Variance of Time to Recruitment for a Two Grade Manpower System with Independent and Non-Identically Inter - Decision Times and Correlated Wastages with Thresholds having Different Distributions

S. Sendhamizh Selvi, S.Jenita

Abstract: ***In this paper, the problem of time to recruitment in an organization with two grades when it is subjected to loss of manpower due to the policy decisions taken by the organization is studied. As the exit of personnel is unpredictable, a recruitment policy involving two thresholds for each grade there are optional and mandatory, is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach two mathematical models are constructed using the univariate policy of recruitment. Performance measures namely mean and variance of the time to recruitment are obtained for model I when (i) loss of man powers are exchangeable and constantly correlated exponential random variables (ii) inter-decision times form a sequence of independent and non-identically distributed exponential random variables and iii) optional and mandatory thresholds follows extended exponential distribution. In model II, optional and mandatory thresholds follows SCBZ property.***

Keywords: Manpower planning, shock models, univariate recruitment policy, extended exponential distribution, hypo-exponential distribution, SCBZ property, exchangeable and constantly correlated exponential random variables

I. INTRODUCTION

Consider an organization having two grades in which depletion of manpower occurs at every decision epoch. In the univariate policy of recruitment, based on shock model approach, recruitment is made as and when the cumulative loss of manpower crosses the threshold. Many models have been discussed using different kinds of wastage and different types of distributions for the thresholds. Such models could be seen in [1],[2],[3],[5] and [6]. In [4], for a single graded system, a univariate CUM recruitment policy involving two thresholds optional and mandatory is suggested and mean and variance of time to recruitment is obtained when (i) loss of manpower and inter-decision times are independent and identically distributed exponential random variables (ii) thresholds are extended exponential and SCBZ Property. In [8], for two graded system the authors have obtained mean time to recruitment when loss of manpower and inter-decision times are independent and identically distributed exponential random variables with threshold having SCBZ property.

In [9], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) loss of manpower and inter-decisions times are independent and non-identically distributed exponential random variables (ii) thresholds optional and mandatory follows exponential random variables. In [10] and [11] the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving the optional and mandatory thresholds having extended exponential distribution and SCBZ property. In [12], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving non-identically distributed random variables with optional and mandatory thresholds having extended exponential distribution and SCBZ property.

The objective of the present paper is to obtain the mean and variance of time to recruitment for a two graded system using the univariate cumulative recruitment policy considering optional and mandatory thresholds follows extended exponential distribution and SCBZ property for both the grades with independent and non-identically distributed inter-decision times and correlated wastages.

Notations:

X_i : Loss of man hours due to the i^{th} decision epoch i=1, 2, 3…… forming a sequence of exchangeable and constantly correlated exponential random variables with correlated ρ.

M(.) : Distribution function of X_i

m(.) : Probability density function of X_i with mean α, $g(x) = \frac{1}{\alpha} e^{-(x/\alpha)}$

S_k : Cumulative loss of manpower in the first k-decisions (k=1, 2, 3….), $S_k = \sum_{i=1}^{k} X_i$

Revised Manuscript Received on May 07, 2019.

S. Sendhamizh Selvi, Assistant Professor, PG & Research Department of Mathematics, Government Arts College, Trichy-22

S.Jenita, Assistant Professor, PG & Research Department of Mathematics, Bishop Heber College, Trichy-17

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$G_k(.)$: Distribution function of sum of k distributedconstantly correlated and exchangeableexponential random variables.

$g_k(.)$: Probability density function of S_k,$g_k^*(.)$:k-fold convolution of S_k

ρ : Correlation between X_i&X_j,i≠j,

α : Mean of inter-decision times, $\alpha = \frac{b}{1-\rho}$

ψ (n,x) : $\int_0^x e^{-\epsilon}\epsilon^{n-1}d\epsilon$

U_k : Inter-decision times are independent and non-identically distributed exponential random variables Between(k-1)th and kth decisions with parametersβ_k (β_k>0)

$F_k(.)$: Distribution function of U_k, $f_k(.)$: Probability density function of $U_k$with mean $\frac{1}{\beta_k}$ (β_k>0)

Y_1, Y_2 : A continuous random variables denoting the optional thresholds for grade 1 and 2 respectively.

Z_1, Z_2 : A continuous random variables denoting the mandatory thresholds for grade 1 and 2 respectively.

It is assumed that $Y_1<Z_1$ and $Y_2<Z_2$, Y=Max (Y_1, Y_2) and Z=Max (Z_1, Z_2).

