Logarithmic Mean Labeling of Some Chain Graphs

A.Durai Baskar, A. Rajesh Kannan, P. Manivannan, R.Rathajeyalakshmi

Abstract: *A function f is called a logarithmic mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f:V(G)\rightarrow\{1,2,3,\dots,q+1\}$ is injective and the induced function $f^*:E(G)\rightarrow\{1,2,3,\dots,q\}$ defined as*

$$f^*(uv)=\left\lfloor\frac{f(v)-f(u)}{\ln f(v)-\ln f(u)}\right\rfloor,\ for\ all\ uv\in E(G),$$

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph. In this paper, we study the logarithmic meanness of some chain graphs.

Key Words. *labeling, logarithmic mean labeling, logarithmic mean graph.*

I. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected and simple graph. $G(V,E)$ Let be a graph with p vertices and q edges. For notations and terminology, we follow [6]. For a detailed survey on graph labeling we refer to [5].

Path on n vertices is denoted by P_n and a Cycle on n vertices is denoted by C_n. $G\odot S_m$ is the graph obtained from G by attaching m pendant vertices at each vertex of G. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1\times G_2$ has p_1p_2 vertices which are $\{(u,v):u\in G_1, v\in G_2\}$ and any two vertices (u_1,v_1) and (u_2,v_2) are adjacent in $G_1\times G_2$ if either $u_1=u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1=v_2$. The study of graceful graphs and graceful labeling methods was first introduced by Rosa [7] and so many authors are working in the area of graph labeling [4,5], motivated these we introduce a new type of labeling called logarithmic mean labeling A function f is called a logarithmic mean labeling of a graph $G(V,E)$ if $f:V(G)\rightarrow\{1,2,3,\dots,q+1\}$ is injective and the induced

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Function
$f^*:E(G)\rightarrow\{1,2,3,\dots,q\}$
defined as $f^*(uv)=\left\lfloor\frac{f(v)-f(u)}{\ln f(v)-\ln f(u)}\right\rfloor,\ for\ all\ uv\in E(G),$
is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph.

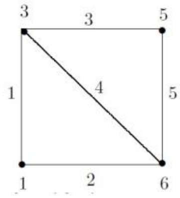

Figure 1 . A logarithmic mean graph of K_4-e

In this paper, we have discussed the logarithmic mean labeling of some chain graphs.

Theorem 2.1 $G^*(p_1,p_2,\dots,p_n)$ is a logarithmic mean graph for any p_j, for $1\le j\le n$.

Proof. Let $\{v_i^{(j)};1\le j\le n, 1\le i\le p_j\}$ be the vertices of the n number of cycles.

For $1\le j\le n-1$, the j^{th} and $(j+1)^{th}$ cycles are identified by a vertex $v^{(j)}_{\frac{p_j+3}{2}}$ and $v_1^{(j+1)}$ while p_j is odd and $v^{(j)}_{\frac{p_j+2}{2}}$ and $v_1^{(j+1)}$ while p_j is even.

We define $f:V(G^*(p_1,p_2,\dots,p_n))\rightarrow\{1,2,3,\dots,\sum_{j=1}^{n}p_j+1\}$ as follows:

We define $f:V(G^*(p_1,p_2,\dots,p_n)\rightarrow\left\{1,2,3,\dots,\sum_{j=1}^{n}p_j+1\right\}$ as follows:

$$f(v_i^{(1)})=\begin{cases}2i-1 & 1\le i\le\lfloor\frac{p_1}{2}\rfloor+1\\ 2p_1-2(i-2) & \lfloor\frac{p_1}{2}\rfloor+2\le i\le p_1\end{cases}\quad\text{and}$$

for $2\le j\le n$,

$$f(v_i^{(j)})=\begin{cases}\sum_{k=1}^{j-1}p_k+2i-1 & 2\le i\le\lfloor\frac{p_j}{2}\rfloor+1\\ \sum_{k=1}^{j-1}p_k+2(i-1) & i=\lfloor\frac{p_j}{2}\rfloor+2\text{ and }p_j\text{ is odd}\\ \sum_{k=1}^{j-1}p_k+2(i-2) & i=\lfloor\frac{p_j}{2}\rfloor+2\text{ and }p_j\text{ is even}\\ \sum_{k=1}^{j-1}p_k+2p_j-2(i-2) & i=\lfloor\frac{p_j}{2}\rfloor+3\le i\le p_j.\end{cases}$$

