Heat and Mass Transfer on MHD Two Phase Blood Flow through a Stenosed Artery with Permeable Wall

D.Karthikeyan, G.Jeevitha

Abstract: ***This paper analyzed the heat and mass transfer effects on the two-phase model of the unsteady pulsatile blood flow when it flows through the stenosed artery with permeable wall under the effects of radiation and chemical reaction. We derive a mathematical model for the mixed convection problem of two-phase blood flow as nonlinear partial differential equations and get the exact solutions interms of Bessel functions for the velocity, temperature and concentration profiles. The effects of various parameters on flow characteristics for two phase blood flow through stenosed artery are depicted in graphs.***

Keywords: ***Bessel differential equation, Blood flow, core and plasma regions***

I. INTRODUCTION

The presence of a constriction in the lumen of an arteries are narrowed by the development of atherosclerotic plaques that protrude into the lumen, resulting arterial stenosis. Artery disturbs the normal blood flow and causes arterial diseases (myocardial infarction and cerebral strokes). It is known that hydrodynamic factors play a vital role in the development and progression of arterial stenosis.

Bugliarello and Sevilla (1970) and Cokelet (1972) have reported that for blood flowing through narrow blood vessels, there is a peripheral layer of plasma(Newtonian fluid) and a core region of suspension of all the red cells as a non-Newtonian fluid. To analyze the two-phase blood flow behavior under these hemodynamical disturbances Ponalagusamy (2016) stabilized the two fluid model for blood flow through a tapered stenotic artery considering core region as couple stress fluid and a peripheralregion of plasma as a Newtonian fluid. The author reported that wall shear stress is highin the case of converging tapered stenosis and it is low in the case of non-tapered divergingstenosis. Further, for the case of symmetric and axisymmetric stenosis Sankar (2011) proposed a mathematical model for two-phase blood flow and investigated that presence of RBC-depleted peripheral layer near the wall helps in the functioning of the diseased arterial system.

In this paper analyzed the heat and mass transfer on the two-phase model of the unsteady oscillatory pulsatile blood flow through the vertical stenosed artery with permeable wall under the effects of radiation and chemical reaction.

We derive a mathematical model for the mixed convection problem of two-phase blood flow as nonlinear partial differential equations and get the exact solutions interms of bessel functions for the velocity, temperature and concentration profiles. The effects of various parameters on flow characteristics for two phase blood flow through stenosed artery are depicted in graphs.

II. FORMULATION OF THE PROBLEM

Consider the continuum model of unsteady, incompressible, oscillatory two-phase blood flow through a vertically stenosed coronary artery of length L in the presence of applies magnetic field M as shown in Fig.1. In the artery of radius r, the two-phase model of blood consists a core region of radius r_c which contains erythrocutes a suspension of the uniform hematocrit of viscosity $\overline{\mu_c}$ and the RBC-depleted plasma layer of radius r_p having viscosity $\overline{\mu_p}$. Artery is assumed as cylindrically shaped as $(\overline{u_c},\overline{v_c},\overline{w_c})$ are the velocity for core region and $(\overline{u_c},\overline{v_c},\overline{w_c})$ represent the velocity vectors for plasma region in $(\bar{r},\bar{\theta},\bar{z})$ directions. Shear rates are assumed as high enough so that for both the regions blood is treated as Newtonian fluid. The temperature of the outer wall of the artery is maintained as $\overline{T_w}$, which is high enough to induce radiative heat transfer and concentration of the blood particles near the wall is assumed as $\overline{C_w}$. To analyze the two-phase model of blood flow viscosity for core and plasma regions are defined separately as

$$\bar{\mu}(\bar{r}) = \begin{cases} \bar{\mu}_c \text{ for } 0 \le r \le r_c(\bar{z}) \\ \bar{\mu}_p \text{ for } r_c(\bar{z}) \le r \le r_p(\bar{z}) \end{cases} \tag{1}$$

