On Difference Cordial Labeling of Networks

D. Florence Isido, V.M.Chitra

Abstract: LetG be a(p,q) . Let $f: V(G) \to \{1,2,3,\ldots,p\}$ be a function. For each edge uv assign the label $|f(u)-f(v)|$, f is called a difference cordial labeling if f is one to one map and $|e_f(0)-e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and labeled not with 1 respectively. A graph with difference cordial labeling is called difference cordial graph. In this paper we investigate the difference cordial labeling of the butterfly network, benes network, path union of cycles and corona product of cycles with K_1 , $K_2$and K_3 .

Keywords: Butterfly graph, benes graph, path union of cycles, corona product of cycles with K_1, $K_2$and K_3 and difference cordial labeling.

I. INTRODUCTION

Rosa introduced the concept of labeling in 1967.Graph labeling is the assignment of integers tovertices or edges or bothof a graphsubject to certain conditions..

Graph labeling has applications in Coding Theory, Radar location codes, Missile guidance codes, optimal circuit design, X-Ray Crystallographic analysis, cloud computing, data mining andconflict resolution.

Cahit introduced cordial labeling in 1987 and R.Ponraj, S.Sathish Narayanan and R.Kala introduced difference cordial labeling in 2013 and investigated difference cordial labeling of paths, cycles, bipartite graph, complete graph etc.In 2015Seoud and Salman studieddifference cordial labeling the Ladder graph, triangular ladder, diagonal ladder, step ladder, two sided step ladder.

The difference cordial labeling of butterfly network, benes network, path union of cycles and corona product of cycles with K_1 , $K_2$and K_3 are defined and discussed.

II. PRELIMINARIES

Definition 1: The n dimensional*butterfly network* , denoted by $BF(n)$, has a vertex set $V = \{(x;i); x \in V(Q_n), 0 \leq i \leq n$. Two vertices $(x;i)$ and $(y;j)$ are linked by an edge in $BF(n)$ if and only if $j = i+1$ and either

(i). $x = y$ or

(ii). x differs from y in precisely the j^{th} bit.

For $= y$, the edge is said to be a straight edge, otherwise the edge is a cross edge. For fixed $i$the vertex $(x;i)$ is a vertex on level i.

Definition 2: Then dimensional *benes network* consists of back to back butterfly network and it has $2n+1$ levels, $(2n+1)2^n$ vertices and $n2^{n+2}$ edges.

Definition 3: A *path union of graph*is obtained by from n copies $G_1, G_2, G_3 \ldots G_n$ of a graph G by adding an edge from $G_i$to G_{i+1},$i = 1,2\ldots n-1$.

Definition 4: *Corona product of graphs* of two graphs G and H is obtained by taking one copy of G and $|V(G)|$ copies of H and joining each vertex of the i[th] copy of H to the i[th] vertex of G ,where $1 \leq i \leq |V(G)|$.

III. RESULTS AND DISCUSSIONS

Theorem 1: Anybutterfly network $BF(n)$ is difference cordial graph.

Proof:

The butterfly network has $(n+1)2^n$ vertices and $n\,2^{n+1}$ edges. In level 0 and level n , vertices are of degree 2 and in other levels, vertices are of degree 4.

Define $f: V(G) \to \{1,2,3,\ldots(n+1)2^n\}$

$f(v_{ij}) = (n+1)(j-1)+(i+1), 0 \leq i \leq n, 1 \leq j \leq 2^n$

By the above labeling the straight edges are labeled with 1 and cross edges are laeled not with 1. The n dimensional butterfly network has $n(2^n)$ edges with label 1. The total number of edge is $n2^{n+1}$. Therefore the edges with label not with 1 is$n2^{n+1} - n2^n = n2^n$

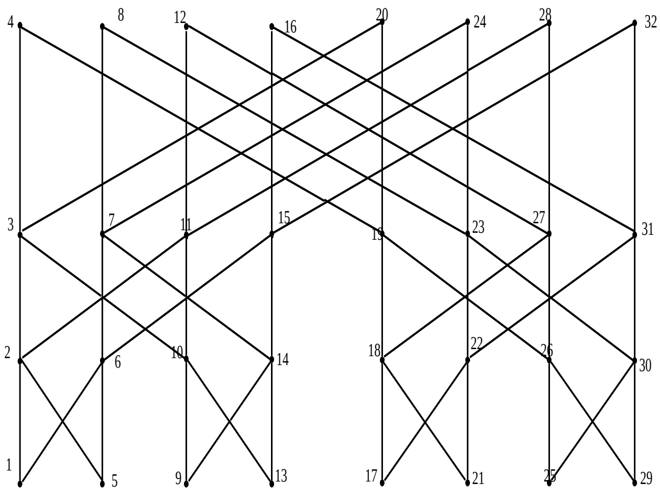

Fig. 1 Butterfly network BF (3)

$e_f(0)$	$e_f(1)$
$n2^n$	$n2^n$

$|e_f(0)-e_f(1)| = 0$

Hence any butterfly network is difference cordial graph.