Then the induced edge labeling is as follows:

$$f^*(v_i^{(1)}v_{i+1}^{(1)})=\begin{cases}2i-1 & 1\le i\le\left\lfloor\frac{p_1}{2}\right\rfloor\\ 2i-1 & i=\left\lfloor\frac{p_1}{2}\right\rfloor+1\text{ and }p_1\text{ is odd}\\ 2p_1-2(i-1) & i=\left\lfloor\frac{p_1}{2}\right\rfloor+1\text{ and }p_1\text{ is even}\\ 2p_1-2(i-1) & \left\lfloor\frac{p_1}{2}\right\rfloor+2\le i\le p_1-1,\end{cases}$$

$$f^*(v_{p_1}^{(1)}v_1^{(1)})=2,$$

for $2\le j\le n$,

$$f^*(v_i^{(j)}v_{i+1}^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} p_k + 2i - 1 & 1 \le i \le \lfloor \frac{p_j}{2} \rfloor \\ \sum_{k=1}^{j-1} p_k + 2i - 1 & i = \lfloor \frac{p_j}{2} \rfloor + 1 \text{ and } p_j \text{ is odd} \\ \sum_{k=1}^{j-1} p_k + 2p_j - 2(i-1) & i = \lfloor \frac{p_j}{2} \rfloor + 1 \text{ and } p_j \text{ is even} \\ \sum_{k=1}^{j-1} p_k + 2p_j - 2(i-1) & \lfloor \frac{p_j}{2} \rfloor + 2 \le i \le p_j - 1 \end{cases}$$

and $f^*(v_{p_j}^{(j)}v_1^{(j)}) = \sum_{k=1}^{j-1} p_k + 2.$

Hence, f is a logarithmic mean labeling of $G^*(p_1, p_2, \dots, p_n)$. Thus the graph $G^*(p_1, p_2, \dots, p_n)$ is a logarithmic mean graph.

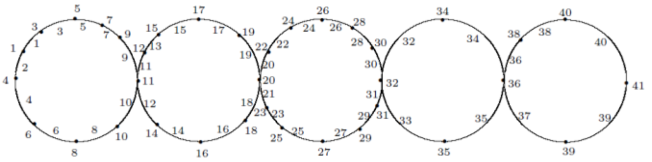

Figure 2.1 . A logarithmic mean labeling of $G^*(10,9,12,4,5)$

Theorem 2.2 $G'(p_1, p_2, \dots, p_n)$ is a logarithmic mean graph if all p_j's are odd or all p_j's are even, for $1 \le j \le n$.

Proof. Let $\{v_i^{(j)}; 1 \le j \le n, 1 \le i \le p_j\}$ be the vertices of the n number of cycles.

Case (i) p_j is odd, for $1 \le j \le n$.

For $1 \le j \le n-1$, the j^{th} and $(j+1)^{th}$ cycles are identified by the edges $v^{(j)}_{\frac{p_{j+1}}{2}} v^{(j)}_{\frac{p_{j+3}}{2}}$ and $v_1^{(j+1)} v_{p_{j+1}}^{(j+1)}$ while j is odd and $v^{(j)}_{\frac{p_{j-1}}{2}} v^{(j)}_{\frac{p_{j+1}}{2}}$ and $v_1^{(j+1)} v_{p_{j+1}}^{(j+1)}$ while j is even.