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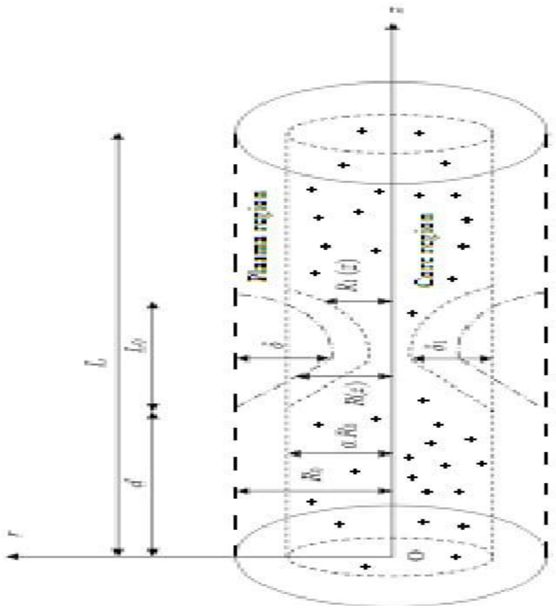

Fig. 1 Geometry of the two-phase vertical stenosed artery

Geometry of the stenosis in plasma region, which is assumed to be symmetric is defined as,

$$\frac{\overline{R(z)}}{\overline{R_0}} = \begin{cases} 1-\frac{\delta_h}{2R_0}\left[1+\cos\frac{2\pi}{L_0}\left(z-d-\frac{L_0}{2}\right)\right]; d \le z \le d+L_0 \\ 1; otherwise \end{cases} \quad (2)$$

In core region geometry of the stenosis is defined as,

$$\frac{\overline{R_1(z)}}{\overline{R_0}} = \begin{cases} \alpha-\frac{\alpha\delta_h}{2R_0}\left[1+\cos\frac{2\pi}{L_0}\left(z-d-\frac{L_0}{2}\right)\right]; d \le z \le d+L_0 \\ \alpha; otherwise \end{cases} \quad (3)$$

where $\overline{L}_0$ is the length of the stenosis as shown in Figure 1 and α is the ratio of the central core radius to the normal artery radius, $\alpha\overline{R}_0$ is the radius of the core region of the normal artery.

So, the equations of core region in terms of these dimensionless parameters can be written as

$$\left(\frac{R_e}{\rho_0}\right)\frac{\partial u_c}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{\mu_0}\left(\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial u_c}{\partial r}\right) - M^2 u_c + \left(\frac{G_r}{\rho_0}\right)\theta_c + \left(\frac{G_m}{\rho_0}\right)\sigma_c \quad (4)$$

$$\frac{P_e K_0}{\rho_0 s_0}\left(\frac{\partial \theta_c}{\partial t}\right) = \left(\frac{\partial^2 \theta_c}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_c}{\partial r}\right) - \frac{K_0}{\alpha_0}N^2\theta_c \quad (5)$$

$$R_e\left(\frac{\partial \sigma_c}{\partial t}\right) = \frac{1}{D_0}\left(\frac{1}{S_c}\right)\left(\frac{\partial^2 \sigma_c}{\partial r^2} + \frac{1}{r}\frac{\partial \sigma_c}{\partial r}\right) - \frac{E}{E_0}\sigma_c \quad (6)$$

