Super C-Logarithmic Meanness of Graphs Obtained from Paths

A.Durai Baskar, S.Saratha Devi

Abstract: ***Two functions namely f and f^* is defined for a graph G whose order $|V| = p$ and size $|E| = q$ as $f: V(G) \to \{1, 2, \dots, |V| + |E|\}$ is an injective and $f^*[uv] = \left\lceil \frac{f(v)-f(u)}{\ln f(v) - \ln f(u)} \right\rceil$ is an induced bijective function of f respectively. Then f is called a Super C-logarithmic mean labeling if $f(V(G)) \cup \{f^*(uv); uv \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a Super C-logarithmic mean labeling is called a Super C-logarithmic mean graph. In this manuscript, some of the graphs like path, total graph of a path, middle graph of a path, triangular ladder, the graph $P_n \odot S_m$ for $m \leq 3$, the graph $TW(P_n)$, the graph $[P_n; S_1]$, subdivision of $P_n \odot K_1$ and the arbitrary subdivision of $K_{1,3}$ admits Super C-logarithmic mean labeling.***

Keywords: Labeling, logarithmic mean labeling, logarithmic mean graph.

I. INTRODUCTION

In this paper, only finite, simple and undirected graphs are considered. For terminology, definitions we follow [6] and for survey [5].

The concept of geometric mean labeling was introduced and studied the geometric mean labeling of some standard graphs [1, 2]. The concept of super geometric labeling was first introduced by A. Durai Baskar et al. and studied the super geometric mean labeling of some special classes of graphs [3, 4].

Motivated by the works on super geometric mean labeling, we introduced a new type of labeling called Super C-logarithmic mean labeling. The logarithmic mean of any two numbers need not be an integer. To assign the edge label as an integer based on the logarithmic mean, we may use either flooring function or ceiling function. In this paper, we consider the ceiling function of our discussion.

A vertex labeling of is an assignment $f: V(G) \to \{1,2,3, \dots, p+$ be an injection. For a vertex labeling , the induced edge labeling is defined as $f^*(uv) = \left\lceil \frac{f(v)-f(u)}{\ln f(v) - \ln f(} \right.$ Then f is called a Super C-logarithmic mean labeling if $f(V(G)) \cup \{f^*(uv); uv \in E(G)\} = \{1,2,3, \dots, p+q\}$. A graph that admits a Super C-logarithmic mean labeling is called a Super C-logarithmic mean graph.

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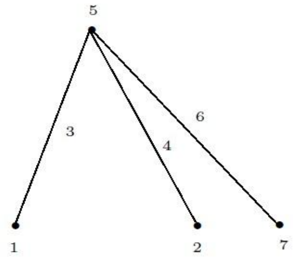

Fig. 1.1 A Super C-logarithmic mean labeling of S_3

In this paper, we have established the Super C-logarithmic meanness of the graph $P_n \odot S_m$ for $m \leq 3$, square graph of a path, total graph of a path, middle graph of a path, triangular ladder, the graph $[P_n; S_1]$, subdivision of $P_n \odot K_1$, the graph $TW(P_n)$ and the arbitrary subdivision of $K_{1,3}$.

II. MAIN RESULTS

Theorem 2.1 *Any P_n allows Super C-logarithmic mean labeling.*

Proof. Let the vertices of P_n be $v_1, v_2, \dots, v_n$. We define $f: V(P_n) \cup E(P_n) \to \{1,2, \dots, 2n-1\}$ as follows:

$f(v_i) = 2i - 1$, for i belongs to $[1, n]$

Then the induced edge labeling is as follows:

$f^*(v_i v_{i+1}) = 2i$, for i belongs to $[1, n-1]$

Hence f is a Super C-logarithmic mean labeling of the path P_n. Thus the P_n is a Super C-logarithmic mean graph.