We define $f: V(G'(p_1, p_2, \dots, p_n)) \to \{1,2,3, \dots, \sum_{j=1}^{n} p_j - n + 2\}$ as follows:

$$f(v_i^{(1)}) = \begin{cases} 1 & i = 1 \\ 2i & 2 \le i \le \lfloor \frac{p_1}{2} \rfloor + 1 \\ 2p_1 + 3 - 2i & \lfloor \frac{p_1}{2} \rfloor + 2 \le i \le p_1 \end{cases} \quad and$$

$for\ 2 \le j \le n,$

$$f(v_i^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} p_k - j + 2i + 2 & 2 \le i \le \lfloor \frac{p_j}{2} \rfloor \text{ and } j \text{ is even} \\ \sum_{k=1}^{j-1} p_k + 2p_j + 3 - j - 2i & \lfloor \frac{p_j}{2} \rfloor + 1 \le i \le p_j - 1 \text{ and } j \text{ even} \\ \sum_{k=1}^{j-1} p_k - j + 2i + 1 & 2 \le i \le \lfloor \frac{p_j}{2} \rfloor + 1 \text{ and } j \text{ is odd} \\ \sum_{k=1}^{j-1} p_k + 2p_j + 4 - j - 2i & \lfloor \frac{p_j}{2} \rfloor + 2 \le i \le p_j - 1 \text{ and } j \text{ odd.} \end{cases}$$

Then the induced edge labeling is as follows:

$$f^*(v_i^{(1)}v_{i+1}^{(1)}) = \begin{cases} 2i & 1 \le i \le \lfloor \frac{p_1}{2} \rfloor \\ 2p_1 + 1 - 2i & \lfloor \frac{p_1}{2} \rfloor + 1 \le i \le p_1 - 1, \end{cases}$$

$f^*(v_{p_1}^{(1)}v_1^{(1)}) = 1$ and

for $2 \le j \le n$,

$$f^*(v_i^{(j)}v_{i+1}^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} p_k - j + 2i + 2 & 1 \le i \le \lfloor \frac{p_j}{2} \rfloor \text{ and } j \text{ is even} \\ \sum_{k=1}^{j-1} p_k + 2p_j + 1 - j - 2i & \lfloor \frac{p_j}{2} \rfloor + 1 \le i \le p_j - 1 \text{ and } j \text{ even} \\ \sum_{k=1}^{j-1} p_k - j + 2i + 1 & 1 \le i \le \lfloor \frac{p_j}{2} \rfloor \text{ and } j \text{ is odd} \\ \sum_{k=1}^{j-1} p_k + 2p_j + 2 - j - 2i & \lfloor \frac{p_j}{2} \rfloor + 1 \le i \le p_j - 1 \text{ and } j \text{ odd.} \end{cases}$$

Case (ii) p_j is even, for $1 \le j \le n$.

For $1 \le j \le n-1$, the j^{th} and $(j+1)^{th}$ cycles are identified by the edges $v^{(j)}_{\frac{p_j}{2}} v^{(j)}_{\frac{p_j+2}{2}}$ and $v_1^{(j+1)} v_{p_{j+1}}^{(j+1)}$.

Wedefine $f: V(G'(p_1, p_2, \dots, p_n)) \to \{1,2,3, \dots, \sum_{j=1}^{n} p_j - n + 2\}$ as follows:

$$f(v_i^{(1)}) = \begin{cases} 1 & i = 1 \\ 2i & 2 \le i \le \lfloor \frac{p_1}{2} \rfloor \\ 2p_1 + 3 - 2i & \lfloor \frac{p_1}{2} \rfloor + 1 \le i \le p_1 \end{cases} \quad and$$

$for\ 2 \le j \le n,$

$$f(v_i^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} p_k - j + 2i + 1 & 2 \le i \le \lfloor \frac{p_j}{2} \rfloor\ 2mm \\ \sum_{k=1}^{j-1} p_k + 2p_j + 4 - j - 2i & \lfloor \frac{p_j}{2} \rfloor + 1 \le i \le p_j - 1. \end{cases}$$

