A Performance on MMD Graphs

R.Revathi, R. Mary Jeya Jothi

Abstract: ***A graph $G(V,E)$ with n vertices is said to have modular multiplicative divisor (MMD) labeling if there exist a bijection $f : V(G) \to \{1,2,\ldots,n\}$ and the induced function $f^* : E(G) \to \{0,1,2,\ldots,n-1\}$ where $f^*(uv) = f(u)f(v)(\bmod n)$ for all $uv \in E(G)$ such that the sum of all edge weights is a multiple of n. This paper studies MMD labeling of larger families of graphs namely, the join of $C_n^{\,+}$ with union of $2m$ disjoint copies of K_1 and corona of star S_n with union of $2m$ disjoint copies of K_1.***

Keywords: ***Graph labeling, corona, join of two graphs, MMD labeling, graph $C_n^{\,+}$***

I. INTRODUCTION

Throughout this paper we consider simple, finite, undirected and connected graphs and refer Harary [1] for notation in graph theory. Survey paper [2] reveals many results on labeling of graphs. Graph labeling has more number of applications in number theory. Application of number theory in graph coloring was discussed by Jothi [6]. Motivated by the result of Marbun[3], we [5] construct large families of graphs using the operations corona, union and addition. Also we [4] characterize certain families of MMD graphs. This paper study the existence of MMD labeling of corona of star S_n with disjoint union of $2m$ copies of K_1 and MMD labeling of the join of $C_n^{\,+}$ (adding pendant edge for each vertex of the cycle C_n) with disjoint union of $2m$ copies of K_1.

II. PRELIMINARIES

Definition2.1. Allocating integers to each vertex or edge, or both under some restrictions is known as graph labeling.

Definition2.2. The *corona* $G \circ H$ of two graphs G (with n number of vertices) and H is constructed from one copy of G and n copies of H, namely $H_1, H_2, \ldots, H_{|V(G)|}$ and join every v_i in $V(G)$ to all vertices in $V(H_i)$. The corona $G \circ H$ of two graphs G and H is illustrated in figure 2.1.

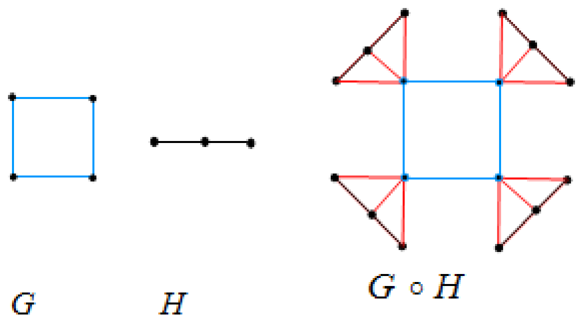

Fig. 2.1 Corona of two graphs *G* and *H*

Definition: 2.3. *Join* of two graphs G and H is obtained by joining each vertex of G with every vertex of H. It is denoted by $G+H$. $V(G+H)$ is equal to $V(G) \cup V(H)$ and $E(G+H)$ is equal to $E(G) \cup E(H) \cup \{e_{xy} / x \in V(G), y \in V(H)\}$. Join of two graphs G and H is shown in figure 2.2.

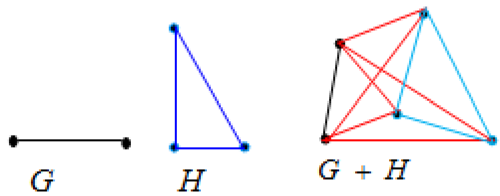

Fig. 2.2 Join of two graphs *G* and *H*

Illustration 2.4

Modular multiplicative divisor graph with $n = 7$ is illustrated in figure 2.3. Addition of all edge weights = 0+0+5+2+1+6 =14 ≡ 0(mod 7).

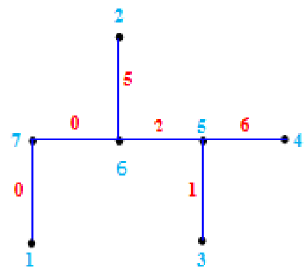

Fig. 2.3 MMD graph

Definition: 2.5 The *star graph* S_p is the complete bipartite graph $K_{1,p}$ with $p+1$ vertices and p edges.