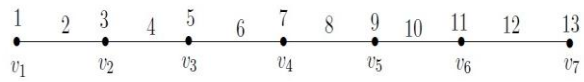

Fig. 1.1 A Super C-logarithmic mean labeling of S_3

Theorem 2.2 *P_n^2 admits Super C-logarithmic mean labeling, for $n > 2$.*

Proof. Let the vertices of P_n be $v_1, v_2, \dots, v_n$ .We define $f: V(P_n^2) \cup E(P_n^2) \to \{1,2,3, \dots, 3n-3\}$ as follows:

$f(v_1) = 1,$

$$f(v_i) = \begin{cases} 3i-2 & 3 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 3i-3 & 2 \leq i \leq n-1 \text{ and } i \text{ is even} \end{cases}$$

and $f(v_n) = 3n - 3.$

Then the induced edge labeling is as follows:

$f^*(v_i v_{i+1}) = 3i - 1,$ for i belongs to $[1, □ - 1]$ and

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$f^*(v_i v_{i+2}) =$
$\begin{cases} 3i+1 & 1 \le i \le n-2 \text{ and } i \text{ is odd} \\ 3i & 2 \le i \le n-2 \text{ and } i \text{ is even} \end{cases}$

Hence, f is a Super C-logarithmic mean labeling of P_n^2. Thus the graph P_n^2 is a Super C-logarithmic mean graph, for $n \geq 3$.

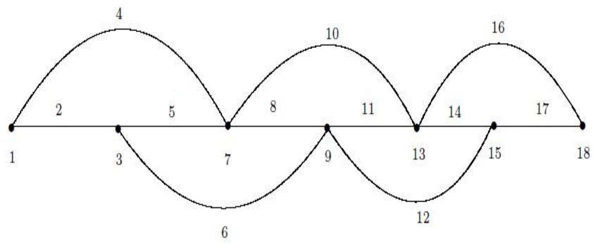

Fig. 2.2 A Super C-logarithmic mean labeling of P_7^2

Theorem 2.3 *The total graph $T(P_n)$ is a Super C-logarithmic mean graph, for $n \geq 3$.*

Proof. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}; 1 \le i \le n-1\}$ be the vertex set and edge set of the path P_n. Then $V(T(P_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}\}$ and

$E(T(P_n)) = \{v_i, v_{i+1}, e_i v_i, e_i v_{i+1}; 1 \le i \le n-1\} \cup \{e_i e_{i+1}; 1 \le i \le n-2\}$.

We define

$f: V(T(P_n)) \cup E(T(P_n)) \to \{1,2,3,\dots,6n-6\}$ as follows:

$f(v_i) = \begin{cases} 6i-5 & 1 \le i \le 2 \\ 6i-6 & 3 \le i \le n \end{cases}$

and $f(e_i) = \begin{cases} 6i-3 & 1 \le i \le 2 \\ 6i-2 & 3 \le i \le n-1. \end{cases}$

Then the induced edge labeling is as follows:

$f^*(v_i v_{i+1}) = \begin{cases} 6i-2 & 1 \le i \le 2 \\ 6i-3 & 3 \le i \le n \end{cases}$

$f^*(e_i v_i) = 6i-4$, for $1 \le i \le n-1$,

$f^*(e_i v_{i+1}) = 6i-1$, for $1 \le i \le n-1$,

and $f^*(e_i e_{i+1}) = \begin{cases} 6 & i=1 \\ 6i+1 & 2 \le i \le n-1. \end{cases}$

Hence, f is a Super C-logarithmic mean labeling of $T(P_n)$. Thus the graph $T(P_n)$ is a Super C-logarithmic mean graph, for $n \geq 3$.

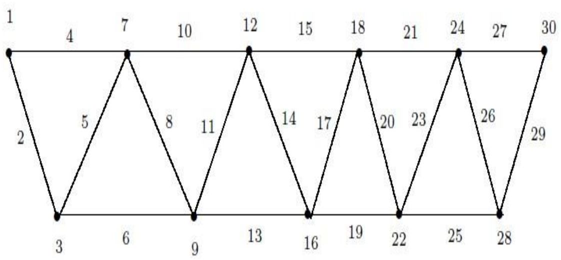

Fig. 2.3 A Super C-logarithmic mean labeling of $T(P_6)$

Theorem 2.4 *$M(P_n)$ is a Super C-logarithmic mean graph, for $n \geq 2$.*

Proof. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}: 1 \le i \le n-1\}$ be the vertex set and edge set of the path P_n. Then

$V(M(P_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}\}$ and

$E(M(P_n)) = \{v_i e_i, e_i v_{i+1}: 1 \le i \le n-1\} \cup \{e_i e_{i+1}: 1 \le i \le n-2\}$.

