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Fibonacci Prime Anti-magic Labeling of Cycle Related Graphs

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***Abstract;** A Fibonacci Prime Anti-magic labeling of a graph G is an injective function g: V (G) →{ f_2, f_3, … f_{n+1}}, where f_n is n^{th} Fibonacci number, such that g (uv) = g.c.d (g(u), g(v)) = 1, for all u, v ∈ V(G) and the induced a function g^* : E(G) → N defined by g*(uv) = (g(u) + g(v)) and all these edge labeling are distinct. A graph which admits Fibonacci Prime Anti-magic labeling is called a Fibonacci Prime Anti-magic graph. In this article we investigate some cycle related graphs are Fibonacci Prime Anti-magic labeling.*

***Keywords:** Fibonacci prime anti-magic graph, cycles, barycentric subdivision, crown graphs.*
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I. INTRODUCTION

We consider finite undirected graphs without loops and multiple edges. For notations and terminology we refer to D.B.West [7]. In section 1 we give basic definitions of graph labeling and an introduction to the prime anti-magic graphs.

Hartsfield and Ringel[2] introduced the concept of anti-magic labeling, which is an assignment of distinct values to different objects in a graph in such a way that when taking certain sums of the labels the sums will all be different. Different kinds of anti-magic graphs were studied by T.Nicholas, S.Somasundaram and V.Vilfred [4].

Definition 1.1

A graph G is called ***anti-magic*** if the q edges of G can be distinctly labeled 1 through q in such a way that when taking the sum of the edge labels incident to each vertex, the sums will all be different.
The notion of prime labeling was originated by Entringer and was discussed in A. Tout [6].

Definition 1.2

A ***prime labeling*** of a graph G of order p is an injective function g : V(G) → { 1, 2, .. , | V(G)| } such that for every pair of adjacent vertices receive co-prime images.
Fibonacci graceful labeling was introduced by Kathiresan and Amutha [3] and different kind of Fibonacci labeling were studied by [1] and [5].

Definition 1.3

The ***Fibonacci numbers*** can be defined by the linear recurrence relation $F_n = F_{n-1} + F_{n-2}$;
n ≥ 3.This generates the infinite sequence of integers beginning
1,1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…

Definition 1.4

A ***Fibonacci prime labeling*** of a graph G = (V, E) with |V (G)| = n is an injective function
g : V(G) → { f_2, f_3, … f_{n+1}}, where f_n is n^{th} Fibonacci number, that induces a function g^* : E(G) → N defined by g^* (uv) = g.c.d (g(u), g(v)) = 1, for all u, v ∈ E(G). The graph which admits Fibonacci prime labeling is called ***Fibonacci prime graph***.

II. MAIN RESULTS

In this chapter we provide Fibonacci Prime Anti-magic labeling schemes for particular families of graphs.

Definition 2.1

A ***Fibonacci Prime Anti-magic labeling*** of a graph G is an injective function g : V(G) → { f_2, f_3, … f_{n+1}}, where f_n is n^{th} Fibonacci number, such that g (uv) = g.c.d (g(u), g(v)) = 1, for all u, v ∈ V(G) and the induced function g^* : E(G) → N defined by g*(uv) = (g(u) + g(v)) and all the edge sums are pair wise distinct.

Theorem 2.2

Cycle C_n is a Fibonacci prime anti-magic graph for n ≥ 3.
Proof:
Let v_1,v_2,…,v_n be the vertices of the cycle C_n.
The edge set of C_n is E (C_n) = { $v_i\ v_{i+1}$/ 1 ≤ i ≤ n – 1} ∪ { $v_n\ v_1$ }.
Define g : V(C_n) → { f_2, f_3, … f_{n+1}} as g (v_i) = f_{i+1} , 1 ≤ i ≤ n, and also satisfies the condition g(uv) = g.c.d (g(u), g(v)) = 1, for all u, v ∈ V(G)
The induced function g^*: E (G) → N defined by g*(uv) = (g(u) + g(v)) and hence all the edge labeling are distinct.
Hence we proved the theorem.
Chandrakala.S and Sekar.C [1] gave the following definitions of $< G, K_{1,m} >$ and barycentric subdivision of a graph G.

Definition2.3

$< \boldsymbol{G, K_{1,m}} >$, m ≥ 1is the graph obtained by attaching $K_{1,m}$ to one vertex of the graph G.

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Definition 2.4

Let G = (V, E) be a graph. Let e = (uv) be an edge of G and w is not a vertex of G. The edge e is subdivided where it is replaced by edge e' = uw and e'' = vw. If every edge of a graph G is subdivided then the resulting graph is called ***barycentric subdivision*** of a graph G.

Theorem 2.5

The graph <C_n,$K_{1,m}$>, m ≥ 1 is a Fibonacci prime anti-magic graph.

Proof:

Let G = <C_n,$K_{1,m}$>.

The vertex set of the cycle $C_n$is V (C_n) = $\{ u_1, u_{2....} u_n \}. Let\ u_1$ be the common vertex of $C_n$and $K_{1,m}$.