Equations for plasma region in dimensionless form are as follows

$$R_e\frac{\partial u_p}{\partial t} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r}\frac{\partial u_p}{\partial r}\right) - M^2 u_p + G_r\theta_p + G_m\sigma_p \quad (7)$$

$$P_e\left(\frac{\partial \theta_p}{\partial t}\right) = \left(\frac{\partial^2 \theta_p}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_p}{\partial r}\right) - N^2\theta_p \quad (8)$$

$$R_e\left(\frac{\partial \sigma_p}{\partial t}\right) = \left(\frac{1}{S_c}\right)\left(\frac{\partial^2 \sigma_p}{\partial r^2} + \frac{1}{r}\frac{\partial \sigma_p}{\partial r}\right) - E\sigma_p \quad (9)$$

where α_0 is the ratio of mean radiation absorption coefficient in the plasma region to mean radiation absorption coefficient in the core region, K_0 is the ratio of thermal conductivity of plasma and core region, s_0 is the specific heat ratio of plasma and core regions and E_0 is the ratio of chemical moles present in the plasma region to chemical moles in the core region. In non-dimensional form factors of thermal radiation, applied magnetic field and chemical reaction are represented by N, M and E for both core and plasma regions.

Corresponding boundary conditions in non-dimensional form to solve the model for both core and plasma regions are given as

$$\begin{cases} u_p = u_B, \dfrac{\partial u_p}{\partial r} = \dfrac{\nu}{\sqrt{D_a}}(u_B - u_{porous}), \theta_p = 1, \sigma_p = 1; r = R(z) \\ u_p = u_c, \theta_p = \theta_c, \sigma_p = \sigma_c; r = R_1(z) \\ \tau_c = \tau_p, \dfrac{\partial \theta_c}{\partial r} = \dfrac{\partial \theta_p}{\partial r}, \dfrac{\partial \sigma_c}{\partial r} = \dfrac{\partial \sigma_p}{\partial r}; r = R_1(z) \\ \dfrac{\partial u_c}{\partial r} = 0, \dfrac{\partial \theta_c}{\partial r} = 0, \dfrac{\partial \sigma_c}{\partial r} = 0; r = 0. \end{cases} \quad (10)$$

III. MATHEMATICAL SOLUTION

Since pumping action of heart results in a pulsatile blood flow, so the pressure gradient can be represented as

$$-\frac{\partial p}{\partial z} = P_0 e^{i\bar{\omega}\bar{t}}$$

In non-dimensional form pressure gradient can be written as

$$-\frac{\partial p}{\partial z} = P_0 e^{it}$$

and flow variables for core and plasma regions in non-dimensional forms can be represented in terms of t as

$$\begin{cases} u_c(r,t) = u_{c_0}(r)e^{it}, u_p(r,t) = u_{p_0}(r)e^{it} \\ \theta_c(r,t) = \theta_{c_0}(r)e^{it}, \theta_p(r,t) = \theta_{p_0}(r)e^{it} \\ \sigma_c(r,t) = \sigma_{c_0}(r)e^{it}, \sigma_p(r,t) = \sigma_{p_0}(r)e^{it} \end{cases} \quad (11)$$

Substituting expression from in to we get time independence momentum, energy and concentration equations of core region

$$\left(\frac{\partial^2 u_{c_0}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{c_0}}{\partial r}\right) - \left(M^2 + \frac{\mu_0 R_e}{\rho_0} i\right) u_{c_0} = -\left(P_0 + \frac{G_r \theta_{c_0}}{\rho_0} + \frac{G_m \sigma_{c_0}}{\rho_0}\right)\mu_0 \quad (12)$$

$$\left(\frac{\partial^2 \theta_{c_0}}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_{c_0}}{\partial r}\right) - \left(\frac{K_0 N^2}{\alpha_0} + i\frac{P_e}{\rho_0}\left(\frac{K_0}{s_0}\right)\right)\theta_{c_0} = 0 \quad (13)$$

$$\left(\frac{\partial^2 \sigma_{c_0}}{\partial r^2} + \frac{1}{r}\frac{\partial \sigma_{c_0}}{\partial r}\right) - \left(iR_e D_0 S_c + \frac{E}{E_0} D_0 S_c\right)\sigma_{c_0} = 0 \quad (14)$$