$H_1(.)$:Distribution function of Y_1 , $H_2(.)$:Distribution function of Y_2

$H_3(.)$:Distribution function of Z_1 , $H_4(.)$:Distribution function of Z_2

W : Continuous random variable denoting the time to recruitment in the organization.

p : Probability that the organization is not going for recruitment whenever the total loss of manpower crossesthe optional threshold.

$V_k(t)$: Probability that exactly k decisions are taken in [0,t)

L(.) : Distribution function of W, l(.) : Probability density function of W, $l^*(.)$: Laplace transform of l(.)

E(W) : Expected time to recruitment

V(W) : Variance of the time to recruitment

CUM policy:Recruitment is done whenever the cumulative loss of manpower crosses mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses optional threshold.

II. MAIN RESULTS

The survival function of W is given by

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, P(S_k<Y) + \sum_{k=0}^{\infty} V_k(t)\, P(S_k\geq Y)\, P(S_k<Z)\, p \quad (1)$$

Model I:Thresholds follows extended exponential distributed random variables

Y_1,Y_2 : Continuous random variables denoting optional thresholds for grade 1and 2 followsextended exponential distribution with parameters $\lambda_1$and $\lambda_2$respectively.

Z_1,Z_2 : Continuous random variables denoting mandatory thresholds for grade 1 and 2 followsextended exponential distribution with parameters $\mu_1$andμ_2 respectively.

Using law oftotal probability and conditioning upon Y, it can be shown that

$$P(S_k<Y) = \int_0^\infty G_k(y)h(y)dy \quad (2)$$

Since X_i's are assumed to be identical, exchangeable and constantly correlated random variables each following the exponential distribution with probability density function$g(x) = \frac{1}{\alpha}e^{-\left(\frac{x}{\alpha}\right)}, (\alpha > 0)$, the cumulative distribution function of the partial sum $S_k = (X_1 + X_2 + \ldots + Xk$ is given by Gurland (1995) as

$$G_k(y) = (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i\varphi(k+i,y/b)}{(1-\rho+k\rho)^{i+1}(k+i-1)!} \quad (3)$$

Where ρ is the constant correlation betweenX_i&X_j, $i \neq j$

Thresholds follows extended exponential distribution, the probability distribution is

$H_1(y) = (1-e^{-\lambda_1 y})^2$ and $H_2(y) = (1-e^{-\lambda_2 y})^2$

H(y) = P(Y_1≤y) P(Y_2≤y) = $H_1(y)$ $H_2(y)$

The probability density function is

$h_1(y) = 2(1-e^{-\lambda_1 y})\lambda_1 e^{-\lambda_1 y}$ and $h_2(y) = 2(1-e^{-\lambda_2 y})\lambda_2 e^{-\lambda_2 y}$

$$h(y) = 2\lambda_1 e^{-\lambda_1 y} + 2\lambda_2 e^{-\lambda_2 y} - 4(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)y} + 2(2\lambda_1+\lambda_2)e^{-(2\lambda_1+\lambda_2)y} +2(\lambda_1+2\lambda_2)e^{-(\lambda_1+2\lambda_2)y} - \lambda_1 e^{-2\lambda_1 y} - \lambda_2 e^{-2\lambda_2 y} - (2\lambda_1 + 2\lambda 2e - 2\lambda 1 + 2\lambda 2y \quad (4)$$

Using (3) and (4) in (2)it can be shown that,

$$P(S_k<Y) = \int_0^\infty (1-\rho)\left[\sum_{i=0}^{\infty}\frac{(k\rho)^i\varphi(k+i,y/b)}{(1-\rho+k\rho)^{i+1}(k+i-1)!}\right] [2\lambda_1 e^{-\lambda_1 y} + 2\lambda_2 e^{-\lambda_2 y} - 4(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)y} + 2(2\lambda_1+\lambda_2)e^{-(2\lambda_1+\lambda_2)y} +2(\lambda_1+2\lambda_2)e^{-(\lambda_1+2\lambda_2)y} - 2\lambda_1 e^{-2\lambda_1 y} - 2\lambda_2 e^{-2\lambda_2 y} - (2\lambda_1+2\lambda_2)e^{-(2\lambda_1+2\lambda_2)y}]\, dy \quad (5)$$

Using Gamma integrals and on further simplification, we get

$$\int_0^\infty \varphi\left(k+i,\frac{y}{b}\right)\lambda_1 e^{-\lambda_1 y}\, dy = \frac{(k+i-1)!}{(1+b\lambda_1)^{k+i}} \quad (6)$$