Let the remaining vertices of $K_{1,m}$ be { $v_1, v_2, ... v_m$}.

Hence V (G) = $\{ u_1, u_2, ... u_n , v_1, v_2, ... v_m\}$ and E(G) = { $(u_i, u_{i+1}), 1 \le i \le n , u_{n+1} = u_1$}∪

{ (u_1, v_i), 1 ≤ i ≤ m }.

Then |V (G)| = n + m and |E(G)| = n + m.

Define g: V(G) → { f_2, f_3, … f_{n+m+1}} as follows:

g (u_i) = f_{i+1} , 1 ≤ i ≤ n

g (v_i) = f_{n+i} , 1 ≤ i ≤ m + 1 and also satisfies the condition

g.c.d {g(u_i), g(u_{i+1}) }= g.c.d {f_{i+1} , f_{i+2}}= 1 for 1 ≤ i ≤ n – 1,

g.c.d {g(u_n), g(u_1) }= g.c.d {f_{n+1} , f_2}= g.c.d {f_{n+1} , 1) }= 1,

g.c.d {g(u_1), g(v_i) }= g.c.d {f_2 , f_{n+i+1}}= 1 for 1 ≤ i ≤ m and also the edge sums are pair wise distinct.

Thus the theorem follows.

Theorem 2.6

Barycentric subdivision of the Cycle $C_n[C_n]$ is a Fibonacci prime anti-magic graph for all n.

Proof:

Let $\{u_1, u_2, ... u_n\}$ be the vertices of the cycle $C_n$and { $u_1', u_2', ... u_n'$}be the newly inserted vertices and obtain barycentric subdivision of cycle C_n. Join each newy inserted vertices of incident edges by an edge we get the new graph $C_n[C_n]$. Let G = $C_n[C_n]$, contains 2n vertices and 3n edges.

The vertices of G is V(G) = { u_i/ 1 ≤ i ≤ n } ∪{ u_i' / 1 ≤ i ≤ n }.

The edge set of G is E(G) = { $u_i\ u_i'$ / 1 ≤ i ≤ n} ∪ {$u_{i+1}' u_i$ / 1 ≤ i ≤ n - 1} ∪{ $u_1' u_n$ } ∪

{$u_i'\ u_{i+1}'$/ 1 ≤ i ≤ n - 1} ∪{ $u_1' u_n$ }.

The vertex labels are defined by

g(u_i') = f_{2i} , 1 ≤ i ≤ n, g(u_i) = f_{2i+1} , 1 ≤ i ≤ n, such that g(uv) = g.c.d (g(u), g(v)) = 1, for all u, v ∈ V(G)

The edge labeling g^* : E(G) → N is defined by g*(uv) = (g(u) + g(v)) for all uv ∈ E(G) and hence all the edge labeling are distinct.

Hence the proof.

Theorem 2.7

The Crown graph C_n^* is a Fibonacci prime anti-magic graph.

Proof:

Let G be a crown graphC_n^*.

Let V (G) = $\{ u_1, u_2, ... u_n , v_1, v_2, ... v_m\}$ and E(G) = { $u_i v_i$/ 1 ≤ i ≤ n - 1}∪ { ($u_n u_1$)}.

Define g: V(G) → { f_2, f_3, … f_{2n+1}} by

g (u_i) = f_{2i+1} , 2 ≤ i ≤ n

g (v_i) = f_{2i+1} , 2 ≤ i ≤ n and also satisfies the condition

g.c.d {g(u_i), g(v_i) }= g.c.d {f_{2i} , f_{2i+1}}= 1 for 1 ≤ i ≤ n ,

g.c.d {g(u_i), g(u_{i+1}) }= g.c.d {f_{2i} , f_{2i+1}}= 1 for 1 ≤ i ≤ m and also the edge sums are pair wise distinct.

Hence G is Fibonacci prime anti-magic graph.

III. CONCLUSION

We proved that cycle related graphs and crown graphs are Fibonacci prime anti-magic graphs. We extend the study to other families of graph.

REFERENCES

1. S.Chandrakala and C.Sekar, Fibonacci Prime Labeling of Cycle Related Graphs, International Journal for Research in Engineering Application and management, Vol.4, 2018, 805 – 807.
2. N.Hartsfield and G.Ringel, Labeling graphs, pearls in graph theory, academic press (1990), 103-121.
3. M.Kathiresan and S.Amutha, Fibonacci graceful graphs, Ars combinatorial,97(2010), 41-50.
4. Nicholas.T, Somasundaram.S and Vilfred.V, On (a,d)-antimagic Special trees, Unicyclic graphs and Complete bipartite graphs, Ars.Combin.70(2004),207-220.
5. Rokad,A.H. and G.V. Ghodasara (2016). Fibonacci cordial labelling of some special graphs, 133-144.
6. Tout.A, A.N.Dabboucy and K.Howalla, Prime labeling of graphs, National Academy Science letters, 11(1982), 365-368.
7. West.D.B, Introduction to Graph Theory, Prentice Hall of India, New Delhi, 2005.