Applications of Δ^*-Closed Sets in Bitopological Spaces

K.Meena, K.Sivakamasundari, V.M.Chitra

Abstract: The motive is to extend the separation axioms of Δ^*-closed sets concept in topological spaces to bitopological spaces and establish few types of space s denoted by(k, m)- ${}_{\Delta^*}T_{\delta}$,(k, m)- ${}_{\Delta^*}T_{\delta g^*}$,(k, m)- ${}_{g\delta}T_{\Delta^*}$ and(k, m)-${}_{\delta g‡}T\Delta^*$-spaces. Also their interrelations are discussed.

Keywords: Δ^*, gδ, ${}_{\delta g‡}$ and(k, m)-Δ^*-closed sets.
MSC:54E55, 54D10.

I. INTRODUCTION

The conception of bitopological spaces was initiated in 1963by J.C.Kelly [4]. The concept has been extended by different topologistsinvarious aspects. The classical theorems in general topological spaces have been converted into particular cases of the similar theorems for bitopological spaces. In topological spaces, the perception of generalised closed sets [6] was extended to bitopological spaces by Fukutake [1]in theyear 1986 .

In thepresent study, (k, m)- ${}_{\Delta^*}T_{\delta}$ -space, (k, m)- ${}_{\Delta^*}T_{\delta g^*}$ -space, (k, m)- ${}_{g\delta}T_{\Delta^*}$ -space and (k, m)-${}_{\delta g}{}^{‡}T_{\Delta}{}^*$-space are establishedin bitopological spacesand the interrelations between them are analyzed.

II. PRELIMINARIES

- The definitions of Δ^***[**primarily the notation was $\delta(\delta g)^*$**]**, gδ, $\delta g^‡$, (k, m) g-closed&(k, m)δg^*-closedsets can be viewed in [5], [2] [3], [1] & [7] respectively.
- The concepts involving the spaces (k, m)-T_b-space, (k, m)-T_c-space and (k, m)-T_d-space in a bitopological space (B, τ_k, τ_m) can be studied using [8]**.**

III. APPLICATIONS OF Δ^*-CLOSED SETS

Definition1 Consider a bitopological space (B, τ_k, τ_m). Let R be a subset of (B, τ_k, τ_m) and T is τ_k -δg-open in (G, τ_k).

Then R is defined to be a **(k, m)-Δ^*-closed set** if τ_m-$\delta cl(R) \subseteq T$ whenever R⊆T,k = 1, 2 and k ≠ m. The notation $D_{\Delta^*}(k, m)$ represents the family of all (k, m)-Δ^*-closed sets in (B, τ_k, τ_m).

Definition 2 Abitopological space (B, τ_k, τ_m) is defined to be a

1) **(k, m)- ${}_{\Delta^*}T_{\delta}$ -space** if every (k, m)-Δ^*-closed set is a τ_m-δ-closed set.
2) **(k, m)- ${}_{\Delta^*}T_{\delta g^*}$ -space** if every (k, m)-Δ^*-closed set is a (k, m)-δg^*-closed set.
3) **(k, m)- ${}_{g\delta}T_{\Delta^*}$ -space** if every (k, m)-gδ-closed set is a (k, m)-Δ^*-closed set.
4) **(k, m)-${}_{\delta g}{}^{‡}T_{\Delta}{}^*$-space** if every (k, m)-${}_{\delta g}{}^{‡}$-closed set is a (k, m)-Δ^*-closed set.

Proposition1 Every (k, m)- ${}_{\Delta^*}T_{\delta}$ -space is a ${}_{\Delta^*}T_{\delta g^*}$ -space .The reverse implication is not possible.

Proof : Let R be (k, m)-Δ^*-closed in (B, τ_k, τ_m). Since (B, τ_k, τ_m) is a ${}_{\Delta^*}T_{\delta}$ -space, R is τ_m -δ-closed. Since eachτ_m-δ-closed is (k, m)-δg^*-closed, R is (k, m)-δg^*-closed. Thus (B, τ_k, τ_m) is a ${}_{\Delta^*}T_{\delta g^*}$ -space.