Consider the first term of the equation (5), and using (6) it can be shown that,

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}\lambda_1 e^{-\lambda_1 y}dy= \frac{(1-\rho)}{(1+b\lambda_1)^{k-1}}\left[\frac{1}{(1+b\lambda_1)(1-\rho+k\rho)-k\rho}\right] = B_{1k} \text{ (say)}$$

Similarly making similar computation for other term of (5), we can write

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}\lambda_2 e^{-\lambda_2 y}dy = \frac{(1-\rho)}{(1+b\lambda_2)^{k-1}}\left[\frac{1}{(1+b\lambda_2)(1-\rho+k\rho)-k\rho}\right]= B_{2k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)y}dy = \frac{(1-\rho)}{(1+b(\lambda_1+\lambda_2))^{k-1}}\left[\frac{1}{(1+b(\lambda_1+\lambda_2))(1-\rho+k\rho)-k\rho}\right]= B_{3k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(2\lambda_1+\lambda_2)e^{-(2\lambda_1+\lambda_2)y}dy =$$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$\frac{(1-\rho)}{(1+b(2\lambda_1+\lambda_2))^{k-1}}\left[\frac{1}{(1+b(2\lambda_1+\lambda_2))(1-\rho+k\rho)-k\rho}\right] = B_{7k}$

$\int_0^{\infty}(1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\lambda_1 + 2\lambda_2)e^{-(\lambda_1+2\lambda_2)y}dy = \frac{(1-\rho)}{(1+b(\lambda_1+2\lambda_2))^{k-1}}\left[\frac{1}{(1+b(\lambda_1+2\lambda_2))(1-\rho+k\rho)-k\rho}\right] =$

$B_{8k}\int_0^{\infty}(1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}2\lambda_1 e^{-2\lambda_1 y}dy = \frac{(1-\rho)}{(1+b(2\lambda_1))^{k-1}}\left[\frac{1}{(1+b2\lambda_1)(1-\rho+k\rho)-k\rho}\right] = B_{9k}$

$\int_0^{\infty}(1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}2\lambda_2 e^{-2\lambda_2 y}d = \frac{(1-\rho)}{(1+b(2\lambda_2))^{k-1}}\left[\frac{1}{(1+b2\lambda_2)(1-\rho+k\rho)-k\rho}\right] = B_{10k}$

$$\int_0^{\infty}(1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(2\lambda_1 + 2\lambda_2)e^{-(2\lambda_1+2\lambda_2)y}dy = \frac{(1-\rho)}{(1+b(2\lambda_1+2\lambda_2))^{k-1}}\left[\frac{1}{(1+b(2\lambda_1+2\lambda_2))(1-\rho+k\rho)-k\rho}\right] = B_{11k}$$

Using above results in equation (5) it can be shown that,

$P(S_k<Y) = 2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k}$ (7)

Similarly

$P(S_k<Z) = 2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k}$ (8)

Where B_{4k}, B_{5k}, B_{6k}, B_{12k}, B_{13k}, B_{14k}, B_{15k}, B_{16k} are obtained as above

(ie). $B_{4k} = \frac{(1-\rho)}{(1+b\mu_1)^{k-1}}\left[\frac{1}{(1+b\mu_1)(1-\rho+k\rho)-k\rho}\right]$, $B_{5k} = \frac{(1-\rho)}{(1+b\mu_2)^{k-1}}\left[\frac{1}{(1+b\mu_2)(1-\rho+k\rho)-k\rho}\right]$

$B_{6k} = \frac{(1-\rho)}{(1+b(\mu_1+\mu_2))^{k-1}}\left[\frac{1}{(1+b(\mu_1+\mu_2))(1-\rho+k\rho)-k\rho}\right]$, $B_{12k} = \frac{(1-\rho)}{(1+b(2\mu_1+\mu_2))^{k-1}}\left[\frac{1}{(1+b(2\mu_1+\mu_2))(1-\rho+k\rho)-k\rho}\right]$

$B_{13k} = \frac{(1-\rho)}{(1+b(\mu_1+2\mu_2))^{k-1}}\left[\frac{1}{(1+b(\mu_1+2\mu_2))(1-\rho+k\rho)-k\rho}\right]$,

$B_{14k} = \frac{(1-\rho)}{(1+b(2\mu_1))^{k-1}}\left[\frac{1}{(1+b2\mu_1)(1-\rho+k\rho)-k\rho}\right]$

