Logarithmic Mean Labeling of Some Cycle Related Graphs

A.Durai Baskar, A. Rajesh Kannan, R.Rathajeyalakshmi, P. Manivannan

Abstract: *A function f is called a logarithmic mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f: V(G) \to \{1,2,3,\ldots,q+1\}$ is injective and the induced function $f^*: E(G) \to \{1,2,3,\ldots,q\}$ defined as*

$$f^*(uv) = \left\lfloor \frac{f(v)-f(u)}{\ln f(v) - \ln f(u)} \right\rfloor, \text{ for all } uv \in E(G),$$

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph. In this paper, we study the logarithmic meanness of some cycle related graphs like the cycle C_n for $n \geq 3$, union of a cycle C_m and a path P_n, union of any two cycles C_m and C_n, the graph $C_3 \times P_n$ and the graph $C_n \circ K_1$.

KeyWords: *labeling, logarithmic mean labeling, logarithmic mean graph.*

I. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [4]. For a detailed survey on graph labeling we refer to [3].Path on n vertices is denoted by P_n and a Cycle on n vertices is denoted by C_n. $G \odot S_m$ is the graph obtained from G by attaching m pendant vertices at each vertex of G. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has $p_1 p_2$ vertices which are $\{(u,v): u \in G_1, v \in G_2\}$ and any two vertices(u_1,v_1) and (u_2,v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The study of graceful graphs and graceful labeling methods was first introduced by Rosa [7] and so many authors are working in the area of graph labeling [1,2,5,6], motivated these we introduce a new type of labeling called logarithmic mean labeling.

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A function f is called a logarithmic mean labeling of a graph $G(V,E)$ if $f: V(G) \to \{1,2,3,\ldots,q+1\}$ is injective and the induced function $f^*: E(G) \to \{1,2,3,\ldots,q\}$ defined as $f^*(uv) = \left\lfloor \frac{f(v)-f(u)}{\ln f(v) - \ln f(u)} \right\rfloor$, for all $uv \in E(G)$, is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph.

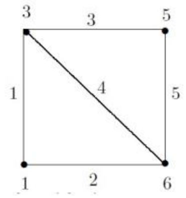

Fig.1 A logarithmic mean graph of $K_4 - e$

In this paper, we have discussed the logarithmic mean labeling of the cycle C_n for $n \geq 3$, union of a cycle C_m and a path P_n, union of any two cycles C_m and C_n, the graph $C_3 \times P_n$ and the graph $C_n \circ K_1$.

2 Main Results

Theorem 2.1 Every cycle is a logarithmic mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n.

We define $f: V(C_n) \to \{1,2,\ldots,n+1\}$ as follows

$$f(v_i) = \begin{cases} i & 1 \leq i \leq \left\lfloor \frac{n+2}{\ln(n+3)} \right\rfloor - 1 \\ i+1 & \left\lfloor \frac{n+2}{\ln(n+3)} \right\rfloor \leq i \leq n. \end{cases}$$

Then the induced edge labeling is as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} i & 1 \leq i \leq \left\lfloor \frac{n+2}{\ln(n+3)} \right\rfloor - 1 \\ i+1 & \left\lfloor \frac{n+2}{\ln(n+3)} \right\rfloor \leq i \leq n-1 \end{cases}$$

and $f^*(v_1 v_n) = \left\lfloor \frac{n+2}{\ln(n+3)} \right\rfloor$.

Hence, f is a logarithmic mean labeling of the cycle C_n. Thus the cycle C_n is a logarithmic mean graph.

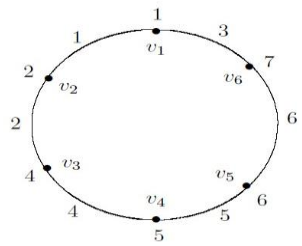

Fig. 2.1 A logarithmic mean labeling of C_6.

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Theorem 2.2 The graph $C_m \cup P_n$ is a logarithmic mean graph.

Proof. Let $u_1, u_2, \dots, u_m$ and $v_1, v_2, \dots, v_n$ be the vertices of the cycle C_m and the path P_n respectively.
We define $f: V(C_m \cup P_n) \to \{1,2,3,\dots,m+n\}$ as follows:

$$f(u_i) = \begin{cases} m+n+2-2i & 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor \\ n & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \\ n-m-1+2i & \left\lfloor \frac{m}{2} \right\rfloor + 2 \le i \le m, \end{cases}$$

$f(v_i) = i,$ for $1 \le i \le n-1$ and $f(v_n) = n+1$.
Then the induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} m+n-2i & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ n & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is odd} \\ n+1 & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is even} \\ n-m-1+2i & \left\lfloor \frac{m}{2} \right\rfloor + 2 \le i \le m-1 \end{cases}$$

$f^*(u_1 u_m) = m+n-1$
and $f^*(v_i v_{i+1}) = i,$ for $1 \le i \le n-1$.