Same, in of plasma region substitute values from

$$\left(\frac{\partial^2 u_{p_0}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{p_0}}{\partial r}\right) - \left(M^2 + R_e i\right) u_{p_0} = -\left(P_0 + \frac{G_r \theta_{p_0}}{\rho_0} + \frac{G_m \sigma_{p_0}}{\rho_0}\right) \quad (15)$$

$$\left(\frac{\partial^2 \theta_{p_0}}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_{p_0}}{\partial r}\right) - \left(N^2 + iP_e\right)\theta_{p_0} = 0 \quad (16)$$

$$\left(\frac{\partial^2 \sigma_{p_0}}{\partial r^2} + \frac{1}{r}\frac{\partial \sigma_{p_0}}{\partial r}\right) - \left(iR_e S_c + E S_c\right)\sigma_{p_0} = 0 \quad (17)$$

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To find the exact solution of heat transfer and of core and plasma regions under the given boundary conditions we apply the definition of Bessel differential equation by assuming

$$\alpha_1 = -\left(\frac{K_0 N^2}{\alpha_0} + i\frac{P_e}{\rho_0}\left(\frac{K_0}{s_0}\right)\right) \text{ and } \alpha_2 = -\left(N^2 + iP_e\right)$$

So, the equations and change in form of

$$\frac{\partial^2\theta_{c_0}}{\partial r^2} + \frac{1}{r}\frac{\partial\theta_{c_0}}{\partial r} + \alpha_1\theta_{c_0} = 0 \tag{18}$$

$$\frac{\partial^2\theta_{p_0}}{\partial r^2} + \frac{1}{r}\frac{\partial\theta_{p_0}}{\partial r} + \alpha_2\theta_{p_0} = 0 \tag{19}$$

Now, the exact solution for temperature profile of core region under the given boundary conditions is calculated as

$$\theta_{c_0} = \left(X_1\left[\frac{\sqrt{\alpha_2}Y_1\left(\sqrt{\alpha_2}R_1\right)}{X_4 Y_0\left(\sqrt{\alpha_2}R\right)} - \frac{\sqrt{\alpha_1}X_2 J_1\left(\sqrt{\alpha_1}R_1\right)}{X_4}\right] + X_2\right) J_0\left(\sqrt{\alpha_1}(r)\right) \tag{20}$$

where values of are expressed as

$$X_1 = \left(\frac{J_0\left(\sqrt{\alpha_2}R_1\right)}{J_0\left(\sqrt{\alpha_1}R_1\right)} - \frac{J_0\left(\sqrt{\alpha_2}R\right)Y_0\left(\sqrt{\alpha_2}R_1\right)}{Y_0\left(\sqrt{\alpha_2}R\right)J_0\left(\sqrt{\alpha_1}R_1\right)}\right), \quad X_2 = \left(\frac{Y_0\left(\sqrt{\alpha_2}R_1\right)}{J_0\left(\sqrt{\alpha_1}R_1\right)Y_0\left(\sqrt{\alpha_2}R\right)}\right)$$

$$X_3 = \left(J_1\left(\sqrt{\alpha_2}R_1\right) - \frac{J_0\left(\sqrt{\alpha_2}R\right)}{Y_0\left(\sqrt{\alpha_2}R\right)}Y_1\left(\sqrt{\alpha_2}R_1\right)\right), \quad X_4 = \sqrt{\alpha_1}X_1 J_1\left(\sqrt{\alpha_1}R_1\right) - \sqrt{\alpha_2}X_3$$

Solution for the temperature profile of the plasma region under the given boundary conditions is calculated as

$$\theta_{p_0} = \left[\left(\frac{\sqrt{\alpha_2}Y_1\left(\sqrt{\alpha_2}R_1\right)}{X_4 Y_0\left(\sqrt{\alpha_2}R\right)} - \frac{\sqrt{\alpha_1}X_2 J_1\left(\sqrt{\alpha_1}R_1\right)}{X_4}\right)\left(J_0\left(\sqrt{\alpha_2}r\right) - \frac{J_0\left(\sqrt{\alpha_2}R\right)}{Y_0\left(\sqrt{\alpha_2}R\right)}Y_0\left(\sqrt{\alpha_2}r\right)\right)\right] + \frac{Y_0\left(\sqrt{\alpha_2}r\right)}{Y_0\left(\sqrt{\alpha_2}R\right)} \tag{21}$$