$B_{15k} = \frac{(1-\rho)}{(1+b(2\mu_2))^{k-1}}\left[\frac{1}{(1+b2\mu_2)(1-\rho+k\rho)-k\rho}\right]$, $B_{16k} = \frac{(1-\rho)}{(1+b(2\mu_1+2\mu_2))^{k-1}}\left[\frac{1}{(1+b(2\mu_1+2\mu_2))(1-\rho+k\rho)-k\rho}\right]$

Using (7) and (8) in (1) and on simplification,

$$P(W>t) = \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)]\{(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k}) + p(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k}) - p(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k})(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k})\} \quad (9)$$

From renewal theory and using L(t) = 1- P(W>t) and l(t) = $\frac{d}{dt}$L(t), $l^*(s)$ = Laplace transform of l(t), we get

$$l^*(s) = \sum_{k=0}^{\infty}[f_k^*(s)-f_k^*(s)]\{(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k}) + p(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k}) - p(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k})(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k})\} \quad (10)$$

It is known that

$$E(W) = -\left[\frac{d}{ds}l^*(s)\right]_{s=0} \quad (11)$$

$$E(W^2) = \left[\frac{d^2}{ds^2}l^*(s)\right]_{s=0} \quad (12)$$

$$Var(W) = E(W^2) - (E(W))^2 \quad (13)$$

After simplification using inter-decision times are independent and non-identically distributed random variables,

It can be shown that

$$E(w) = \sum_{k=0}^{\infty}\frac{1}{\beta_{k+1}}\{(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k}) + p(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k}) - p(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k})(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k})\} \quad (14)$$

Equation (14) gives the meantime to recruitment for maximum model.

$$\text{Now, } E(w^2) = \sum_{k=0}^{\infty}\frac{2}{(\beta_{k+1})^2}\{(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k}) + p(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k}) - p(2B_{1k}+2B_{2k}-4B_{3k}+2B_{7k}+2B_{8k}-B_{9k}-B_{10k}-B_{11k})(2B_{4k}+2B_{5k}-4B_{6k}+2B_{12k}+2B_{13k}-B_{14k}-B_{15k}-B_{16k})\} \quad (15)$$

Using (14) and (15) in (13) , we get variance of time to recruitment for maximum model.

III. NUMERICAL ILLUSTRATIONS

The analytical expressions for the performance measures namely mean and variance of the time to recruitment are analyzed numerically by varying a parameter at a time and keeping others parameters fixed. The effect of nodal parameterρ for loss of man hoursand p on the performance measures are shown in the following tables.

The parameter of inter-decision times are fixed. The value of correlationρ and p vary.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

λ_1=0.01, λ_2=0.12, μ_1=0.02, μ_2=0.025, $\beta_1 = 0.5, \beta_2 = 0.6, \beta_3 = 0.7, \beta_4 = 0.8, \beta_5 = 0.9, \beta_6 = 1.2$, b=1, α = 1.06,

Table. 1 The meantime to recruitment when nodal parameters ρ and p varying as given below

ρ / p	0.3	0.33	0.35	0.37	0.39
0.4	8.2866	8.2873	8.2878	8.2882	8.2886
0.5	8.2846	8.2858	8.2866	8.2874	8.2881
0.6	8.2796	8.2817	8.2831	8.2846	8.2860
0.7	8.2660	8.2699	8.2725	8.2751	8.2777
0.8	8.2216	8.2292	8.2343	8.2393	8.2444

Table. 2 The variance of time to recruitment when nodal parameters ρ and p varying as given below

ρ / p	0.3	0.33	0.35	0.37	0.39
0.4	38.2335	38.2247	38.2188	38.2130	38.2071
0.5	38.2541	38.2386	38.2282	38.2178	38.2075
0.6	38.3062	38.2786	38.2603	38.2419	38.2236
0.7	38.4523	38.4025	38.3693	38.3361	38.3029
0.8	38.9343	38.8396	38.7765	38.7133	38.6501

Findings:

We observe the following, from table 1

1. As probability value p alone increases, meantime to recruitment increase.
2. Ascorrelationρalone increases, meantime to recruitment decreases.

From table 2

1. As probability value p alone increases, variance of time to recruitment decrease.
2. As correlation ρ alone increases, variance of the time to recruitment increase.

Model II: Thresholds follows SCBZ property

Y_1,Y_2 : Continuous random variables denoting optional thresholds for grade1and 2 follows SCBZpropertywith parameters (θ_1,θ_2, θ_3, θ_4, λ_1,λ_2) respectively.