Hence, f is a logarithmic mean labeling of the graph $C_m \cup P_n$. Thus the graph $C_m \cup P_n$ is a logarithmic mean graph, for any $m \ge 3$ and $n \ge 2$.

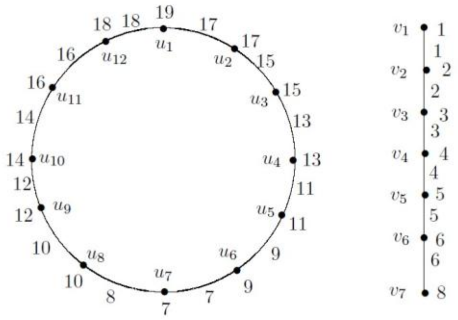

Fig. 2.2 A logarithmic mean labeling of $C_{12} \cup P_7$

The graph $C_m \cup nT, n \ge 2$ cannot be a logarithmic mean graph. But the graph $C_m \cup T$ may be a logarithmic mean graph. The T-graph T_n is obtained by attaching a pendant vertex to a neighbour of the pendant vertex of a path on $(n-1)$ vertices.

Theorem 2.3 For a T-graph T_n, $T_n \cup C_m$ is a logarithmic mean graph, for $n \ge 2$ and $m \ge 3$.

Proof. Let $u_1, u_2, \dots, u_{n-1}$ be the vertices of the path P_{n-1} and u_n be the pendant vertex identified with u_2. Let $v_1, v_2, \dots, v_m$ be the vertices of the cycle C_m.
$\therefore V(T_n \cup C_m) = V(C_m) \cup V(P_n) \cup \{u_n\}$ and
$E(T_n \cup C_m) = E(C_m) \cup E(P_n) \cup \{u_2 u_n\}$.
We define $f: V(T_n \cup C_m) \to \{1,2,3,\dots,m+n\}$ as follows:

$$f(v_i) = \begin{cases} m+n+2-2i & 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor \\ n & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \\ n-m-1+2i & \left\lfloor \frac{m}{2} \right\rfloor + 2 \le i \le m, \end{cases}$$

$f(u_i) = i+1,$ for $1 \le i \le n-2$,

$f(u_{n-1}) = n-1$ ad $f(u_n) = 1$.
Then the induced edge labeling is as follows:
$f^*(v_i v_{i+1}) =$

$$\begin{cases} m+n-2i & 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor \text{ 2mm} \\ n & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is odd 2mm} \\ n+1 & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is even 2mm} \\ n-m-1+2i & \left\lfloor \frac{m}{2} \right\rfloor + 2 \le i \le m-1, \end{cases} \text{ and2mm}$$

$f^*(u_i u_{i+1}) = i+1,$ for $1 \le i \le n-2$,

$f^*(u_2 u_n) = 1$ and $f^*(v_1 v_m) = m+n-1$.

Hence f is a logarithmic mean labeling of $T_n \cup C_m$. Thus the graph $T_n \cup C_m$ is a logarithmic mean graph, for $n \ge 2$ and $m \ge 3$.

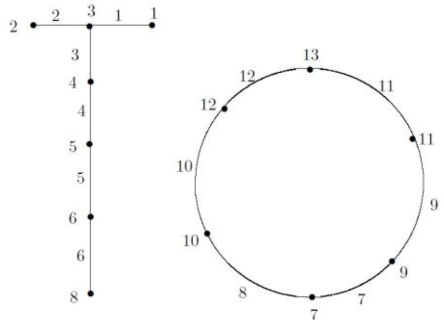

Fig. 2.3 A logarithmic mean labeling of $T_7 \cup C_6$

Theorem 2.4 Union of any two cycles C_m and C_n is a logarithmic mean graph.

Proof. Let $u_1, u_2, \dots, u_m$ and $v_1, v_2, \dots, v_n$ be the vertices of the cycles C_m and C_n respectively. We define $f: V(C_m \cup C_n) \to \{1,2,3,\dots,m+n+1\}$ as follows:

$$f(u_i) = \begin{cases} i & 1 \le i \le \left\lfloor \frac{m+1}{\ln(m+2)} \right\rfloor - 1 \\ i+1 & \left\lfloor \frac{m+1}{\ln(m+2)} \right\rfloor \le i \le m-1, \end{cases}$$

$f(u_m) = m+2$ and

$$f(v_i) = \begin{cases} m+n+3-2i & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ m+1 & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \\ m-n+2i & \left\lfloor \frac{n}{2} \right\rfloor + 2 \le i \le n. \end{cases}$$

Then the induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} i & 1 \le i \le \left\lfloor \frac{m+1}{\ln(m+2)} \right\rfloor - 1 \\ i+1 & \left\lfloor \frac{m+1}{\ln(m+2)} \right\rfloor \le i \le m-1, \end{cases}$$

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$$f^*(v_i v_{i+1}) = \begin{cases} m+n+1-2i & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ m+1 & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ m+2 & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ m-n+2i & \left\lfloor \frac{n}{2} \right\rfloor + 2 \le i \le n-1, \end{cases}$$

$$f^*(u_1 u_m) = \left\lfloor \frac{m+1}{\ln(m+2)} \right\rfloor$$

and $f^*(v_1 v_n) = m + n$.