Now, the final expressions for temperature profile considering unsteady flow for core and plasma regions respectively are as follows

$$\theta_c = \left[\left(X_1\left(\frac{\sqrt{\alpha_2}Y_1\left(\sqrt{\alpha_2}R_1\right)}{X_4 Y_0\left(\sqrt{\alpha_2}R\right)} - \frac{\sqrt{\alpha_1}X_2 J_1\left(\sqrt{\alpha_1}R_1\right)}{X_4} + X_2\right) J_0\left(\sqrt{\alpha_1}(r)\right)\right)\right] e^{it} \tag{22}$$

$$\theta_p = \left[\left(\frac{\sqrt{\alpha_2}Y_1\left(\sqrt{\alpha_2}R_1\right)}{X_4 Y_0\left(\sqrt{\alpha_2}R\right)} - \frac{\sqrt{\alpha_1}X_2 J_1\left(\sqrt{\alpha_1}R_1\right)}{X_4}\right)\left(J_0\left(\sqrt{\alpha_2}r\right) - \frac{J_0\left(\sqrt{\alpha_2}R\right)}{Y_0\left(\sqrt{\alpha_2}R\right)}Y_0\left(\sqrt{\alpha_2}r\right)\right)\right] e^{it} + \frac{Y_0\left(\sqrt{\alpha_2}r\right)}{Y_0\left(\sqrt{\alpha_2}R\right)} e^{it} \tag{23}$$

where $J_n(x)$ is simply the Bessel function of first kind and $Y_n(x)$ represents the Bessel function of second kind for integer value of n.

To solve subject to the boundary conditions we assume,

$$\beta_1 = -\left(iR_e D_0 S_c + \frac{E}{E_0}D_0 S_c\right) \text{ and } \beta_2 = -\left(iR_e S_c + ES_c\right)$$

So the Equations become

$$\frac{\partial^2\sigma_{c_0}}{\partial r^2} + \frac{1}{r}\frac{\partial\sigma_{c_0}}{\partial r} + \beta_1\sigma_{c_0} = 0 \tag{24}$$

$$\frac{\partial^2\sigma_{p_0}}{\partial r^2} + \frac{1}{r}\frac{\partial\sigma_{p_0}}{\partial r} + \beta_2\sigma_{p_0} = 0 \tag{25}$$

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Now, apply the definition of Bessel differential equation to calculate the value of concentration profiles under the given boundary conditions for both core as well as plasma regions. So, Solution for concentration profile for core region is calculated as

$$\sigma_{c_0} = \left(X_5 \left[\frac{\sqrt{\beta_2} Y_1\left(\sqrt{\beta_2} R_1\right)}{X_8 Y_0\left(\sqrt{\beta_2} R\right)} - \frac{\sqrt{\beta_1} X_6 J_1\left(\sqrt{\beta_1} R_1\right)}{X_8} \right] + X_6 \right) J_0\left(\sqrt{\beta_1}(r)\right) \quad (26)$$

where the values of X_5, X_6 and X_8 are assumed as

$$X_5 = \left(\frac{J_0\left(\sqrt{\beta_2} R_1\right)}{J_0\left(\sqrt{\beta_1} R_1\right)} - \frac{J_0\left(\sqrt{\beta_2} R\right) Y_0\left(\sqrt{\beta_2} R_1\right)}{Y_0\left(\sqrt{\beta_2} R\right) J_0\left(\sqrt{\beta_1} R_1\right)} \right), \quad X_6 = \left(\frac{Y_0\left(\sqrt{\beta_2} R_1\right)}{J_0\left(\sqrt{\beta_1} R_1\right) Y_0\left(\sqrt{\beta_2} R\right)} \right)$$