Z_1,Z_2 : Continuous random variables denoting mandatory thresholds for grade1 and 2 follows SCBZ property with parameters (θ_5,θ_6, θ_7, θ_8, μ_1,μ_2) respectively. It is assumed that $Y_1<Z_1$ and $Y_2<Z_2$

As in Rao and Talwalkar (1990), the distribution of Y_1,Y_2 and Z_1,Z_2 are given by

$H_1(y) = 1 - p_1 e^{-(\theta_1+\lambda_1)y} - q_1 e^{-\theta_2 y}$, $H_2(y) = 1- p_2e^{-(\theta_3+\lambda_2)y} - q_2e^{-\theta_4 y}$

$h_1(y) = (\theta_1 + \lambda_1)p_1 e^{-(\theta_1+\lambda_1)y} + \theta_2 q_1 e^{-\theta_2 y}$, $h_2 = (\theta_3 + \lambda_2)p_2e^{-(\theta_3+\lambda_2)y}+\theta_4q_2e^{-\theta_4y}$

$H_3(z) = 1 - p_3 e^{-(\theta_5+\mu_1)z} - q_3 e^{-\theta_6 z}$, $H_4(z) = 1 - p_4 e^{-(\theta_7+\mu_2)z} - q_4 e^{-\theta_8 z}$

$h_3(z) = (\theta_5 + \mu_1)p_3 e^{-(\theta_5+\mu_1)z} + \theta_6 q_3 e^{-\theta_6 z}$, $h_4(z) = (\theta_7 + \mu_2)p_4 e^{-(\theta_7+\mu_2)z} + \theta_8\, q_4 e^{-\theta_8 z}$

Where $p_1 = \frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda_1}$, $q_1 = \frac{\lambda_1}{\theta_1-\theta_2+\lambda_1}$, $p_2 = \frac{\theta_3-\theta_4}{\theta_3-\theta_4+\lambda_2}$, $q_2 = \frac{\lambda_2}{\theta_3-\theta_4+\lambda_2}$

$p_3 = \frac{\theta_5-\theta_6}{\theta_5-\theta_6+\mu_1}$, $q_3 = \frac{\mu_1}{\theta_5-\theta_6+\mu_1}$, $p_4 = \frac{\theta_7-\theta_8}{\theta_7-\theta_8+\mu_2}$ $q_4 = \frac{\mu_2}{\theta_7-\theta_8+\mu_2}$

$H(y) = P(Y_1 \le y)\, P(Y_2 \le y)$

$= (1- p_1e^{-(\theta_1+\lambda_1)y} - q_1e^{-\theta_2 y})\,(1- p_2e^{-(\theta_3+\lambda_2)y} - q_2e^{-\theta_4 y})$

$= 1- p_2e^{-(\theta_3+\lambda_2)y} - q_2e^{-\theta_4 y} - p_1e^{-(\theta_1+\lambda_1)y} + p_1p_2e^{-(\theta_1+\lambda_1)y}e^{-(\theta_3+\lambda_2)y}$
$+ p_1q_2e^{-(\theta_1+\lambda_1)y}e^{-\theta_4 y} - q_1e^{-\theta_2 y} + q_1p_2e^{-(\theta_3+\lambda_2)y}e^{-\theta_2 y} + q_1q_2e^{-\theta_2 y}e^{-\theta_4 y}$

$H(y) = 1- p_2e^{-(\theta_3+\lambda_2)y} - q_2e^{-\theta_4 y} - p_1e^{-(\theta_1+\lambda_1)y} + p_1p_2e^{-(\theta_1+\theta_3+\lambda_1+\lambda_2)y}$
$+ p_1q_2e^{-(\theta_1+\theta_4+\lambda_1)y} - q_1e^{-\theta_2 y} + q_1p_2e^{-(\theta_2+\theta_3+\lambda_2)y} + q_1q_2e^{-(\theta_2+\theta_4)y}$

$h(y) = (\theta_3 + \lambda_2)p_2e^{-(\theta_3+\lambda_2)y} + \theta_4q_2e^{-\theta_4 y} + (\theta_1 + \lambda_1)p_1e^{-(\theta_1+\lambda_1)y}$
$-(\theta_1+\theta_3+\lambda_1+\lambda_2)p_1p_2e^{-(\theta_1+\theta_3+\lambda_1+\lambda_2)y} - (\theta_1+\theta_4+\lambda_1)p_1q_2e^{-(\theta_1+\theta_4+\lambda_1)y}$
$+\theta_2q_1e^{-\theta_2 y} - (\theta_2+\theta_3+\lambda_2)q_1p_2e^{-(\theta_2+\theta_3+\lambda_2)y} - (\theta_2+\theta_4)q_1q_2e^{-(\theta_2+\theta_4)y}$ (16)