We define

$f: V(M(P_n)) \cup E(M(P_n)) \to \{1,2,3,\dots,5n-5\}$ as follows: $f(v_i) = \begin{cases} 1 & i=1 \\ 5i-5 & 2 \le i \le n \end{cases}$

and $f(e_i) = 5i-2$, for $1 \le i \le n-1$.

Then the induced edge labeling is as follows:

$f^*(e_i e_{i+1}) = 5i+1$, for $1 \le i \le n-2$,

$f^*(e_i v_i) = 5i-3$, for $1 \le i \le n-1$

and $f^*(e_i v_{i+1}) = 5i-1$, for $1 \le i \le n-1$.

Hence, f is a Super C-logarithmic mean labeling of $M(P_n)$. Thus the graph $M(P_n)$ is a Super C-logarithmic mean graph for $n \geq 2$.

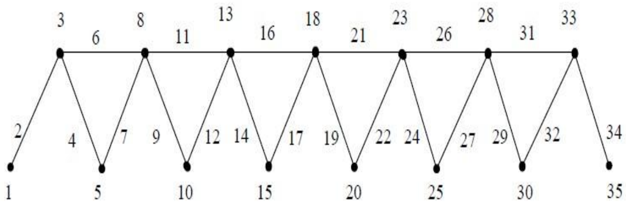

Fig. 2.4 A Super C-logarithmic mean labeling of $M(P_8)$

Theorem 2.5 *TL_n is a Super C-logarithmic mean graph, for $n \geq 2$.*

Proof. Let the vertex set of TL_n be $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ and the edge set of TL_n be $\{u_i u_{i+1}, u_i v_{i+1}, v_i v_{i+1}: 1 \le i \le n-1\} \cup \{u_i v_i: 1 \le i \le n\}$.

Then TL_n has $2n$ vertices and $4n-3$ edges.

We define $f: V(TL_n) \cup E(TL_n) \to \{1,2,3,\dots,6n-3\}$ as follows:

$f(v_i) = 6i-5$, for $1 \le i \le n$

and $f(u_i) = 6i-3$, for $1 \le i \le n$.

Then the induced edge labeling is as follows:

$f^*(v_i v_{i+1}) = 6i-2$ for $1 \le i \le n-1$,

$f^*(u_i u_{i+1}) = 6i$ for $1 \le i \le n-1$,

$f^*(u_i v_i) = 6i-4$ for $1 \le i \le n$

and $f^*(u_i v_{i+1}) = 6i-1$, for $1 \le i \le n-1$.

Hence, f is a Super C-logarithmic mean labeling of TL_n. Thus the graph TL_n is a Super C-logarithmic mean graph for $n \geq 2$.

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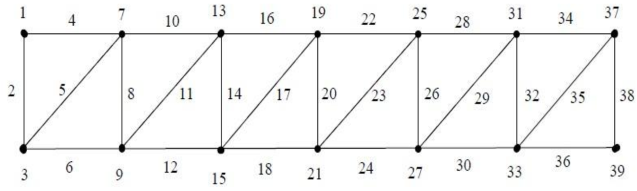

Fig. 2.5 A Super C-logarithmic mean labeling of TL_7

Theorem 2.6 *The graph $P_n \odot S_m$ is a Super C-logarithmic mean graph, for $n \geq 1$ and $m \leq 3$.*

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}, \dots, v_m^{(i)}$ be the pendant vertices at each vertex u_i of the path P_n, for $1 \leq i \leq n$.

Case 1. $m = 1$. We define $f: V(P_n \odot S_1) \cup E(P_n \odot S_1) \rightarrow \{1,2,3, \dots, 4n-1\}$ as follows:

$f(u_i) = 4i - 1$, for $1 \leq i \leq n$ and

$$f\left(v_1^{(i)}\right) = \begin{cases} 1 & i = 1 \\ 4i-4 & 2 \leq i \leq n \end{cases}$$

Then the induced edge labeling is as follows:

$f^*(u_i u_{i+1}) = 4i + 1$, for $1 \leq i \leq n-1$ and

$f^*\left(v_1^{(i)} u_i\right) = 4i - 2$, for $1 \leq i \leq n$.

