Applications of $\Delta^*$-Closed Sets in Bitopological Spaces

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Abstract: The motive is to extend the separation axioms of $\Delta^*$-closed sets concept in topological spaces to bitopological spaces and establish few types of space $s$ denoted by $(k, m)$-$\Delta^* \delta g^*$-space and $(k, m)$-$\delta g \Delta^*$-space. Also their interrelations are discussed.

Keywords: $\Delta^*$, $\delta g^*$, $(k, m)$-$\Delta^*$-closed sets.

I. INTRODUCTION

The conception of bitopological spaces was initiated in 1963 by J.C.Kelly [4]. The concept has been extended by different topologists in various aspects. The classical theorems in general topological spaces have been converted into particular cases of the similar theorems for bitopological spaces. In topological spaces, the perception of generalised closed sets [6] was extended to bitopological spaces by Fukutake [1] in the year 1986.

In the present study, $(k, m)$-$\Delta^* \delta g^*$-space, $(k, m)$-$\delta g \Delta^*$-space and $(k, m)$-$\delta g \Delta^*$-space are established in bitopological spaces and the interrelations between them are analyzed.

II. PRELIMINARIES

- The definitions of $\Delta [\text{primarily the notation was } \delta (\delta g)]$, $\delta g$, $\delta g^*$, $(k, m)$-g-closed and $(k, m)$-$\delta g^*$-closed sets can be viewed in [5], [2], [3], [1] & [7] respectively.
- The concepts involving the spaces $(k, m)$-$T^\delta_\gamma$-space, $(k, m)$-$T^*\delta_\gamma$-space and $(k, m)$-$T^\delta_\gamma$-space in a bitopological space $(B, \tau^1_k, \tau^1_m)$ can be studied using [8].

III. APPLICATIONS OF $\Delta^*$-CLOSED SETS

Definition 1 Consider a bitopological space $(B, \tau^1_k, \tau^1_m)$. Let $R$ be a subset of $(B, \tau^1_k, \tau^1_m)$ and $T$ is $\tau^1_k$-$\delta g^*$-open in $(G, \tau^1_k)$.

Then $R$ is defined to be a $(k, m)$-$\Delta^*$-closed set if $\tau^1_m - \delta c l (R) \subseteq T$ whenever $R \subseteq T, k = 1, 2$ and $k \neq m$. The notation $D^* (k, m)$ represents the family of all $(k, m)$-$\Delta^*$-closed sets in $(B, \tau^1_k, \tau^1_m)$.

Definition 2 Abitopological space $(B, \tau^1_k, \tau^1_m)$ is defined to be a

1) $(k, m)$-$\Delta^* \delta g^*$-space if every $(k, m)$-$\Delta^*$-closed set is a $\tau^1_m$-$\delta$-closed set.

2) $(k, m)$-$\Delta^* \delta g^*$-space if every $(k, m)$-$\Delta^*$-closed set is a $(k, m)$-$\delta g^*$-closed set.

3) $(k, m)$-$\Delta^* \delta g^*$-space if every $(k, m)$-$\delta g^*$-closed set is a $(k, m)$-$\Delta^*$-closed set.

4) $(k, m)$-$\delta g \Delta^*$-space if every $(k, m)$-$\delta g \Delta^*$-closed set is a $(k, m)$-$\Delta^*$-closed set.

Proposition 1 Every $(k, m)$-$\Delta^* \delta g^*$-space is a $\Delta^* \delta g^*$-space. The reverse implication is not possible.

Proof: Let $R$ be a $(k, m)$-$\Delta^*$-closed in $(B, \tau^1_k, \tau^1_m)$. Since $(B, \tau^1_k, \tau^1_m)$ is a $\Delta^* \delta g^*$-space, $R$ is $\tau^1_m$-$\delta$-closed. Since each $\tau^1_m$-$\delta$-closed is a $(k, m)$-$\delta g^*$-closed, $R$ is $(k, m)$-$\delta g^*$-closed. Thus $(B, \tau^1_k, \tau^1_m)$ is a $\Delta^* \delta g^*$-space.

Counter example 1 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}\}$ and $\tau_2 = \{\phi, B, \{d_2\}, \{d_1, d_2, d_3\}\}$. Then $\delta c l (G, \tau_1) = \{\phi, B\}$; $\delta c l (G, \tau_2) = \{\phi, B, \{d_2\}, \{d_1, d_2, d_3\}\}$.