Using (3) and (16) in (2) it can be shown that

$$P(S_k < Y) = \int_0^\infty (1-\rho)\left[\sum_{i=0}^{\infty}\frac{(k\rho)^i\varphi(k+i,y/b)}{(1-\rho+k\rho)^{i+1}(k+i-1)!}\right]$$
$$[(\theta_3+\lambda_2)p_2e^{-(\theta_3+\lambda_2)y} + \theta_4q_2e^{-\theta_4 y} + (\theta_1+\lambda_1)p_1e^{-(\theta_1+\lambda_1)y}$$
$$-(\theta_1+\theta_3+\lambda_1+\lambda_2)p_1p_2e^{-(\theta_1+\theta_3+\lambda_1+\lambda_2)y} - (\theta_1+\theta_4+\lambda_1)p_1q_2e^{-(\theta_1+\theta_4+\lambda_1)y}$$
$$+\theta_2q_1e^{-\theta_2 y} - (\theta_2+\theta_3+\lambda_2)q_1p_2e^{-(\theta_2+\theta_3+\lambda_2)y} - (\theta_2+\theta_4)q_1q_2e^{-(\theta_2+\theta_4)y}]\,dy \quad (17)$$

From (6) and (17), it can be shown that

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\theta_1+\lambda_1)p_1e^{-(\theta_1+\lambda_1)y}dy$$
$$= \frac{(1-\rho)p_1}{(1+b(\theta_1+\lambda_1))^{k-1}}\left[\frac{1}{(1+b(\theta_1+\lambda_1))(1-\rho+k\rho)-k\rho}\right] = C_{1k}(say)$$

Similarly

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\theta_3+\lambda_2)p_2 e^{-(\theta_3+\lambda_2)y}dy$$

$$=\frac{(1-\rho)p_2}{(1+b(\theta_3+\lambda_2))^{k-1}}\left[\frac{1}{(1+b(\theta_3+\lambda_2))(1-\rho+k\rho)-k\rho}\right] = C_{2k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}\theta_2 q_1 e^{-\theta_2 y}dy$$

$$=\frac{(1-\rho)q_1}{(1+b\theta_2)^{k-1}}\left[\frac{1}{(1+b\theta_2)(1-\rho+k\rho)-k\rho}\right] = C_{3k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}\theta_4 q_2 e^{-\theta_4 y}dy$$

$$=\frac{(1-\rho)q_2}{(1+b\theta_4)^{k-1}}\left[\frac{1}{(1+b\theta_4)(1-\rho+k\rho)-k\rho}\right] = C_{7k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\theta_1+\theta_3+\lambda_1+\lambda_2)p_1 p_2 e^{-(\theta_1+\theta_3+\lambda_1+\lambda_2)y}dy$$

$$=\frac{(1-\rho)p_1p_2}{(1+b(\theta_1+\theta_3+\lambda_1+\lambda_2))^{k-1}}\left[\frac{1}{(1+b(\theta_1+\theta_3+\lambda_1+\lambda_2))(1-\rho+k\rho)-k\rho}\right] = C_{8k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\theta_1+\theta_4+\lambda_1)p_1 q_2 e^{-(\theta_1+\theta_4+\lambda_1)y}dy$$

$$=\frac{(1-\rho)p_1q_2}{(1+b(\theta_1+\theta_4+\lambda_1))^{k-1}}\left[\frac{1}{(1+b(\theta_1+\theta_4+\lambda_1))(1-\rho+k\rho)-k\rho}\right] = C_{9k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\theta_2+\theta_3+\lambda_2)q_1 p_2 e^{-(\theta_2+\theta_3+\lambda_2)y}dy$$

$$=\frac{(1-\rho)q_1p_2}{(1+b(\theta_2+\theta_3+\lambda_2))^{k-1}}\left[\frac{1}{(1+b(\theta_2+\theta_3+\lambda_2))(1-\rho+k\rho)-k\rho}\right] = C_{10k}$$

$$\int_0^\infty (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\varphi(k+i,y/b)}{(k+i-1)!}(\theta_2+\theta_4)q_1 q_2 e^{-(\theta_2+\theta_4)y}dy$$

$$=\frac{(1-\rho)q_1q_2}{(1+b(\theta_2+\theta_4))^{k-1}}\left[\frac{1}{(1+b(\theta_2+\theta_4))(1-\rho+k\rho)-k\rho}\right] = C_{11k}$$