Case 2. $m = 2$.

We define $f: V(P_n \odot S_2) \cup E(P_n \odot S_2) \rightarrow \{1,2,3, \dots, 6n-1\}$ as follows:

$f(u_i) = 6i - 3$, for $1 \leq i \leq n$,

$f(v_1^{(i)}) = 6i - 5$, for $1 \leq i \leq n$ and

$f\left(v_2^{(i)}\right) = 6i - 1$, for $1 \leq i \leq n$.

Then the induced edge labeling is as follows:

$f^*(u_i u_{i+1}) = 6i$, for $1 \leq i \leq n-1$,

$f^*(v_1^{(i)} u_i) = 6i - 4$, for $1 \leq i \leq n$ and

$f^*\left(v_2^{(i)} u_i\right) = 6i - 2$, for $1 \leq i \leq n$.

Case 3. $m = 3$.

We define $f: V(P_n \odot S_3) \cup E(P_n \odot S_3) \rightarrow \{1,2,3, \dots, 8n-1\}$ as follows:

$f(u_i) = 8i - 3$, for $1 \leq i \leq n$,

$$f(v_1^{(i)}) = \begin{cases} 1 & i = 1 \\ 8i-8 & 2 \leq i \leq n \end{cases}$$

$f(v_2^{(i)}) = 8i - 6$, for $1 \leq i \leq n$ and $f\left(v_3^{(i)}\right) = 8i - 1$, for $1 \leq i \leq n$.

Then the induced edge labeling is as follows:

$f^*(u_i u_{i+1}) = 8i + 1$, for $1 \leq i \leq n-1$,

$f^*(v_1^{(i)} u_i) = 8i - 5$, for $1 \leq i \leq n$,

$f^*(v_2^{(i)} u_i) = 8i - 4$, for $1 \leq i \leq n$ and

$f^*\left(v_3^{(i)} u_i\right) = 8i - 2$, for $1 \leq i \leq n$.

Hence, f is a Super C-logarithmic mean labeling of $P_n \odot S_m$. Thus the graph $P_n \odot S_m$ is a Super C-logarithmic mean graph, for $n \geq 1$ and $m \leq 3$.

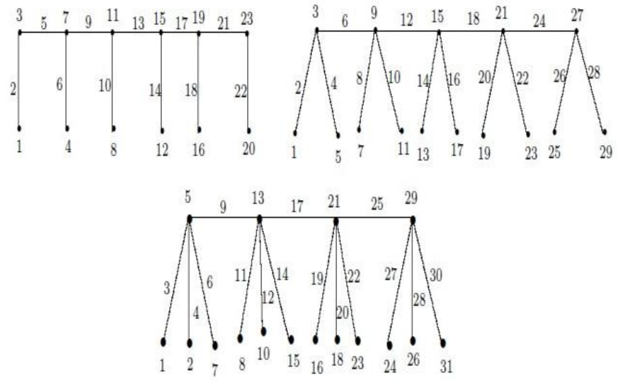

Fig. 2.6 A Super C-logarithmic mean labeling of $P_6 \odot S_1, P_5 \odot S_2$ and $P_4 \odot S_3$

Theorem 2.7 *$TW(P_n)$ is a Super C-logarithmic mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}$ be the pendant vertices at each vertex u_i of the path P_n, for $2 \leq i \leq n-1$. Then $V(TW(P_n)) = V(P_n) \cup \left\{v_1^{(i)}, v_2^{(i)} : 2 \leq i \leq n-1\right\}$ and $E(TW(P_n)) = E(P_n) \cup \left\{u_i v_1^{(i)}, u_i v_2^{(i)} : 2 \leq i \leq n-1\right\}$.

We define $f: V(TW(P_n)) \cup E(TW(P_n)) \rightarrow \{1,2,3, \dots, 6n-9\}$ as follows: $f(u_i) = \begin{cases} 1 & i = 1 \\ 6i-7 & 2 \leq i \leq n-2, \end{cases}$

$$f\left(v_1^{(i)}\right) = \begin{cases} 2 & i = 2 \\ 6i-9 & 3 \leq i \leq n-2, \end{cases}$$

$f\left(v_2^{(i)}\right) = 6i - 5$ for $2 \leq i \leq n-2$,

$f(u_{n-1}) = 6n - 11, f(u_n) = 6n - 9$,

$f\left(v_1^{(n-1)}\right) = 6n - 16$ and $f\left(v_2^{(n-1)}\right) = 6n - 14$.