Counter example 1 Let B = {d_1, d_2, d_3}, τ_1= {ϕ, B, {d_1}} and τ_2= {ϕ, B, {d_1}, {d_2}, {d_1, d_2}, {d_1, d_3}}. Then $\delta\mathcal{C}(G, \tau_1)$ = {ϕ, B} ; $\delta\mathcal{C}(G, \tau_2)$ = {ϕ, B, {d_2}, {d_1, d_3}} ; $D_{\Delta^*}(1,2)$ = {ϕ, B, {d_2}, {d_1, d_3}, {d_2, d_3}} and $D_{\delta g^*}(1,2)$ = {ϕ, B, {d_2}, {d_1, d_3}, {d_2, d_3}}. Hence the result.

Proposition2 The (k, m)- ${}_{\Delta^*}T_{\delta}$ -space is independent of (k, m)-T_b-space and (k, m)-T_c-space.

Counter example 3.6 Let B = {d_1, d_2, d_3}, τ_1= {ϕ, B, {d_1}} and τ_2 = {ϕ, B, {d_1}, {d_2, d_3}}.Then$GO(B, \tau_1)$ = {ϕ, B, {d_1}, {d_2}, {d_3}, {d_1, d_2}, {d_2, d_3}, {d_1, d_3}};$\delta\mathcal{C}(B, \tau_2)$ = {ϕ, B, {d_1}, {d_2, d_3}}; $D_{\Delta^*}(1,2)$ = {ϕ, B, {d_1}, {d_2, d_3}}; $D_{gs}(1,2)$ = {ϕ, B, {d_1}, {d_2}, {d_3}, {d_1, d_2}, {d_2, d_3}, {d_1,

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$d_3\}\}$; $D_{g^*}(1,2) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$.Hence (B, τ_k, τ_m) is a (1, 2)-${}_{\Delta^*}T_{\delta}$-space but not a (1, 2)-T_b-space.

Counter example 3 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_1, d_2\}, \{d_1, d_3\}\}$. Then$GO(B,\tau_2) = \{\phi, G, \{d_1\},\{d_1, d_2\}, \{d_1, d_3\}\}$; $\delta\mathcal{C}(B, \tau_1) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$; $D_{\Delta^*}(2,1) = \{\phi, B, \{d_1\}, \{d_2\}, \{d_3\}, \{d_1, d_2\}, \{d_2, d_3\}, \{d_1, d_3\}\}$; $D_{gs}(2,1) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$; $D_{g^*}(2,1) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$.Hence (B, τ_2, τ_1) is both (2, 1)-T_b-space as well as (2, 1)-T_c-space but not a (2, 1)-${}_{\Delta^*}T_{\delta}$-space.

Proposition3 The (k, m)-${}_{\Delta^*}T_{\delta}$-space is independent with (k, m)-T_d-space.

Counter example4 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$. Then $\delta\mathcal{C}(B, \tau_2) = \{\phi, B, \{d_3\}, \{d_1,d_3\}, \{d_2, d_3\}\}$; $D_{\Delta^*}(1,2) = \{\phi, B, \{d_3\}, \{d_2, d_3\}, \{d_1, d_3\}\}$; $D_{gs}(1,2) = \{\phi, B, \{d_1\}, \{d_2\}, \{d_3\}, \{d_1, d_2\}, \{d_2, d_3\}, \{d_1, d_3\}\}$; $D_g(1,2) = \{\phi, B, \{d_2\}, \{d_3\}, \{d_1, d_2\}, \{d_2, d_3\}, \{d_1, d_3\}\}$.Hence (B, τ_1, τ_2) is a (1, 2)-${}_{\Delta^*}T_{\delta}$-space but not a (1, 2)-T_d-space.

Counter example5 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_1, d_2\}, \{d_1,d_3\}\}$.Then $\delta(B, \tau_1) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$; $D_{\Delta^*}(2,1) = \{\phi, B, \{d_1\}, \{d_2\}, \{d_3\}, \{d_1, d_2\}, \{d_2, d_3\}, \{d_1, d_3\}\}$; $D_{gs}(2,1) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$; $D_g(2,1) = \{\phi, B, \{d_1\}, \{d_2, d_3\}\}$.Hence (B, τ_2, τ_1) is a (2, 1)-T_d-space but not (2, 1)-${}_{\Delta^*}T_{\delta}$-space.

Proposition4 The (k, m)-${}_{\Delta^*}T_{\delta}$-space is independent with(k, m)-${}_{g\delta}T_{\delta g^*}$-space.