Theorem 2: Any benes network is a difference cordial graph.

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Proof:

The n dimentional network has $(2n+1)2^n$ vertices and $n\,2^{n+2}$ edges. In level 0 and level n-1 , degree of vertices is 2 and in other levels, degree of vertices is 4.

Define $f: V(G) \to \{1, 2, 3, \ldots, (2n+1)2^n\}$

$f(v_{ij}) = (2n+1)(j-1) + (i+1), 0 \le i \le 2n, 1 \le j \le 2^n$.

The induced edge labeling,

$f^*(v_{ij}, v_{(i+1)j}) = 1, 0 \le i \le 2n, 1 \le j \le 2^n$

The benes network consists of $n(2^{n+2})$ edges, in that there are $2n(2^n) = n(2^{n+1})$ straight edges labeled with 1. Therefore the edges labeled not with 1 are equal to

$n\,(2^{n+2}) - n(2^{n+1}) = n(2^{n+1})$.

Here$|e_f(0) - e_f(1)| = 0$Hence benes network is difference cordial graph.

$e_f(0)$	$e_f(0)$
$n(2^{n+1})$	$n(2^{n+1})$

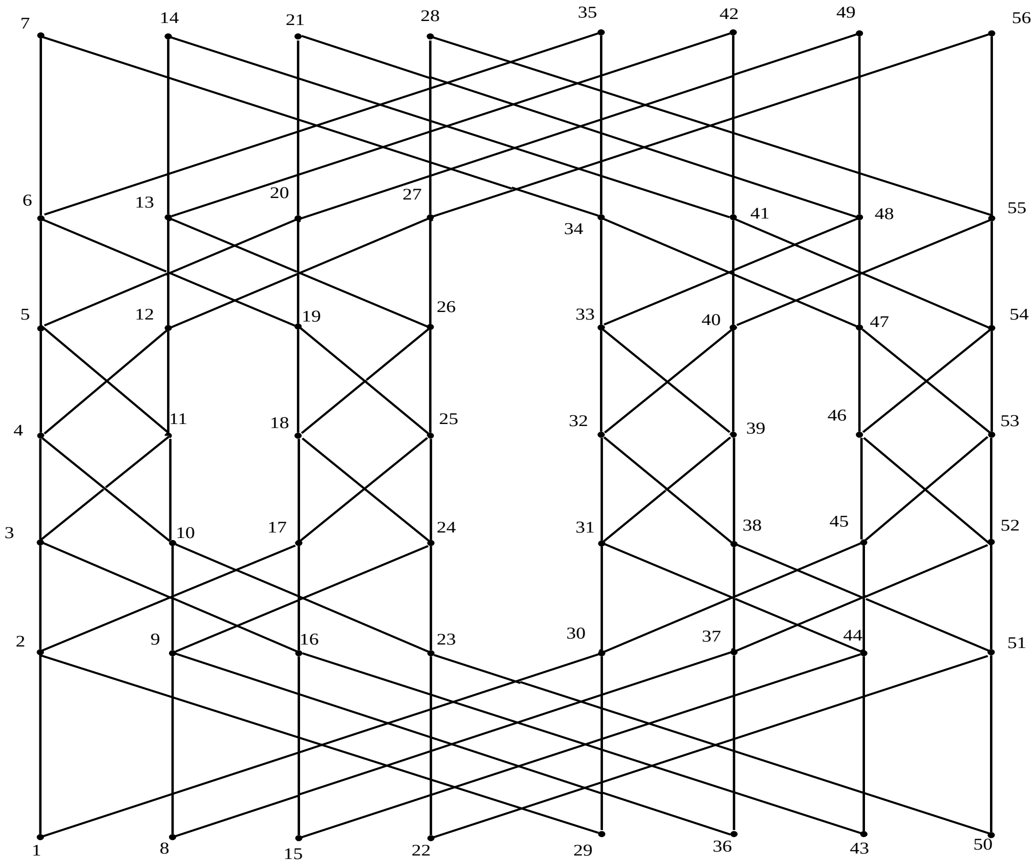

Fig. 2 Benes network BB (3)

Theorem 3: Any path union of cycle C_m is a difference cordial graph.