$D^* (1, 2) = \{\phi, B, \{d_2\}, \{d_1, d_3\}, \{d_2, d_3\}\}$ and $\Delta^* (1, 2) = \{\phi, B, \{d_2\}, \{d_1, d_3\}, \{d_2, d_3\}\}$. Hence the result.

Proposition 2 The $(k, m)$-$\Delta^* \delta g^*$-space is independent of $(k, m)$-$T^\delta_\gamma$-space and $(k, m)$-$T^\gamma_\delta$-space.

Counter example 3 Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \{d_1\}\}$ and $\tau_2 = \{\phi, B, \{d_2\}, \{d_1, d_3\}\}$. Then $G (B, \tau_1) = \{\phi, B, \{d_1\}, \{d_2\}, \{d_1, d_2, d_3\}\}$.

$D^* (1, 2) = \{\phi, B, \{d_2\}, \{d_1, d_3\}, \{d_2, d_3\}\}$ and $\Delta^* (1, 2) = \{\phi, B, \{d_2\}, \{d_1, d_3\}, \{d_2, d_3\}\}$. Hence the result.
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$\text{g}_* (1, 2) = \{\phi, \ B, \ (d_1), \ (d_2, d_3)\}$. Hence

\[(B, \tau_k, \tau_m) \text{ is a } (1, 2)-\Delta^\ast T\delta\text{-space but not a } (1, 2)-T_{\delta}\text{-space.} \]

Counter example 3

Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$ and $\tau_2 = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Then $\Delta C(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$; $\Delta* C(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Let $\delta g(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$; $D \Delta* (1, 2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}; D \Delta* (1, 2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Hence \(\tau_2\) is a \( (1, 2)\)-\(\Delta^\ast T\delta\text{-space but not a } (2, 1)-\Delta^\ast T\delta\text{-space.} \)

Proposition 3

The \((k, m)\)-\(\Delta^\ast T\delta\text{-space is independent with } (k, m)\)-\(T\delta\text{-space.} \)

Counter example 4

Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$ and $\tau_2 = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Then $\delta C(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$; $\delta C(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Let $\delta g(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$; $\delta g(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Hence \(\tau_2\) is a \( (1, 2)\)-\(\Delta^\ast T\delta\text{-space and not a } (1, 2)-\Delta^\ast T\delta\text{-space.} \)

Counter example 5

Let $B = \{d_1, d_2, d_3\}$, $\tau_1 = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$ and $\tau_2 = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Then $\delta C(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$; $\delta C(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Let $\delta g(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$; $\delta g(B, \tau_2) = \{\phi, B, \ (d_1), \ (d_2, d_3)\}$. Hence \(\tau_2\) is a \( (1, 2)\)-\(\Delta^\ast T\delta\text{-space but not a } (1, 2)-\Delta^\ast T\delta\text{-space.} \)

Proposition 4

The \((k, m)\)-\(\Delta^\ast T\delta\text{-space is independent with } (k, m)\)-\(T\delta\text{-space.} \)

Counter example 6

Let $B = \{d_1, d_2, d_3\}, \tau_1 = \{\phi, B, \ (d_1)\}$ and $\tau_2 = \{\phi, B, \ (d_1)\}$. Then $\delta C(B, \tau_2) = \{\phi, B, \ (d_1)\}; \Delta (1, 2) = \{\phi, B, \ (d_1)\}, \{d_1, d_2\}; D \delta g (1, 2) = \{\phi, B, \ (d_1)\}, \{d_1, d_2\}; D \delta g (1, 2) = \{\phi, B, \ (d_1)\}, \{d_1, d_2\}$. Hence \(\tau_2\) is a \( (1, 2)\)-\(\Delta^\ast T\delta\text{-space but not a } \delta g\text{-space.} \)
D_{T_1}(1,2) = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\}, \{d_1d_2\})\};
D_{T_2}(1,2) = \{(\phi, B, \{d_1\}, \{d_1d_2\}, \{d_1d_2\})\}. Hence (B, T_1, T_2) is (1, 2)-T_{2_{\delta}} and (1, 2)-T_{\delta^g}-spaces but not a (1, 2)-T_{\delta^g}.T_{\delta^g}-space.

**Remark 3** The (k, m)-\(\Delta^*\)-space and (k, m)-\(\delta^g\)-space are independent.

**Counter example 10** Let B = \{d_1, d_2, d_3\}, \(T_1 = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\})\}\) \& \(T_2 = \{(\phi, B, \{d_1\}, \{d_2\})\}\). Then \(D_{T_1}(1,2) = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\}, \{d_1d_2\})\}; D_{T_2}(1,2) = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\}, \{d_1d_2\})\}. Hence (B, T_1, T_2) is a (1, 2)-\(\Delta^*\)-space but not a (1, 2)-\(\delta^g\)-space.