Hence, f is a logarithmic mean labeling of the graph $C_m \cup C_n$. Thus the graph $C_m \cup C_n$ is a logarithmic mean graph, for any $m, n \ge 3$.

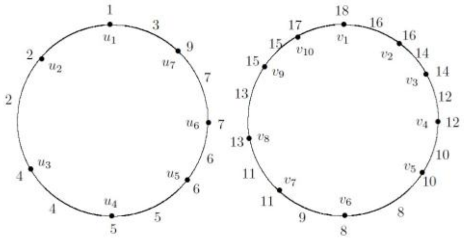

Fig. 2.4 A logarithmic mean labeling of $C_7 \cup C_{10}$

Theorem 2.5 $C_3 \times P_n$ is a logarithmic mean graph, for any n.

Proof. Let $V(C_3 \times P_n) = \{v_1^{(i)}, v_2^{(i)}, v_3^{(i)}; 1 \le i \le n\}$ be the vertex set of $C_3 \times P_n$ and $E(C_3 \times P_n) = \{v_1^{(i)} v_2^{(i)}, v_2^{(i)} v_3^{(i)}, v_1^{(i)} v_3^{(i)}; 1 \le i \le n\} \cup \{v_1^{(i)} v_1^{(i+1)}, v_2^{(i)} v_2^{(i+1)}, v_3^{(i)} v_3^{(i+1)}; 1 \le i \le n-1\}$ be the edge set of $C_3 \times P_n$.

We define $f: V(C_3 \times P_n) \to \{1, 2, 3, \ldots, 6n-2\}$ as follows:

$$f(v_1^{(j)}) = \begin{cases} 9j-8 & 1 \le j \le 2 \\ 8j-11 & 3 \le j \le 4, \end{cases}$$

$$f(v_2^{(j)}) = \begin{cases} 6j-3 & 1 \le j \le 2 \\ 3j+7 & 3 \le j \le 4, \end{cases}$$

$$f(v_3^{(j)}) = \begin{cases} 5+j & 1 \le j \le 2 \\ 7j-6 & 3 \le j \le 4 \text{ and} \end{cases}$$

$f(v_i^{(j)}) = f(v_i^{(j-3)}) + 18$, for $1 \le i \le 3$ and $5 \le j \le n$.

Then the induced edge labeling is as follows:

$$f^*(v_1^{(j)} v_2^{(j)}) = \begin{cases} 1 & j = 1 \\ 5j-1 & 2 \le j \le 3 \\ f^*(v_1^{(j-3)} v_2^{(j-3)}) + 18 & 4 \le j \le n, \end{cases}$$

$$f^*(v_2^{(j)} v_3^{(j)}) = \begin{cases} 3j+1 & 1 \le j \le 2 \\ 5j & 3 \le j \le 4 \\ f^*(v_2^{(j-3)} v_3^{(j-3)}) + 18 & 5 \le j \le n, \end{cases}$$

$$f^*(v_1^{(j)} v_3^{(j)}) = \begin{cases} 6j-4 & 1 \le j \le 2 \\ 8j-11 & 3 \le j \le 4 \\ f^*(v_1^{(j-3)} v_3^{(j-3)}) + 18 & 5 \le j \le n, \end{cases}$$

$$f^*(v_1^{(j)} v_1^{(j+1)}) = \begin{cases} 8j-5 & 1 \le j \le 2 \\ 8(j-1) & 3 \le j \le 4 \\ f^*(v_1^{(j-3)} v_1^{(j-2)}) + 18 & 5 \le j \le n-1, \end{cases}$$

$$f^*(v_2^{(j)} v_2^{(j+1)}) = \begin{cases} 5 & j = 1 \\ 5j+2 & 2 \le j \le 4 \\ f^*(v_2^{(j-3)} v_2^{(j-2)}) + 18 & 5 \le j \le n-1 \text{ and} \end{cases}$$

$$f^*(v_3^{(j)} v_3^{(j+1)}) = \begin{cases} 4j+2 & 1 \le j \le 2 \\ 5j+3 & 3 \le j \le 4 \\ f^*(v_3^{(j-3)} v_3^{(j-2)}) + 18 & 5 \le j \le n-1. \end{cases}$$

Hence f is a logarithmic mean labeling of $C_3 \times P_n$. Thus the graph $C_3 \times P_n$ is a logarithmic mean graph, for any n.