The path union of cycles has *mn* vertices and$mn + n - 1$ edges.

Define $f: V(G) \to \{1, 2, 3, \ldots, mn\}$

Case 1 : When *m* is even:

When i is odd , $f(v_{i1}) = 2i, f(v_{i2}) = 2i - 1, 1 \le i \le n$.

When i is even , $f(v_{i1}) = 2i - 1, f(v_{i2}) = 2i, 1 \le i \le n$.

$f(v_{i(2j-1)}) = 2(n-1) + 2j + (i-1)(m-2), 1 \le i \le n, 2 \le j \le \frac{m}{2}$

$f(v_{i(2j)}) = 2(n-1) + 2j - 1 + (i-1)(m-2), 1 \le i \le n, 2 \le j \le \frac{m}{2}$

$|f(v_{i1}) - f(v_{i2})| = |2i - (2i-1)| = 1, 1 \le i \le n.$

$|f(v_{i(2j-1)}) - f(v_{i(2j)})|$
$= |(2(n-1) + 2j + (i-1)(m-2))$
$- (2(n-1) + (2j-1) + (i-1)(m-2))| = 1, 1 \le i \le n, 2 \le j \le \frac{m}{2}$

$|f(v_{i(2j)}) - f(v_{i(2j+1) mod\, m})| = |f(v_{i(2j)}) - f(v_{i(2(j+1)-1)})|$
$= |(2(n-1) + (2j-1) + (i-1)(m-2))$
$- (2(n-1) + 2(j+1) + (i-1)(m-2))| = 3$

$|f(v_{(2i-1)1}) - f(v_{(2i)1})| = |2i - (2i-1)| = 1, 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$|f(v_{(2i)1}) - f(v_{(2i+1)1})| = |f(v_{(2i)1}) - f(v_{(2(i+1)-1)1})|$

$= |(2i-1) - (2(i+1))| = 3, 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

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The induced edge labeling,

$f^*(v_{i1}, v_{i2}) = 1, 1 \le i \le n$

$f^*(v_{i(2j-1)}, v_{i(2j)}) = 1, 1 \le i \le n, 2 \le j \le \frac{m}{2}$

$f^*(v_{i(2j)}, v_{i((2j+1)(mod\ m))}) = 0, 1 \le i \le n, 1 \le j \le \frac{m}{2}$

$f^*(v_{(2i-1)1}, v_{(2i)1}) = 1, 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f^*(v_{(2i)1}, v_{(2i+1)1}) = 0, 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

Case 1 a: When n is even

$e_f(1) = n\left(\frac{m}{2} - 1\right) + n + \frac{n}{2} = n\left(\frac{m-2}{2}\right) + 3\frac{n}{2} = n\left(\frac{m-2+3}{2}\right) = n\left(\frac{m+1}{2}\right)$

$e_f(0) = (m+1)n - 1 - (m+1)\frac{n}{2} = (m+1)\frac{n}{2} - 1$

$|e_f(0) - e_f(1)| = 1$

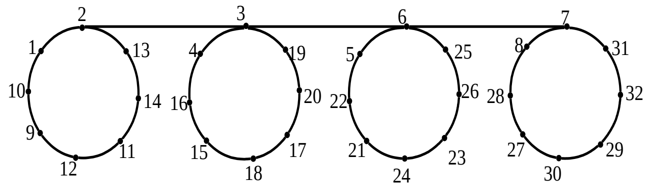

Fig. 3 Path union of cycles (m is even)

$e_f(1)$	$e_f(0)$
$(m+1)\frac{n}{2}$	$(m+1)\frac{n}{2} - 1$

Case 1 b: When n is odd

$e_f(1) = n\left(\frac{m}{2} - 1\right) + n + \frac{n-1}{2} = n\left(\frac{m-2}{2}\right) + 3\frac{n}{2} - \frac{1}{2} = n\left(\frac{m-2+3}{2}\right) - \frac{1}{2}$

$= n\left(\frac{m+1}{2}\right) - \frac{1}{2}$

$e_f(0) = (m+1)n - 1 - \frac{n(m+1)-1}{2} = \frac{n(m+1)-1}{2}$

$e_f(1)$	$e_f(0)$
$\frac{n(m+1)-1}{2}$	$\frac{n(m+1)-1}{2}$

$|e_f(0) - e_f(1)| = 0$

Case 2: When *m* is odd: $1 \le i \le n, 2 \le j \le \left\lfloor \frac{m}{2} \right\rfloor$