Then the induced edge labeling is as follows:

$$f^*(v_i^{(1)}v_{i+1}^{(1)}) = \begin{cases} 2i & 1 \le i \le \lfloor \frac{p_1}{2} \rfloor \\ 2p_1 + 1 - 2i & \lfloor \frac{p_1}{2} \rfloor + 1 \le i \le p_1 - 1, \end{cases}$$

$$f^*(v_{p_1}^{(1)}v_1^{(1)}) = 1\ and$$

For $2 \le j \le n$,

$$f^*(v_i^{(j)}v_{i+1}^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} p_j - j + 2i + 1 & 1 \le i \le \lfloor \frac{p_j}{2} \rfloor\ 2mm \\ \sum_{k=1}^{j-1} p_k + 2p_j + 2 - j - 2i & \lfloor \frac{p_j}{2} \rfloor + 1 \le i \le p_j - 1. \end{cases}$$

Hence, f is a logarithmic mean labeling of $G'(p_1, p_2, \dots, p_n)$. Thus the graph $G'(p_1, p_2, \dots, p_n)$ is a logarithmic mean graph.

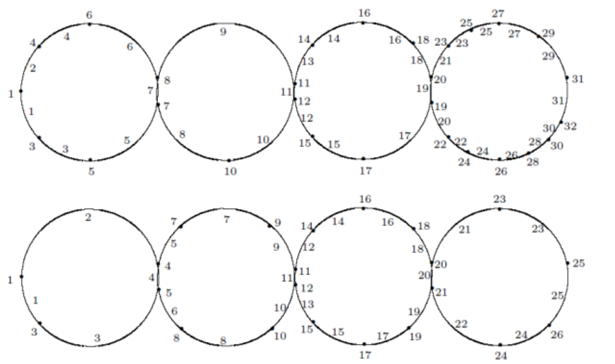

Figure 2.2: A logarithmic mean labeling of $G'(7,5,9,13)$ and $G'(4,8,10,6)$

Theorem 2.3 $\hat{G}(p_1, m_1, p_2, m_2, \ldots, m_{n-1} p_n)$ is a logarithmic mean graph for any p_j's and m_j's.

Proof. Let $\{v_i^{(j)}; 1 \le j \le n, 1 \le i \le p_j\}$ and $\{u_i^{(j)}; 1 \le j \le n-1, 1 \le i \le m_j\}$ be the n number of cycles and $(n-1)$ number of paths respectively.

For $1 \le j \le n-1$, the j^{th} cycle and j^{th} path are identified by a vertex $v_{\frac{p_j}{2}+2}^{(j)}$ and $u_1^{(j)}$ while p_j is even and $v_{\frac{p_j+3}{2}}^{(j)}$ and $u_1^{(j)}$ while p_j is odd. And the j^{th} path and $(j+1)^{th}$ cycle are identified by a vertex $u_{m_j}^{(j)}$ and $v_1^{(j+1)}$.

Wedefine
$f: V(\hat{G}(p_1, m_1, p_2, m_2, \ldots, m_{n-1}, p_n)) \to \{1,2,3, \ldots, \sum_{j=1}^{n-1} (p_j + m_j) + p_n - n + 2\}$

as follows:

$$f(v_i^{(1)}) = \begin{cases} 2i-1 & 1 \le i \le \left\lfloor \frac{p_1}{2} \right\rfloor + 1 \\ 2p_1 + 4 - 2i & \left\lfloor \frac{p_1}{2} \right\rfloor + 2 \le i \le p_1, \end{cases}$$

$f(u_i^{(1)}) = p_1 + i$, for $2 \le i \le m_1$,

for $2 \le j \le n$,

$$f(v_i^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} (p_k + m_k) + 2i - j & 2 \le i \le \left\lfloor \frac{p_j}{2} \right\rfloor + 1 \\ \sum_{k=1}^{j-1} (p_k + m_k) + 2i - j - 1 & i = \left\lfloor \frac{p_j}{2} \right\rfloor + 2 \text{ and } p_j \text{ is odd} \\ \sum_{k=1}^{j-1} (p_k + m_k) + 2i - j - 3 & i = \left\lfloor \frac{p_j}{2} \right\rfloor + 2 \text{ and } p_j \text{ is even} \\ \sum_{k=1}^{j-1} (p_k + m_k) + 2p_j - 2i - j + 5 & \left\lfloor \frac{p_j}{2} \right\rfloor + 3 \le i \le p_j \end{cases}$$

and for $3 \le j \le n$,

$$f(u_i^{(j-1)}) = \sum_{k=1}^{j-2} (p_k + m_k) + p_{j-1} + i + 2 - j, \text{ for } 2 \le i \le m_{j-1}.$$