Then the induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} 3 & i = 1 \\ 6i-4 & 2 \leq i \leq n-3, \end{cases}$$

$f^*\left(u_i v_1^{(i)}\right) = 6i - 8$ for $2 \leq i \leq n-2$,

$f^*\left(u_i v_2^{(i)}\right) = 6i - 6$ for $2 \leq i \leq n-1$,

$f^*(u_{n-2} u_{n-1}) = 6n - 15, f^*(u_{n-1} u_n) = 6n - 10$,

and $f^*\left(u_{n-1} v_1^{(n-1)}\right) = 6n - 13$.

Hence, f is a Super C-logarithmic mean labeling of $TW(P_n)$. Thus the graph $TW(P_n)$ is a Super C-logarithmic mean graph, for $n \geq 3$.

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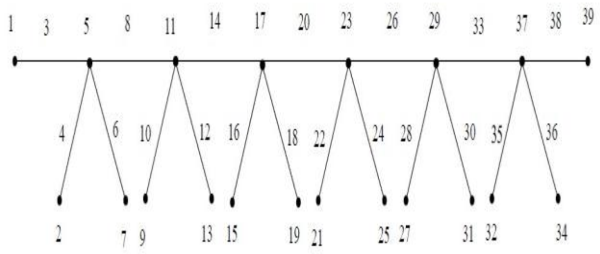

Fig. 2.7 A Super C-logarithmic mean labeling of $TW(P_8)$

Theorem 2.8 $[P_n; S_1]$ *is a Super C-logarithmic mean graph, for* $n \geq 1$.

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}, \dots, v_m^{(i)}$ be the pendant vertices at each vertex u_i of the path P_n, for $1 \leq i \leq n$. We define $f: V([P_n; S_1]) \cup E([P_n; S_1]) \to \{1,2,3,\dots,6n-1\}$ as follows:

$$f(u_i) = \begin{cases} 5 & i = 1 \\ 6i-5 & 2 \leq i \leq n, \end{cases}$$

$f\left(v_1^{(i)}\right) = 6i - 3$ for $1 \leq i \leq n$,

$$f\left(v_2^{(i)}\right) = \begin{cases} 1 & i = 1 \\ 6i & 2 \leq i \leq n-1 \end{cases}$$

and $f\left(v_2^{(n)}\right) = 6n - 1$.

Then the induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} 6 & i = 1 \\ 6i-2 & 2 \leq i \leq n-1, \end{cases}$$

$$f^*\left(u_i v_1^{(i)}\right) = \begin{cases} 4 & i = 1 \\ 6i-4 & 2 \leq i \leq n, \end{cases}$$

$$f^*\left(v_1^{(i)} v_2^{(i)}\right) = \begin{cases} 2 & i = 1 \\ 6i-1 & 2 \leq i \leq n-1 \end{cases}$$

and $f^*\left(v_1^{(n)} v_2^{(n)}\right) = 6n - 2$.

Hence, f is a Super C-logarithmic mean labeling of $[P_n; S_1]$. Thus the graph $[P_n; S_1]$ is a Super C-logarithmic mean graph, for $n \geq 1$.

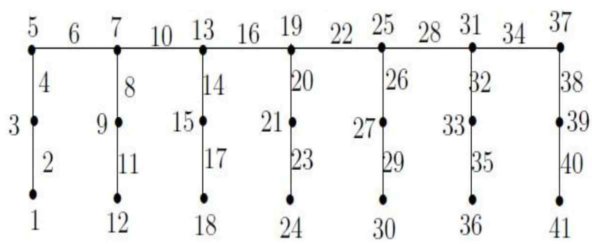

Fig. 2.8 A Super C-logarithmic mean labeling of $[P_7; S_1]$

Theorem 2.9 $S(P_n \odot K_1)$ *is a Super C-logarithmic mean graph, for* $n \geq 1$.