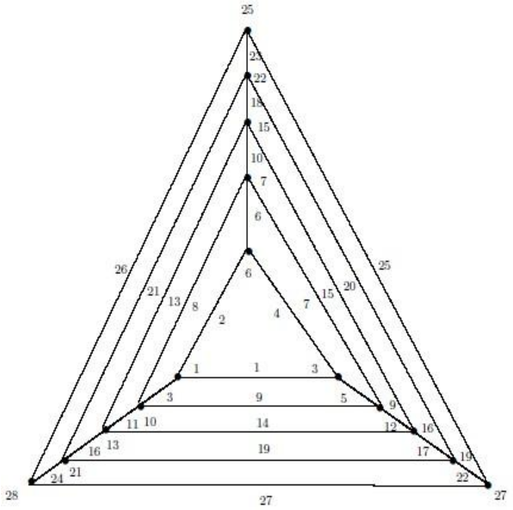

Fig. 2.5 A logarithmic mean labeling of $C_3 \times P_5$

Theorem 2.6 $C_n \odot K_1$ is a logarithmic mean graph, for $n \ge 3$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n and let u_i be the pendant vertices attached at each v_i, for $1 \le i \le n$. Consider the graph $C_n \odot K_1$, for $n \ge 4$.

Case (i) $\left\lfloor \frac{2n}{\ln(2n+1)} \right\rfloor$ is odd.

We define $f: V(C_n \odot K_1) \to \{1, 2, 3, \ldots, 2n+1\}$ as follows:

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 2i & 2 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \\ 2i+1 & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 2 \le i \le n \end{cases} \text{ and}$$

$$f(u_i) = \begin{cases} 2 & i = 1 \\ 2i-1 & 2 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \\ 2i+1 & i = \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \\ 2i & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 2 \le i \le n. \end{cases}$$

Then the induced edge labeling is as follows:

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$$f^*(v_iv_{i+1}) = \begin{cases} 2i & 1 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \\ 2i+1 & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \le i \le n-1, \end{cases}$$

$$f^*(v_1v_n) = \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \text{ and}$$

$$f^*(u_iv_i) = \begin{cases} 2i-1 & 1 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \\ 2i & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \le i \le n. \end{cases}$$

Case (ii) $\left\lfloor \frac{2n}{\ln(2n+1)} \right\rfloor$ is even.

We define $f: V(C_n \odot K_1) \to \{1,2,3,\ldots,2n+1\}$ as follows:

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 2i & 2 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \\ 2i+1 & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \le i \le i \le n \end{cases} \quad \text{and}$$

$$f(u_i) = \begin{cases} 2 & i = 1 \\ 2i-1 & 2 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \\ 2i & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \le i \le n. \end{cases}$$

Then the induced edge labeling is as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 2i & 1 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor - 1 \\ 2i+1 & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*(v_1v_n) = \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \text{ and}$$

$$f^*(u_iv_i) = \begin{cases} 2i-1 & 1 \le i \le \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor \\ 2i & \left\lfloor \frac{4n}{\ln(2n+1)} \right\rfloor + 1 \le i \le n. \end{cases}$$

Hence, the graph $C_n \odot K_1$, for $n \ge 4$ admits a logarithmic mean labeling.For $n = 3$, a logarithmic mean labeling of $C_3 \odot K_1$ is as shown in Figure 2.6.

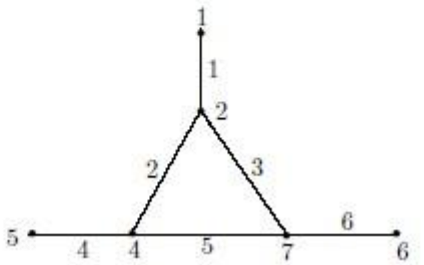

Fig. 2.6 A logarithmic mean labeling of $C_3 \odot K_1$

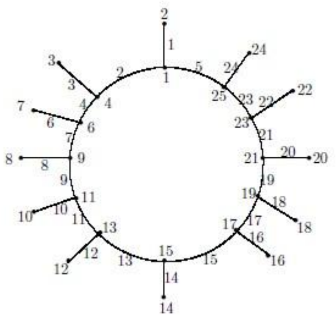

Fig. 2.7 A logarithmic mean labeling of $C_{12} \odot K_1$

II. CONCLUSION

This paper, exhibits the logarithmic meanness of some cycle related graphs like the cycle C_n for $n \ge 3$.

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