Counter example 6 Let $B = \{d_1, d_2, d_3\}$,$\tau_1 = \{\phi, B, \{d_1\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$. Then $\delta(B, \tau_2) = \{\phi, B, \{d_3\}, \{d_1,d_3\}, \{d_2, d_3\}\}$; $D_{\Delta^*}(1,2) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{g\delta}(1,2) = \{\phi, B, \{d_1\}, \{d_2\}, \{d_3\}, \{d_1, d_2\}, \{d_2, d_3\}, \{d_1, d_3\}\}$; $D_{\delta g^*}(1,2) = \{\phi, B, \{d_3\}, \{d_1, d_3\},\{d_2, d_3\}\}$.Hence (B, τ_1, τ_2) is a (1, 2)-${}_{\Delta^*}T_{\delta}$-space but not a ${}_{g\delta}T_{\delta g^*}$-space .

Counter example 7 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$. Then $\delta\mathcal{C}(B, \tau_1) = \{\phi,B\}$; $D_{\Delta^*}(2,1) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{g\delta}(2,1) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{\delta g^*}(2,1) = \{\phi, B, \{d_3\}, \{d_1, d_3\},\{d_2, d_3\}\}$.Hence $(B, \tau_2, \tau_1)$is a (2, 1)-${}_{g\delta}T_{\delta g^*}$-space but not a (2, 1)-${}_{\Delta^*}T_{\delta}$-space.

Proposition 5 Every(k, m)-${}_{g\delta}T_{\delta g^*}$-space is a(k, m)-${}_{\Delta^*}T_{\delta g^*}$-space.

Proof: Assume that Ris a (k, m)-Δ^*-closed set in (G, τ_k, τ_m). Each (k, m)-Δ^*-closed is (k, m)-gδ-closed. By hypothesis, R is (k, m)-δg^*-closed. Hence (B, τ_k, τ_m) is a (k, m)-${}_{\Delta^*}T_{\delta g^*}$-space.

Example 1 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$.Then $D_{\Delta^*}(2,1) = \{\phi, B, \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{\delta g^*}(2,1) = \{\phi, B, \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{g\delta}(2,1) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$.Thus (B, τ_2, τ_1) is a (2, 1)-${}_{g\delta}T_{\delta g^*}$-space and (2, 1)-${}_{\Delta^*}T_{\delta g^*}$-space.

Remark2 The spaces (k, m)-T_b-space, (k, m)-T_c-space and (k, m)-T_d-space are independent with (k, m)-${}_{\delta g^{\ddagger}}T_{\Delta^*}$-space.

Counter example8 Let $B = \{d_1, d_2, d_3\}$,$\tau_1 = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}\}$.Then $D_{\Delta^*}(1,2) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$;$D_{\delta g^{\ddagger}}(1, 2) = \{\phi, G, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{gs}(1,2) = \{\phi, B, \{d_2\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_{g^*}(1,2) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$; $D_g(1,2) = \{\phi, B, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$.Hence (B, τ_1, τ_2) is a (1, 2)-${}_{\delta g^{\ddagger}}T_{\Delta^*}$-space but not (1, 2)-$T_b$, (1, 2)-$T_c$and (1, 2)-$T_d$-spaces.

Counter example 9 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1, d_2\}\}$ and $\tau_2 = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2\}, \{d_1, d_3\}\}$.Then $D_{\Delta^*}(1,2) = \{\phi, B, \{d_2\}, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$;$D_{\delta g^{\ddagger}}(1, 2) = \{\phi, B, \{d_1\}, \{d_2\}, \{d_3\}, \{d_1, d_2\},\{d_1, d_3\}, \{d_2, d_3\}\}$;

$D_{gs}(1,2) = \{\phi, B, \{d_2\}, \{d_3\}, \{d_1, d_3\}, \{d_2, d_3\}\}$;

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$D_{g^*}(1,2)$ = {ϕ, B, {d_2}, {d_3}, {d_1, d_3}, {d_2, d_3}}; $D_g(1,2)$ = {ϕ, B, {d_2}, {d_3}, {d_1, d_3},{d_2, d_3}}.Hence (B, τ_1, τ_2) is (1, 2)-T_b, (1, 2)-T_c and (1, 2)-T_d-spaces but not a (1, 2)-$_{\delta g}{}^{\ddagger}T_{\Delta}{}^{*}$-space.

Remark 3 The (k, m)-$_{\Delta^*}T_{\delta g^*}$ and (k, m)-$_{\delta g}{}^{\ddagger}T_{\Delta}{}^{*}$-spaces are independent.

