i y_j$ from a clique X_r in G_l. A paired vertex of $x_i y_j$ is $y_j x_i$ which is chosen from the clique Y_r in G_l such that $x_i y_j$ $y_j x_i \in E(G_l)$ and the vertices $x_i y_j$ dominating set of G_l. Let X_r be a clique in G_l which corresponds to a vertex x_i of degree r. The partitions of S are

Let A = {$x_i y_j$, $y_j x_i$

Let B = {$x_i y_j$, $y_j x_i$

Let C = {$x_i y_j$, $y_j x_i$

and $y_j x_i$ dominated exactly two cliques of same degree X_r and Y_r in G_l. Likewise select the pair of vertices ($x_i y_j$, $y_j x_i$) among the same degree of cliques which dominate exactly two different cliques then $|S| \leq |A| + |B| + |C| \leq n+1$.

III. CONCLUSION

For any graph G of order n, the paired equitable domination number of inflated graph of G is at most n+1 where as the paired domination of inflated graph of G is n if and only if G has a perfect matching.

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