Proof. Let $V(P_n \odot K_1) = \{u_i, v_i : 1 \leq i \leq n\}$. Let x_i be the vertex which divides the edge $u_i v_i$, for $1 \leq i \leq n$ and y_i be the vertex which divides the edge $u_i v_{i+1}$, for $1 \leq i \leq n-1$. Then

$V(S(P_n \odot K_1)) = \{u_i, v_i, x_i, y_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$

$E((P_n \odot K_1)) = \{u_i x_i, v_i x_i : 1 \leq i \leq n\} \cup \{u_i y_i, y_i u_{i+1} : 1 \leq j \leq n-1\}$

We define

$f: V(S(P_n \odot K_1)) \cup E(S(P_n \odot K_1)) \to \{1,2,3,\dots,8n-3\}$

as follows:

$$f(u_i) = \begin{cases} 5 & i = 1 \\ 8i-7 & 2 \leq i \leq n, \end{cases}$$

$f(y_i) = 8i - 1$ for $1 \leq i \leq n-1$,

$f(x_i) = 8i - 5$ for $1 \leq i \leq n$,

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 8i-2 & 2 \leq i \leq n-1 \end{cases}$$

and $f(v_n) = 8n - 3$.

Then the induced edge labeling is as follows:

$$f^*(u_i y_i) = \begin{cases} 6 & i = 1 \\ 8i-4 & 2 \leq i \leq n-1, \end{cases}$$

$f^*(y_i u_{i+1}) = 8i$ for $1 \leq i \leq n-1$,

$$f^*(u_i x_i) = \begin{cases} 4 & i = 1 \\ 8i-6 & 2 \leq i \leq n, \end{cases}$$

$$f^*(x_i v_i) = \begin{cases} 2 & i = 1 \\ 8i-3 & 2 \leq i \leq n-1 \end{cases}$$

and $f^*(x_n v_n) = 8n - 4$.

Hence, f is a Super C-logarithmic mean labeling of $S(P_n \odot K_1)$. Thus the graph $S(P_n \odot K_1)$ is a Super C-logarithmic mean graph, for $n \geq 1$.

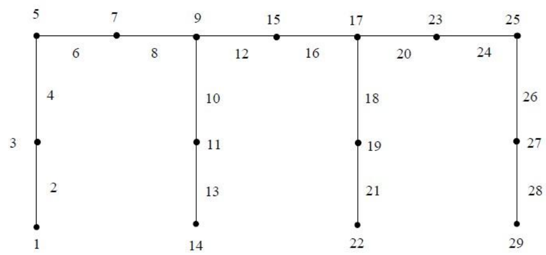

Fig. 2.9 A Super C-logarithmic mean labeling of $S(P_4 \odot K_1)$

Theorem 2.10 *An arbitrary subdivision of $K_{1,3}$ is a Super C-logarithmic mean graph.*

Proof. Let G be an arbitrary subdivision of $K_{1,3}$ and let v_0, v_1, v_2 and v_3 be the vertices of G in which v_0 is the central vertex and v_1, v_2 and v_3 are the pendant vertices of $K_{1,3}$.

Let the edges $v_0 v_1, v_0 v_2$ and $v_0 v_3$ of $K_{1,3}$ be subdivided by p_1, p_2 and p_3 number of vertices respectively.

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Let $v_0, v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \ldots, v_{p_1+1}^{(1)}(=v_1), v_0, v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \ldots, v_{p_2+1}^{(2)}(=v_2)$ and $v_0, v_1^{(3)}, v_2^{(3)}, v_3^{(3)}, \ldots, v_{p_3+1}^{(3)}(=v_3)$ be the vertices of G and $v_0 = v_0^{(i)}$, for $1 \le i \le 3$. Let $e_j^{(i)} = v_{j-1}^{(i)} v_j^{(i)}, 1 \le j \le p_i + 1$ and $1 \le i \le 3$ be the edges of G and it has $p_1 + p_2 + p_3 + 4$ vertices and $p_1 + p_2 + p_3 + 3$ edges with $p_1 \le p_2 \le p_3$.

Case 1. $p_1 = p_2$.