$$X_7 = \left(J_1\left(\sqrt{\beta_2} R_1\right) - \frac{J_0\left(\sqrt{\beta_2} R\right)}{Y_0\left(\sqrt{\beta_2} R\right)} Y_1\left(\sqrt{\beta_2} R_1\right) \right), \quad X_8 = \sqrt{\beta_1} X_5 J_1\left(\sqrt{\beta_1} R_1\right) - \sqrt{\beta_2} X_7$$

Solution for concentration profile of plasma region under the given boundary conditions is calculated as

$$\sigma_{p_0} = \left[\left(\frac{\sqrt{\beta_2} Y_1\left(\sqrt{\beta_2} R_1\right)}{X_8 Y_0\left(\sqrt{\beta_2} R\right)} - \frac{\sqrt{\beta_1} X_6 J_1\left(\sqrt{\beta_1} R_1\right)}{X_8} \right) \left(J_0\left(\sqrt{\beta_2} r\right) - \frac{J_0\left(\sqrt{\beta_2} R\right)}{Y_0\left(\sqrt{\beta_2} R\right)} Y_0\left(\sqrt{\beta_2} r\right) \right) \right] + \frac{Y_0\left(\sqrt{\beta_2} r\right)}{Y_0\left(\sqrt{\beta_2} R\right)} \quad (27)$$

So, the final solutions of concentration profile for unsteady in core and plasma regions respectively are as follows

$$\sigma_c = \left[\left(X_5 \left(\frac{\sqrt{\beta_2} Y_1\left(\sqrt{\beta_2} R_1\right)}{X_8 Y_0\left(\sqrt{\beta_2} R\right)} - \frac{\sqrt{\beta_1} X_6 J_1\left(\sqrt{\beta_1} R_1\right)}{X_8} + X_6 \right) J_0\left(\sqrt{\beta_1}(r)\right) \right) \right] e^{it} \quad (28)$$

$$\sigma_p = \left[\left(\frac{\sqrt{\beta_2} Y_1\left(\sqrt{\beta_2} R_1\right)}{X_8 Y_0\left(\sqrt{\beta_2} R\right)} - \frac{\sqrt{\beta_1} X_6 J_1\left(\sqrt{\beta_1} R_1\right)}{X_8} \right) \left(J_0\left(\sqrt{\beta_2} r\right) - \frac{J_0\left(\sqrt{\beta_2} R\right)}{Y_0\left(\sqrt{\beta_2} R\right)} Y_0\left(\sqrt{\beta_2} r\right) \right) \right] e^{it} + \frac{Y_0\left(\sqrt{\beta_2} r\right)}{Y_0\left(\sqrt{\beta_2} R\right)} e^{it} \quad (29)$$

To find the solution for velocity profile in core region, we put the values of θ_{c_0} and σ_{c_0} from and apply the method variation of parameters for the given non-homogenous differential equations by assuming

$$\gamma_1 = -\left(M^2 + \frac{\mu_0 R_e}{\rho_0} i \right) \quad (30)$$

$$\left(\frac{\partial^2 u_{c_0}}{\partial r^2} + \frac{1}{r} \frac{\partial u_{c_0}}{\partial r} \right) + \gamma_1 u_{c_0} = -\left(P_0 + \frac{G_r \theta_{c_0}}{\rho_0} + \frac{G_m \sigma_{c_0}}{\rho_0} \right) \mu_0 \quad (31)$$