Then the induced edge labeling is as follows:

$$f^*(v_i^{(1)} v_{i+1}^{(1)}) = \begin{cases} 2i-1 & 1 \le i \le \left\lfloor \frac{p_1}{2} \right\rfloor \\ 2i-1 & i = \left\lfloor \frac{p_1}{2} \right\rfloor + 1 \text{ and } p_1 \text{ is odd} \\ 2p_1 - 2i + 2 & i = \left\lfloor \frac{p_1}{2} \right\rfloor + 1 \text{ and } p_1 \text{ is even} \\ 2p_1 - 2i + 2 & \left\lfloor \frac{p_1}{2} \right\rfloor + 2 \le i \le p_1 - 1, \end{cases}$$

$f^*(v_{p_1}^{(1)} v_1^{(1)}) = 2$,

$f^*(u_i^{(1)} u_{i+1}^{(1)}) = p_1 + i$, for $1 \le i \le m_1 - 1$,

for $2 \le j \le n$,

$$f^*(v_i^{(j)} v_{i+1}^{(j)}) = \begin{cases} \sum_{k=1}^{j-1} (p_k + m_k) + 2i - j & 1 \le i \le \left\lfloor \frac{p_j}{2} \right\rfloor \\ \sum_{k=1}^{j-1} (p_k + m_k) + 2i - j & i = \left\lfloor \frac{p_j}{2} \right\rfloor + 1 \text{ and } p_j \text{ is odd} \\ \sum_{k=1}^{j-1} (p_k + m_k) + 2p_j - 2i - j + 3 & i = \left\lfloor \frac{p_j}{2} \right\rfloor + 1 \text{ and } p_j \text{ is even} \\ \sum_{k=1}^{j-1} (p_k + m_k) + 2p_j - 2i - j + 3 & \left\lfloor \frac{p_j}{2} \right\rfloor + 2 \le i \le p_j - 1, \end{cases}$$

$f^*(v_{p_j}^{(j)} v_1^{(j)}) = \sum_{k=1}^{j-1} (p_k + m_k) - j + 3$ and

for $3 \le j \le n$,

$$f^*(u_i^{(j-1)} u_{i+1}^{(j-1)}) = \sum_{k=1}^{j-2} (p_k + mk) + p_{j-1} + i + 2 - j, \text{ for } 1 \le i \le m_{j-1} - 1.$$

Hence, f is a logarithmic mean labeling of $\widehat{G}(p_1, m_1, p_2, m_2 \ldots, m_{n-1}, p_n)$. Thus the graph $\widehat{G}(p_1, m_1, p_2, m_2 \ldots, m_{n-1}, p_n)$ is a logarithmic mean graph. □

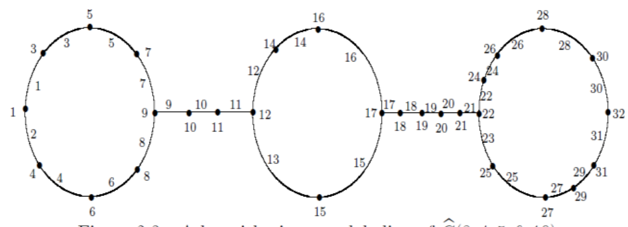

Figure 2.3 : A logarithmic mean labeling of $\widehat{G}(8,4,5,6,10)$

Theorem 2.4 Let G be a graph obtained from a path by identifying any of its edges by an edge of a cycle, then G is a logarithmic mean graph.

Proof. Let $v_1, v_2, \ldots, v_p$ be the vertices of the path on p vertices. Let m be the number of cycles are placed in a path in order to get G and the edges of the j^{th} cycle be identified with the edge (v_{i_j}, v_{i_j+1}) of the path having the length n_j.