$f(v_{i1}) = 2 + n(i-1)$

$f(v_{i2}) = 1 + n(i-1)$

$f(v_{in}) = 3 + n(i-1)$

$f(v_{i(2j-1)}) = n(i-1) + 2j + 1$

$f(v_{i(2j)}) = n(i-1) + 2j$

$|f(v_{i1}) - f(v_{i2})| = |(2 + n(i-1)) - (1 + n(i-1))| = 1, 1 \le i \le n.$

$|f(v_{i1}) - f(v_{in})| = |(2 + n(i-1)) - (3 + n(i-1))| = 1, 1 \le i \le n.$

$|f(v_{i(2j-1)}) - f(v_{i(2j)})| = |(n(i-1) + 2j + 1) - (n(i-1) + 2j)| = 1,$

$1 \le i \le n, 2 \le j \le \frac{m-1}{2}$

$|f(v_{i(2j-1)}) - f(v_{i(2j+1)})| = |f(v_{i(2j-1)}) - f(v_{i(2(j+1)-1)})|$

$= |(n(i-1) + 2j + 1) - (n(i-1) + 2(j+1))| = 3,$

$1 \le i \le n, 2 \le j \le \frac{m-1}{2}$

The induced edge labeling is,

$f^*(v_{i1}, v_{i2}) = 1, 1 \le i \le n$

$f^*(v_{i1}, v_{in}) = 1, 1 \le i \le n$

$f^*(v_{i(2j-1)}, v_{i(2j)}) = 1, 1 \le i \le n, 2 \le j \le \frac{m-1}{2}$

$f^*(v_{i(2j-1)}, v_{i(2j)}) = 1, 2 \le j \le \frac{m-1}{2}$

$f^*(v_{i(2j)}, v_{i(2j+1)}) = 0, 1 \le j \le \frac{m-1}{2}$

Number of edges labeled with 1 is $n\left(\frac{m-1}{2} - 1\right) + 2n = n\left(\frac{m-1}{2} - 1 + 2\right) = n\left(\frac{m-1}{2} + 1\right) = n\left(\frac{m+1}{2}\right)$ and total number of edges are $mn + n - 1$.

So the number of edges labeled not with 1 is

$mn + n - 1 = (n(m+1) - 1) - \left(n\left(\frac{m+1}{2}\right)\right) = \left(\frac{n(m+1)}{2}\right) - 1$

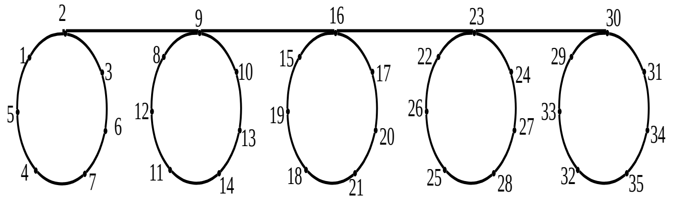

Fig. 4 Path union of cycle (m is odd)

$e_f(0)$	$e_f(1)$
$\frac{n(m+1)}{2} - 1$	$\frac{n(m+1)}{2}$

$|e_f(0) - e_f(1)| = 1$. Hence path union of cycle is a difference cordial graph.

Theorem 4: $C_n \odot K_1$ isa difference cordial graph.

Define $f: V(G) \to \{1, 2, 3, \ldots, 2n\}$

$f(v_{1j}) = (2j - 1), 1 \le j \le n.$

$f(v_{2j}) = 2j, 1 \le j \le n.$

$|f(v_{1j}) - f(v_{2j})| = |(2j-1) - 2j| = 1, 1 \le j \le n$

$|f(v_{1j}) - f(v_{1(j+1)})| = |(2j-1) - (2(j+1) - 1)| = 2, 1 \le j \le n$

It induces the edge labeling

$f^*(v_{1j}, v_{2j}) = 1, 1 \le j \le n$

$f^*(v_{1j}, v_{1(j+1)}) = 0, 1 \le j \le n$

$e_f(0)$	$e_f(1)$
n	n

$|e_f(0) - e_f(1)| = 0.$

Therefore $C_n \odot K_1$ is a difference cordial graph.

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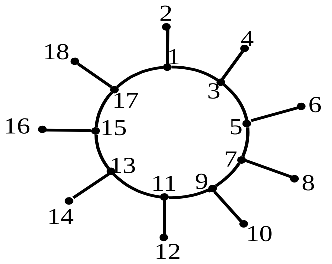

Fig. 5 Corona product of C_9 with K_1

Theorem 5: $C_n \odot K_2$ isa difference cordial graph.