So, the momentum equation convert in the form of which can be treated as an ordinary differential equation. In variation of parameters method first, we calculate the complementary solution for homogenous differential equation by using the definition the Bessel differential equation as

$$u_{c_{0C}} = C_1 J_0\left(\sqrt{\gamma_1} r\right) + C_2 Y_0\left(\sqrt{\gamma_1} r\right)$$

where we have

$$u_{c_{0_1}} = J_0\left(\sqrt{\gamma_1} r\right), u_{c_{0_2}} = Y_0\left(\sqrt{\gamma_1} r\right)$$

The Wronskian of these two functions is

$$W_1 = \begin{vmatrix} J_0\left(\sqrt{\gamma_1} r\right) & Y_0\left(\sqrt{\gamma_1} r\right) \\ -\sqrt{\gamma_1} J_1\left(\sqrt{\gamma_1} r\right) & -\sqrt{\gamma_1} Y_1\left(\sqrt{\gamma_1} r\right) \end{vmatrix} = \frac{2}{\pi r}$$

Now, for finding the complete solution of the non-homogeneous we find

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$$F_1 = -\int \frac{Y_0\sqrt{\gamma_1}r\left(P_0 + \frac{G_r\theta_{c_0}}{\rho_0} + \frac{G_m\sigma_{c_0}}{\rho_0}\right)\mu_0}{W_1}dr \text{ and } G_1 = \int \frac{J_0\sqrt{\gamma_1}r\left(P_0 + \frac{G_r\theta_{c_0}}{\rho_0} + \frac{G_m\sigma_{c_0}}{\rho_0}\right)\mu_0}{W_1}dr$$

The complete solution of the velocity profile for the core region is of the form of

$$u_{c_0} = C_1J_0\left(\sqrt{\gamma_1}r\right) + C_2Y_0\left(\sqrt{\gamma_1}r\right) + F_1J_0\left(\sqrt{\gamma_1}r\right) + G_1Y_0\left(\sqrt{\gamma_1}r\right) \quad (32)$$

Now for of plasma region, we assume that

$$\gamma_2 = -\left(M^2 + R_e i\right) \quad (33)$$

So the convert in terms γ_2 as

$$\left(\frac{\partial^2 u_{p_0}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{p_0}}{\partial r}\right) + \gamma_2 u_{p_0} = -\left(P_0 + \frac{G_r\theta_{p_0}}{\rho_0} + \frac{G_m\sigma_{p_0}}{\rho_0}\right) \quad (34)$$

For given complementary solution for homogeneous differential equation can be calculated as

$$u_{p_{0C}} = C_3J_0\left(\sqrt{\gamma_2}r\right) + C_4Y_0\left(\sqrt{\gamma_2}r\right)$$

So, we have

$$u_{p_{0_1}} = J_0\left(\sqrt{\gamma_2}r\right), u_{p_{0_2}} = Y_0\left(\sqrt{\gamma_2}r\right)$$

The Wronskian of these two functions is

$$W_2 = \begin{vmatrix} J_0\left(\sqrt{\gamma_2}r\right) & Y_0\left(\sqrt{\gamma_2}r\right) \\ -\sqrt{\gamma_2}J_1\left(\sqrt{\gamma_2}r\right) & -\sqrt{\gamma_2}Y_1\left(\sqrt{\gamma_2}r\right) \end{vmatrix} = \frac{2}{\pi r}$$

For solution of the non-homogeneous equation further, we calculate

$$F_2 = -\int \frac{Y_0\sqrt{\gamma_2}r\left(P_0 + \frac{G_r\theta_{c_0}}{\rho_0} + \frac{G_m\sigma_{c_0}}{\rho_0}\right)\mu_0}{W_2}dr \text{ and } G_2 = \int \frac{J_0\sqrt{\gamma_2}r\left(P_0 + \frac{G_r\theta_{c_0}}{\rho_0} + \frac{G_m\sigma_{c_0}}{\rho_0}\right)\mu_0}{W_2}dr$$