**Counter example 11** Let B = \{d_1, d_2, d_3\}, \(T_1 = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\})\}\) \& \(T_2 = \{(\phi, B, \{d_1\}, \{d_2\})\}\). Then \(D_{T_1}(1,2) = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\}, \{d_1d_2\})\}; D_{T_2}(1,2) = \{(\phi, B, \{d_1\}, \{d_2\}, \{d_1d_2\}, \{d_1d_2\})\}. Hence (G, T_1, T_2) is a (1, 2)-\(\delta^g\)-space but not a (1, 2)-\(\Delta^*\)-space.

**Proposition 6** The space(B, \(T_\delta\))which is a (k, m)-\(\delta^g\)-\(\Delta^*\)-space as well as (k, m)-\(\Delta^*\)-space can be viewed as a (k, m)-\(T_\delta\)-space.

**Proof**: By hypothesis, a subset R of B is (k, m)-\(\Delta^*\)-closed and \(T_\delta\)-closed. Hence (G, T_k, T_m) is (k, m)-\(T_\delta\)-space.

**Proposition 7** In (k, m)-\(\Delta^*\)-space, either \(d\) is \(T_m\)-\(\delta\)-open or \(T_k\)-\(\delta\)-closed for each c\(\in\)B.

**Proof**: Let (B, T_k, T_m) be (k, m)-\(\Delta^*\)-\(T_\delta\)-space and \(\{d\}\) be not \(T_k\)-\(\delta\)-closed. Thus \(\{d\}\) is (k, m)-\(\Delta^*\)-closed. Therefore \(\{d\}\) is \(T_m\)-\(\delta\)-closed in (B, T_k, T_m) space. Hence \(\{d\}\) is \(T_m\)-\(\delta\)-open.

**Proposition 8** In a bitopological space(B, \(T_\delta\), \(T_{\delta}\)) the statements (i) \& (ii) are equivalent.

(i)B is (k, m)-\(\Delta^*\)-\(T_\delta\)-space.

(ii) Each subset consisting only one element is either \(T_k\)-\(\delta\)-closed or \(T_m\)-\(\delta\)-open for \(k \neq m\).

**Proof**: (i) \(\Rightarrow\) (ii): Assume that \(\{s\}\) is not \(T_k\)-\(\delta\)-closed subset for some \(s\) \(\in\)G. Then G - \(\{s\}\) is not \(T_k\)-\(\delta\)-open and hence G is the only \(T_k\)-\(\delta\)-open subset containing G - \(\{s\}\). Therefore G - \(\{s\}\) is (k, m)-\(\Delta^*\)-closed. Since (G, T_k, T_m) is (k, m)-\(\Delta^*\)-\(T_\delta\)-space, G - \(\{s\}\) is \(T_m\)-\(\delta\)-closed. Thus \(\{s\}\) is \(T_m\)-\(\delta\)-open.

(ii) \(\Rightarrow\) (i): Let R be a (k, m)-\(\Delta^*\)-closed subset of (G, T_k, T_m) and sc \(T_m\)-\(\delta\)-cl(R).

**Case 1**: If \(\{s\}\) is \(T_k\)-\(\delta\)-closed and \(s\) \(\in\)R then sc\(T_m\)-\(\delta\)-cl(R) \(\cap\) R. Thus \(T_m\)-\(\delta\)-cl(R) \(\cap\) R contains a non empty \(T_k\)-\(\delta\)-closed set \(\{s\}\) which is a contradiction to the fact that R is (k, m)-\(\Delta^*\)-closed. So sc\(\in\)R.

**Case 2**: If \(\{s\}\) is \(T_m\)-\(\delta\)-open, since \(s\) \(\in\)\(T_m\)-\(\delta\)-cl(R), then for every \(T_m\)-\(\delta\)-open set T containing s, we have T \(\cap\) R \(\neq\) \(\phi\). But \(\{s\}\) is \(T_m\)-\(\delta\)-open. Therefore \(\{s\}\) \(\cap\) R \(\neq\) \(\phi\). Hence \(\in\)R. In both cases \(s\) \(\in\)R. Hence R is \(T_m\)-\(\delta\)-closed.

**IV. CONCLUSION**

This paper establish the new type of topological spaces by using the concepts of bit topological spaces.

**REFERENCES**