For $1 \le j \le m$, the vertices of the j^{th} cycle be $v_{i_j,l}, 1 \le l \le n_j$ where $v_{i_j,1} = v_{i_j}$ and $v_{i_j,n_j} = v_{i_j+1}$.

We define $f: V(G) \to \left\{1,2,3,\ldots, \sum_{j=1}^{m} n_j + p - m\right\}$ as follows:

$f(v_k) = k$, for $1 \le k \le i_1$

$f(v_{i_j}) = i_j + \sum_{k=1}^{j-1} (n_k - 2) + j - 1$, for $1 \le j \le m$

$f(v_{i_j+1}) = f(v_{i_j}) + n_j$, for $1 \le j \le m$,

$f(v_{i_j+k}) = f(v_{i_j+1}) + k - 1$, for $2 \le k \le i_{j+1} - i_j - 1$ and $1 \le j \le m-1$

$f(v_{i_m+k+1}) = f(v_{i_m+k}) + k - 1$, for $2 \le k \le p - i_m$

and for $1 \le j \le m$,

$$f(v_{i_j,l}) = \begin{cases} f(v_{i_j}) + l - 1, & 2 \le l \le \left\lfloor \frac{f(v_{i_j+1}) - f(v_{i_j})}{ln(f(v_{i_j+1})) - ln(f(v_{i_j}))} \right\rfloor - f(v_{i_j}) \\ f(v_{i_j}) + l, & \left\lfloor \frac{f(v_{i_j+1}) - f(v_{i_j})}{ln(f(v_{i_j+1})) - ln(f(v_{i_j}))} \right\rfloor - f(v_{i_j}) + 1 \le l \le n_j - 1. \end{cases}$$

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Then the induced edge labeling is obtained as follows:

$$f^*(v_k v_{k+1}) = k, \text{ for } 1 \le k \le i_1 - 1$$

$$f^*(v_{i_j+k} v_{i_j+k+1}) = f(v_{i_j+k}), \text{ for } 1 \le k \le i_{j+1} - i_j - 1 \text{ and } 1 \le j \le m-1,$$

$$f^*(v_{i_m+k} v_{i_m+k+1}) = f(v_{i_m+k}), \text{ for } 1 \le k \le p - i_m - 1 \text{ and}$$

for $1 \le j \le m$,

$$f^*(v_{i_j,l} v_{i_j,l+1}) = \begin{cases} f(v_{i_j}) + l - 1, & 1 \le l \le \left\lfloor \frac{f(v_{i_j+1}) - f(v_{i_j})}{ln(f(v_{i_j+1})) - ln(f(v_{i_j}))} \right\rfloor - f(v_{i_j}) \\ f(v_{i_j}) + l, & \left\lfloor \frac{f(v_{i_j+1}) - f(v_{i_j})}{ln(f(v_{i_j+1})) - ln(f(v_{i_j}))} \right\rfloor - f(v_{i_j}) + 1 \le l \le n_j - 1 \end{cases}$$

$$\text{and } f^*(v_{i_j} v_{i_{j+1}}) = \left\lfloor \frac{f(v_{i_j+1}) - f(v_{i_j})}{ln(f(v_{i_j+1})) - ln(f(v_{i_j}))} \right\rfloor, \text{ for } 1 \le j \le m.$$

Hence, the graph G admits a logarithmic mean labeling. Thus the graph G is obtained from a path by identifying any of its edges by an edge of a cycle, is a logarithmic mean graph.

Corollary 2.5 The triangular snake graph T_n is a logarithmic mean graph.

Corollary 2.6 The alternate triangular snake graph $A(T_n)$ is a logarithmic mean graph.

Corollary 2.7 The quadrilateral snake graph Q_n is a logarithmic mean graph.

Corollary 2.8 The alternate quadrilateral snake graph $A(Q_n)$ is a logarithmic mean graph.

Corollary 2.9 The tadpoles graph T(n, k) is a logarithmic mean graph for n ≥3

II. CONCLUSION

This paper exhibits the properties of logarithmic meanness of some chain graphs**.**

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