Counter example 10 Let B = {d_1, d_2, d_3},τ_1 = {ϕ, B, {d_1, d_2}} and τ_2 = {ϕ, B, {d_1}, {d_2, d_3}}.Then $D_{\Delta^*}(1,2)$ = {ϕ, B, {d_1}, {d_3}, {d_1, d_3}, {d_2, d_3}}; $D_{\delta g^*}(1,2)$ = {ϕ, G, {d_1}, {d_3}, {d_1, d_3}, {d_2, d_3}};$D_{\delta g\ddagger}$ (1, 2) ={ϕ, B, {d_1}, {d_2}, {d_3}, {d_1, d_2},{d_1, d_3}, {d_2, d_3}}.Hence (B, τ_1, τ_2) is a (1, 2)-$_{\Delta^*}T_{\delta g^*}$-space but not a (1, 2)-$_{\delta g}{}^{\ddagger}T_{\Delta}{}^{*}$-space.

Counter example 11 Let B = {d_1, d_2, d_3}, τ_1= {ϕ, B, {d_1}, {d_1, d_2},{d_1, d_3}} and τ_2={ϕ, B, {d_1},{d_2, d_3}}. Then $D_{\Delta^*}(1,2)$ = {ϕ, B, {d_1}, {d_2}, {d_3}, {d_1, d_2},{d_1, d_3},{d_2, d_3}}; $D_{\delta g^*}(1,2)$ = {ϕ, B, {d_1},{d_2, d_3}};$D_{\delta g\ddagger}$ (1, 2) = {ϕ, B,{d_1}, {d_2}, {d_3}, {d_1, d_2},{d_1, d_3},{d_2, d_3}}.Hence (G, τ_1, τ_2) is a (1, 2)-$_{\delta g}{}^{\ddagger}T_{\Delta}{}^{*}$-space but not a (1, 2)-$_{\Delta^*}T_{\delta g^*}$-space.

Proposition 6 The space(B, τ_k , τ_m)which is a (k, m)-$_{g\delta}T_{\Delta^*}$-space as well as (k, m)-$_{\Delta^*}T_{\delta}$ space can be viewed as a (k, m)-T_{δ}-space.

Proof : By hypothesis,a subset Rof B is(k, m)-Δ^*-closed and τ_j-δ-closed. Hence (G, τ_k, τ_m) is a(k, m)-T_{δ}-space.

Proposition 7 In (k, m)-$_{\Delta^*}T_{\delta}$-space, either {d} is τ_m-δ-open or τ_k–δg-closed for each d∊B.

Proof : Let (B, τ_k, τ_m) be a (k, m)-$_{\Delta^*}T_{\delta}$-space and {d} be not τ_k–δg-closed. Thus $\{d\}^c$ is (k, m)-Δ^*-closed. Therefore $\{d\}^c$ is τ_m–δ-closed in (B,τ_k, τ_m) space.Hence {d} is τ_m-δ-open.

Proposition 8 In a bitopologicalspace (B, τ_k , τ_m) the statements (i) & (ii) are equivalent.

(i)B is (k, m)-$_{\Delta^*}T_{\delta}$-space.

(ii) Each subset consisting only one element is either τ_k-δg-closed or τ_m-δ-open for k ≠ m.

Proof: (i) ⟹ (ii): Assume that {s} is not τ_k–δg-closed subset for somes∈G. Then G –{s} is not τ_k–δg-open and hence G is the only τ_k–δg-open set containing G−{s}. Therefore G –{s} is (k, m)-Δ^*-closed. Since (G, τ_k, τ_m) is a (k, m)-$_{\Delta^*}T_{\delta}$-space, G−{s} is τ_m-δ-closed. Thus {s} is τ_m-δ-open.

(ii) ⟹ (i) : Let R be a (k, m)-Δ^*-closed subset of (G, τ_k, τ_m) and s∊ τ_m δcl(R).

Case 1 : If {s} is τ_k-δg-closed and s∉R then s∊τ_j-δcl(R) \ R. Thus τ_m-δcl(R)−Rcontains a non emptyτ_k–δg-closed set {s} which is a contradiction to the fact that R is(k, m)-Δ^*-closed. So s∊R.

Case 2 : If {s} is τ_m-δ-open, since s∈τ_m-δcl(R), then for every τ_m-δ-open set T containing s, we have T ∩ R ≠ ϕ. But {s} is τ_m–δ-open. Therefore {s} ∩ R ≠ ϕ.Hences∈R. In both cases s∈R. Hence R is τ_m-δ-closed.

IV. CONCLUSION

This paper establish the new type of topological spaces by using the concepts of bit topological spaces.

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