Using above results in (17), it can be shown that

$$P(S_k< Y) = C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k} \quad (18)$$

Similarly

$$P(S_k< Z) = C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k} \quad (19)$$

Where $C_{4k}, C_{5k}, C_{6k}, C_{12k}, C_{13k}, C_{14k}, C_{15k}, C_{16k}$ are obtained as above

$$C_{4k}=\frac{(1-\rho)p_3}{(1+b(\theta_5+\mu_1))^{k-1}}\left[\frac{1}{(1+b(\theta_5+\mu_1))(1-\rho+k\rho)-k\rho}\right],\ C_{5k}=\frac{(1-\rho)p_4}{(1+b(\theta_7+\mu_2))^{k-1}}\left[\frac{1}{(1+b(\theta_7+\mu_2))(1-\rho+k\rho)-k\rho}\right],$$

$$C_{6k}=\frac{(1-\rho)q_3}{(1+b\theta_6)^{k-1}}\left[\frac{1}{(1+b\theta_6)(1-\rho+k\rho)-k\rho}\right],\ C_{12k}=\frac{(1-\rho)q_4}{(1+b\theta_8)^{k-1}}\left[\frac{1}{(1+b\theta_8)(1-\rho+k\rho)-k\rho}\right],$$

$$C_{13k}=\frac{(1-\rho)p_3p_4}{(1+b(\theta_5+\theta_7+\mu_1+\mu_2))^{k-1}}\left[\frac{1}{(1+b(\theta_5+\theta_7+\mu_1+\mu_2))(1-\rho+k\rho)-k\rho}\right],$$

$$C_{14k}=\frac{(1-\rho)p_3q_4}{(1+b(\theta_5+\theta_8+\mu_1))^{k-1}}\left[\frac{1}{(1+b(\theta_5+\theta_8+\mu_1))(1-\rho+k\rho)-k\rho}\right],\ C_{15k}=\frac{(1-\rho)q_3p_4}{(1+b(\theta_6+\theta_7+\mu_2))^{k-1}}\left[\frac{1}{(1+b(\theta_6+\theta_7+\mu_2))(1-\rho+k\rho)-k\rho}\right],$$

$$C_{16k}=\frac{(1-\rho)q_3q_4}{(1+b(\theta_6+\theta_8))^{k-1}}\left[\frac{1}{(1+b(\theta_6+\theta_8))(1-\rho+k\rho)-k\rho}\right].$$

Using (18) and (19) in (1) and on simplification,

$$P(W>t) = \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)]\{(C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k} + p(C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k} - p(C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k}(C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k}\} \quad (20)$$

Using (11) to (14) as in model I, it can be shown that

$$E(w)=\sum_{k=0}^{\infty}\frac{1}{\beta_{k+1}}\{(C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k} + p(C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k} - p(C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k})(C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k}\} \quad (21)$$

Equation (21) gives the meantime to recruitment for maximum model.

$$\text{Now, } E(w^2)=\sum_{k=0}^{\infty}\frac{1}{(\beta_{k+1})^2}\{(C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k} + p(C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k} - p(C_{1k}+C_{2k}+C_{3k}+C_{7k}-C_{8k}-C_{9k}-C_{10k}-C_{11k})(C_{4k}+C_{5k}+C_{6k}+C_{12k}-C_{13k}-C_{14k}-C_{15k}-C_{16k}\} \quad (22)$$

Using (21) and (22) in (13) , we get variance of time to recruitment for maximum model.

Numerical Illustrations

The analytical expressions for the performance measures namely mean and variance of the time to recruitment are analyzed numerically by varying a parameter at a time and keeping others parameters fixed. The effect of nodal parameter ρ for loss of man hour and p on the performance measures are shown in the following tables.

The parameter of inter-decision times are fixed. The value of correlation ρ and p vary.