It has $3n$ vertices and $4n$ edges.

Define $f: V(G) \to \{1, 2, 3, \dots 3n\}$

$f(v_{1j}) = 3j - 1, 1 \le j \le n.$

$f(v_{2(2j-1)}) = 3j - 2, 1 \le j \le n.$

$f(v_{2(2j)}) = 3j, 1 \le j \le n.$

$|f(v_{1j}) - f(v_{2(2j)})| = |(3j-1) - 3j| = 1, 1 \le j \le n.$

$|f(v_{1j}) - f(v_{2(2j-1)})| = |(3j-1) - (3j-2)| = 1, 1 \le j \le n.$

$|f(v_{2(2j-1)}) - f(v_{2(2j)})| = |(3j-2) - (3j)| = 2, 1 \le j \le n.$

$|f(v_{1j}) - f(v_{i(j+1)})| = |(3j-1) - (3(j+1)-1)| = 3, 1 \le j \le n.$

It induces the edge labeling

$f^*(v_{1j}, v_{2(2j)}) = 1, 1 \le j \le n$

$f^*(v_{1j}, v_{2(2j-1)}) = 1, 1 \le j \le n$

$f^*(v_{2(2j-1)}, v_{2(2j)}) = 0, 1 \le j \le n$

$f^*(v_{1j}, v_{1(j+1)}) = 0, 1 \le j \le n$

$e_f(1) = n + n = 2n.$

$e_f(0) = n + n = 2n$

$e_f(0)$	$e_f(1)$
$2n$	$2n$

$|e_f(0) - e_f(1)| = 0$. Hence $C_n \odot K_2$ is a difference cordial graph.

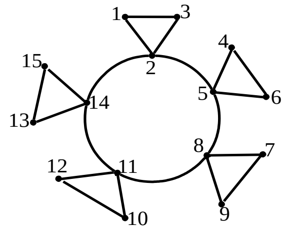

Fig. 6 Corona product of C_5 with K_2

Theorem 6: $C_n \odot K_3$ is nota difference cordial graph.

It has $4n$ vertices and $7n$ edges

Case 1: Label the vertices in C_n as well as the vertices in K_3 consecutively

Number of edges with label 1 is $(n-1) + n(2) = 3n - 1$ and the number of edges not with label 1 is $7n - (3n - 1) = 4n + 1$

Case 2: When each vertex of i^th copy of K_3 is connected to the i^th vertex of C_n, K_4 is formed. Label the vertices of K_4 consecutively. K_4 can have at most three edges with label 1, hence

Number of edges with label 1 is $3n$ and the number of edges not with label 1 is $7n - 3n = 4n$.

In both the cases the labeling is not difference cordial labeling.

Illustration:

Define $f: V(G) \to \{1, 2, 3, \dots, 4n\}$

$f(v_{1j}) = 4j - 2, 1 \le j \le n$

$f(v_{2(2j-1)}) = 4j - 3, 1 \le j \le n.$

$f(v_{2(2j)}) = 4j - 1, 1 \le j \le n.$

$f(v_{3j}) = 4j, 1 \le j \le n.$

$|f(v_{1j}) - f(v_{2(2j)})| = |(4j-2) - (4j-1)| = 1, 1 \le j \le n$

$|f(v_{1j}) - f(v_{2(2j-1)})| = |(4j-2) - (4j-3)| = 1, 1 \le j \le n$

$|f(v_{3j}) - f(v_{2(2j)})| = |(4j) - (4j-1)| = 1, 1 \le j \le n$

$e_f(1) = n + n + n = 3n.$

$e_f(0) = 7n - 3n = 4n.$

$e_f(0)$	$e_f(1)$
$4n$	$3n$

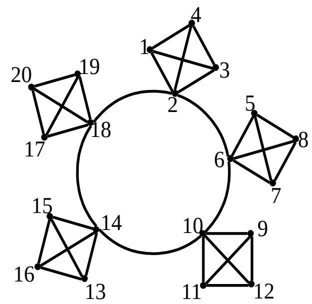

Fig. 7 Corona product of cycle with K_3

$|e_f(0) - e_f(1)| = n$. Hence $C_n \odot K_3$ is not a difference cordial graph

IV. CONCLUSION

The difference cordial labeling of butterfly network, benes network, path union of cycles and corona product of cycles with K_1, K_2 and K_3 are investigated.

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