III. MAIN RESULTS

Any graph G, the graph mG denotes the union of m disjoint copies of G. In this section we discuss MMD labeling of two families of graphs, namely join of $C_n^{\,+}$ with union of $2m$

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disjoint copies of K_1 and the corona of star S_n with union of $2m$ disjoint copies of K_1

Theorem: 3.1 The join of C_n^+ with disjoint union of $2m$ copies of K_1 admits MMD labeling.

Proof:

Let $V(C_n^+)$ be{ $v_1,v_2,v_3,\ldots,v_{2n}$} and $E(C_n^+)$ be {v_iv_{i+1}, $1 \le i \le n-1$} U { v_nv_1} U {v_iv_{n+i}, $1 \le i \le n$}. Lable the vertices of as follows.

Pair the vertices as s_i = {(v_{n+i} , v_{i+1}), $1 \le i \le n-1$} with $d(v_{n+i} , v_{i+1}) = 2$ such that $s_i \cap s_j = \phi$ for $i \neq j$. There exists $\frac{N}{2}-1$ distinct pairs of positive integers (x_i, y_i) namely $\left\{(1, N-1), (2, N-2), \ldots, \left(\frac{N}{2}-1, \frac{N}{2}+1\right)\right\}$ such that $x_i + y_i = N, 1 \le i \le \frac{N}{2}-1.$

Assign $\frac{N}{2}-m-1$ pairs of integers to $\frac{N}{2}-m-1$ pairs of vertices. Lable the vertices of v_1 and v_{2n} from the next pair of integers $\left(\frac{N}{2}-m, \frac{N}{2}+m\right)$. Adding all edge weights of C_n^+, we have

$$\sum_{i=1}^{n-1} f(v_i)f(v_{i+1}) + f(v_1)f(v_n)\sum_{i=1}^{n} f(v_i)f(v_{n+i})$$
$$= N\left[f(v_1)+f(v_2)+\ldots..f(v_{n-1})\right]+N\, f(v_n)$$
$$\equiv (0 \bmod N) \qquad (1)$$

Remaining $2m$ copies of K_1 vertices { $u_1,u_2,\ldots,u_{2m}$} can be labeled with the remaining m pairs of distinct integers. Let G be the join of C_n^+ and disjoint union of $2m$ copies of K_1 with $2(m+n)$ vertices and $2n(2m+1)$ edges. Addition of all remaining edge weights of G is equal to

$$\left[f(v_1)+f(v_2)+\ldots..f(v_{2n})\right]\left[f(u_1)+f(u_2)+\ldots..f(u_{2m})\right]$$
$$mN\left[f(v_1)+f(v_2)+\ldots..f(v_{2n})\right]$$
$$\equiv (0 \bmod N) \qquad (2)$$

From (1) and (2) the theorem follows.

Example: 3.2 The join of C_4^+ with disjoint union of 2 copies of K_1 is illustrated in figure 3.1

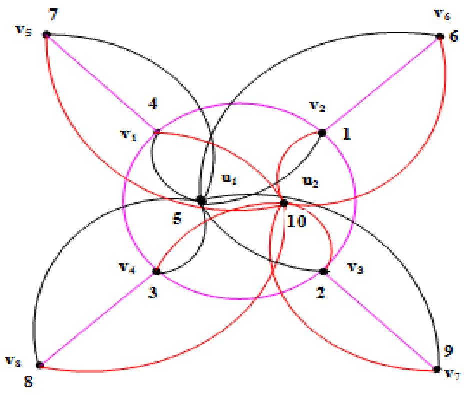

Fig. 3.1 MMD labeling of $C_4^+ + 2K_1$

Corollary: Since $\sum_{i=1}^{2n} f(v_i) = nN$, sum of all edge labels of $C_n^+ + K_1 = nN + nN = 2nN.$

Hence for any positive integer m, the result holds good for $C_n^+ + mK_1$.

The addition of all edge labels of

$C_n^+ + mK_1 = nN + nmN = (m+1)nN.$

Theorem: 3.3 The corona of star S_n with the union of $2m$ disjoint copies of K_1 admits MMD labeling.