$\lambda_1=0.01$, $\lambda_2=0.12$, $\mu_1=0.02$, $\mu_2=0.025$, $\beta_1=0.5, \beta_2=0.6, \beta_3=0.7, \beta_4=0.8, \beta_5=0.9, \beta_6=1.2$

$\theta_1=0.09, \theta_2=0.092,$

$\theta_3=0.07, \theta_4=0.072, \theta_5=0.05, \theta_6=0.053, \theta_7=0.04, \theta_8=0.043$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Table. 1 The meantime to recruitment when nodal parameters ρ and p varying as given below

ρ / p	0.2	0.23	0.26	0.29	0.32
0.4	7.9560	8.0069	8.0577	8.1086	8.1594
0.5	7.8724	7.9383	8.0043	8.0702	8.1361
0.6	7.7241	7.8071	7.8900	7.9730	8.0560
0.7	7.4558	7.5571	7.6584	7.7597	7.8611
0.8	6.9422	7.0603	7.1784	7.2965	7.4147

Table. 2 The variance of time to recruitment when nodal parameters ρ and p varying as given below

ρ / p	0.2	0.23	0.26	0.29	0.32
0.4	41.5753	41.0243	40.4682	39.9069	39.3404
0.5	42.3292	41.6334	40.9289	40.2157	39.4938
0.6	43.6590	42.8253	41.9778	41.1165	40.2415
0.7	45.9384	45.0112	44.0635	43.0953	42.1065
0.8	49.7314	48.8471	47.9349	46.9948	46.0267

IV. CONCLUSION

This paper concludes the following:

1. As probability value p alone increases, meantime to recruitment increase.
2. As correlation ρ alone increases, meantime to recruitment decreases.

From table 2

1. As probability value p alone increases, variance of time to recruitment decrease.
2. As correlation ρ alone increases, variance of the time to recruitment increase.

REFERENCES

1. Akilandeswari.M and Srinivasan.A, (2007) Mean Time to Recruitment for Two Graded Manpower System when Threshold Distribution has SCBZ Property, ActaCienciaIndica, 33M(3), pp: 1113-1118.
2. Barthlomew.D.J. and Morris.B.R..(1971), Aspects of Manpower Planning, Elsevier Publishing Company,New York.
3. Elangovan.R and Sathiyamoorthi.R, (1998), A Shock Model Approach to Determine the Expected Time for Recruitment, Journal of Decision and Mathematical sciences, Vol 2,No.1-3. pp 67-68.
4. Esther Clara.S. (2011), Contributions to the Study on Some Stochastic Models in Manpower Planning, Ph.DThesis, Bharathidasan University.
5. Kasturri.k and Srinivasan.A, (2005), Expected Time to Recruitment for Correlated Inter-decision Times of Exits when Threshold Distribution has SCBZ Property, ActaCienciaIndica. Vol.XXXIM, No.1, pp 277-283.
6. Parthasarathy.S, (2003), On Some Stochastic Models for Manpower Planning Using SCBZ Property, Ph.D, Thesis at Department of Statistics, Annamalai University.
7. Rao.B.R. and Talwalker.S, (1990), Setting the Clock Back to Zero Property for a Class of Life Distribution, Journal of Statistical Planning and Inference, Vol.24, pp 347-352.
8. Sendhamizhselvi.S., (2009), A Study on Expected Time to Recruitment in Manpower Model for a Multigraded System Associated with an Univariate Policy of Recruitment, Ph.D Thesis, Bharathidasan University.
9. SendhamizhSelvi.S and Jenita.S, (2016), Estimation of Mean and Variance of Time to Recruitment in a Two Graded Manpower System with Two Continuous Thresholds for Depletion Having Independent and Non-identically Distributed Random Variables, Proceedings of Heber International Conference on Applications of Actuarial Science, Mathematics, Management and Computer Science, pp 116-127.
10. SendhamizhSelvi.S and Jenita.S, (2016), Estimation of Mean Time to Recruitment in a Two Graded Manpower System with Depletion and Inter-decision Times are Independent and Non-identically Distributed Random Variables, International Journal of Mathematics Trends and Technology (IJMTT), ISSN: 2231-5373, Vol 39, pp: 34-41.
11. SendhamizhSelvi.S and Jenita.S, (2016), Estimation of Mean Time to Recruitment for a Two Graded Manpower System Involving Independent and Non-identically Distributed Random Variables with Thresholds Having SCBZ Property, International Journal of Science and Research (IJSR), ISSN: 2319-7064, Vol 6, Issue 3, pp: 1001-1005.
12. SendhamizhSelvi.S and Jenita.S, (2018), Variance of Time to Recruitment for a Two Graded Manpower System with Different Distribution Thresholds having Non-identically Distributed Wastages and Correlated Inter-Decision Times, Journal of Global Research in Mathematical Archivers,(JGRMA), ISSN: 2320-5822, Vol 5, No. 7, pp: 63-68.