We define

$f: V(G) \cup E(G) \to \{1,2,3, \ldots, 2(p_1 + p_2 + p_3) + 7\}$ as follows:

$f(v_0) = 2(p_1 + p_2) + 5$,

$f(v_j^{(1)}) = 2(p_1 + p_2) + 5 - 4j$, for $1 \le j \le p_1 + 1$,

$f(v_j^{(2)}) = 2(p_1 + p_2) + 6 - 4j$, for $1 \le j \le p_2 + 1$,

and $f(v_j^{(3)}) = 2(p_1 + p_2) + 5 + 2j$, for $1 \le j \le p_3 + 1$.

Then the induced edge labeling is as follows:

$f^*\left(v_j^{(1)} v_{j+1}^{(1)}\right) = 2(p_1 + p_2) + 3 - 4j$, for $1 \le j \le p_1$,

$f^*\left(v_j^{(2)} v_{j+1}^{(2)}\right) = 2(p_1 + p_2) + 4 - 4j$, for $1 \le j \le p_2$,

$f^*\left(v_j^{(3)} v_{j+1}^{(3)}\right) = 2(p_1 + p_2) + 6 + 2j$, for $1 \le j \le p_3$,

$f^*\left(v_0 v_1^{(1)}\right) = 2(p_1 + p_2) + 3$,

$f^*\left(v_0 v_1^{(2)}\right) = 2(p_1 + p_2) + 4$ and

$f^*\left(v_0 v_1^{(3)}\right) = 2(p_1 + p_2) + 6$.

Case 2. $p_1 < p_2 < p_3$.

We define

$f: V(G) \cup E(G) \to \{1,2,3, \ldots, 2(p_1 + p_2 + p_3) + 7\}$ as follows:

$f(v_0) = 2(p_1 + p_2) + 5$,

$f(v_j^{(1)}) = 2(p_1 + p_2) + 6 - 4j$, for $1 \le j \le p_1 + 1$,

$$f(v_j^{(2)}) = \begin{cases} 2(p_1 + p_2) + 5 - 4j & 1 \le j \le p_1 + 1 \\ 2p_2 + 3 - 2j & p_1 + 2 \le j \le p_2 + 1 \end{cases}$$

and $f(v_j^{(3)}) = 2(p_1 + p_2) + 5 + 2j$, for $1 \le j \le p_3 + 1$.

Then the induced edge labeling is as follows:

$f^*\left(v_j^{(1)} v_{j+1}^{(1)}\right) = 2(p_1 + p_2) + 4 - 4j$, for $1 \le j \le p_1$,

$$f^*\left(v_j^{(2)} v_{j+1}^{(2)}\right) = \begin{cases} 2(p_1 + p_2) + 3 - 4j & 1 \le j \le p_1 \\ 2p_2 + 2 - 2j & p_1 + 1 \le j \le p_2, \end{cases}$$

$f^*\left(v_j^{(3)} v_{j+1}^{(3)}\right) = 2(p_1 + p_2) + 6 + 2j$, for $1 \le j \le p_3$,

$f^*\left(v_0 v_1^{(1)}\right) = 2(p_1 + p_2) + 4$,

$f^*\left(v_0 v_1^{(2)}\right) = 2(p_1 + p_2) + 3$

and $f^*\left(v_0 v_1^{(3)}\right) = 2(p_1 + p_2) + 6$.

III. CONCLUSION

Hence, f is a Super C-logarithmic mean labeling of G. Thus the graph G is a Super C-logarithmic mean graph.

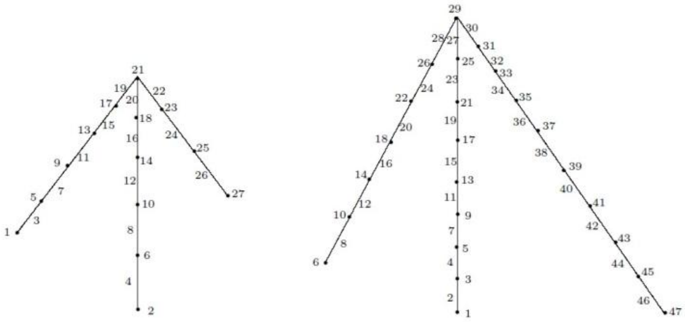

Fig. 2.10 A Super C-logarithmic mean labeling of arbitrary subdivision of $K_{1,3}$

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