Proof: The star graph S_n has $n+1$ vertices { $v_1,v_2,\ldots,v_{n+1}$} and v_{n+1} as the apex vertex. Let G be the join of S_n with the union of $2m$ disjoint copies of K_1 with the vertex set { $v_1,\ldots,v_{n+1}$, $u_1,u_2,\ldots,u_{2m(n+1)}$ }and the edge set {v_iu_j, $1 \le i \le n+1$, $2m(i-1) \le j \le 2mi$}. The graph G has $(n+1)(2m+1)$vertices and $2mn+2m+n$ $=N$(say) edges. Vertices of G are as follows.

$f(u_i) = i$

$f(u_{m+i}) = N-i$

$f(u_{2m+i}) = m+i$

$f(u_{3m+i}) = N-m-i$

$f(u_{4m+i}) = 2m+i$

$f(u_{5m+i}) = N-2m-i$

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$f(u_{2nm+i}) =$

$f(u_{(2n+1)m+i}) = N-nm-i$

$f(v_{n+1}) = N$ for all $1 \le i \le m$. Sum of all edge weights of G is equal to

$f(v_1)[f(u_1) +\ldots+f(u_{2m})]$ +

$f(v_2)[f(u_{2m+1}) +\ldots+f(u_{4m})]$ +

……………………………+

$f(v_{n+1})[f(u_{2mn+1}) +\ldots+f(u_{2m(n+1)})+ f(v_1) +\ldots+f(v_n)]$ which is congruent to N. Hence the result.

Example: 3.4 MMD labeling of corona of star S_5 with disjoint union of 2 copies of K_1

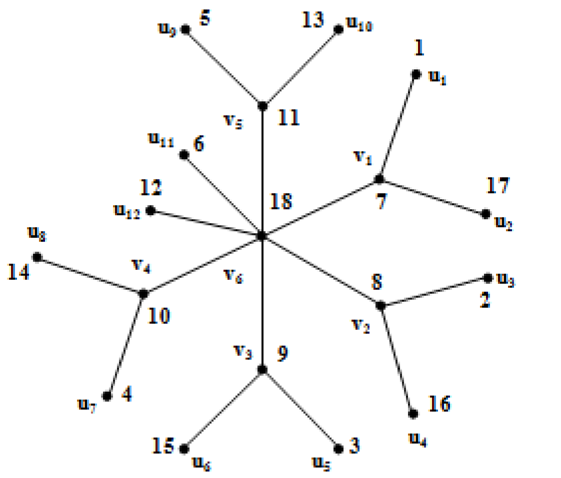

Fig. 3.2 MMD labeling of $S_5 \circ 2K_1$

Construction of the graph S_n corona with the union of $2m$ disjoint copies of K_1 is shown in figure 3.3.

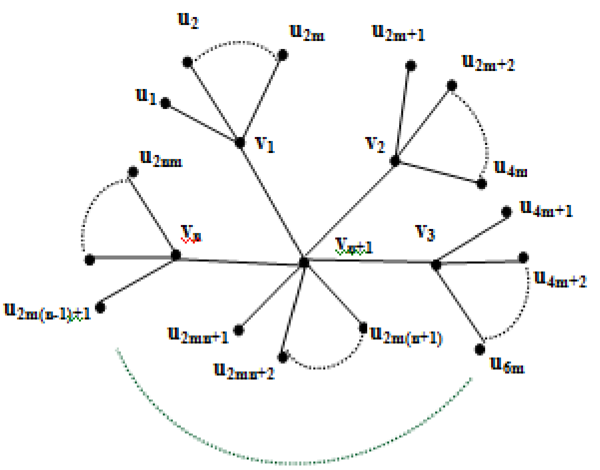

Fig. 3.3 MMD labeling of $S_n \circ 2mK_1$

IV. CONCLUSION AND OPEN PROBLEM

Constructed larger MMD graphs from standard graphs by using graph operations corona and join of two graphs. The problem still remains open for $m \equiv 1 (mod\ 2)$.

Problem: Does there exists MMD labeling for corona of star S_n with the union of m disjoint copies of K_1 for all $n>0$ and $m \equiv 1 (mod\ 2)$?

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