The complete solution of the velocity profile for the plasma region is of the form of

$$u_{p_0} = C_3J_0\left(\sqrt{\gamma_1}r\right) + C_4Y_0\left(\sqrt{\gamma_1}r\right) + F_2J_0\left(\sqrt{\gamma_1}r\right) + G_2Y_0\left(\sqrt{\gamma_1}r\right) \quad (35)$$

Now, we calculate the values of unknowns C_1, C_2, C_3 and C_4 by using boundary condition

First applying the boundary condition $\frac{\partial u_c}{\partial r} = 0$ at $r = 0$ we get $C_2 = 0$

Now, becomes

$$u_{c_0} = C_1J_0\left(\sqrt{\gamma_1}r\right) + F_1J_0\left(\sqrt{\gamma_1}r\right) + G_1Y_0\left(\sqrt{\gamma_1}r\right) \quad (36)$$

After applying all the boundary conditions in to the we get the linear system of C_1, C_3 and C_4 in the form of

$$\begin{pmatrix} J_0\left(\sqrt{\gamma_1}R_1\right) & -J_0\left(\sqrt{\gamma_2}R_1\right) & -Y_0\left(\sqrt{\gamma_2}R_1\right) \\ 0 & S_1 & S_2 \\ -\sqrt{\gamma_1}J_1\left(\sqrt{\gamma_1}R_1\right) & \sqrt{\gamma_2}J_1\left(\sqrt{\gamma_2}R_1\right) & \sqrt{\gamma_2}Y_1\left(\sqrt{\gamma_2}R_1\right) \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} \quad (37)$$

where $S_1 = J_0(\sqrt{\gamma_2}R) + \frac{\sqrt{D_a}}{\nu}J_1(\sqrt{\gamma_2}R)\gamma_2$. $S_2 = Y_0(\sqrt{\gamma_2}R) + \frac{\sqrt{D_a}}{\nu}Y_1(\sqrt{\gamma_2}R)\gamma_2$.

Then D_1, D_2 and D_3 are expressed as

$$D_1 = -A_1(R_1)J_0\left(\sqrt{\gamma_1}R_1\right) - B_1(R_1)Y_0\left(\sqrt{\gamma_1}R_1\right) + A_2(R_1)J_0\left(\sqrt{\gamma_2}R_1\right) + B_2(R_1)Y_0\left(\sqrt{\gamma_2}R_1\right)$$

$$D_2 = -A_2(R)\left[J_0\left(\sqrt{\gamma_2}R\right) + J_1(\sqrt{\gamma_2}R\sqrt{\gamma_2}\frac{\sqrt{D_a}}{\nu}\right] + B_2(R)\left[Y_0\left(\sqrt{\gamma_2}R\right) + Y_1(\sqrt{\gamma_2}R\sqrt{\gamma_2}\frac{\sqrt{D_a}}{\nu}\right] + u_{porous}$$

$$D_3 = -\frac{\partial A_1(R_1)}{\partial r}J_0\left(\sqrt{\gamma_1}R_1\right) + A_1(R_1)\sqrt{\gamma_1}J_1\left(\sqrt{\gamma_1}R_1\right) - \frac{\partial B_1(R_1)}{\partial r}Y_0\left(\sqrt{\gamma_1}R_1\right) + B_1(R_1)\sqrt{\gamma_1}Y_1\left(\sqrt{\gamma_1}R_1\right)$$
$$+\frac{\partial A_2(R_1)}{\partial r}J_0\left(\sqrt{\gamma_2}R_1\right) + \frac{\partial B_2(R_1)}{\partial r}Y_0\left(\sqrt{\gamma_2}R_1\right) - A_2(R_1)\sqrt{\gamma_2}J_1\left(\sqrt{\gamma_2}R_1\right) - B_2(R_1)\sqrt{\gamma_2}Y_1\left(\sqrt{\gamma_2}R_1\right)$$

It can be clearly seen from equation (31) that it is simply the linear system of order 3 X 3 with C_1, C_3 and C_4 unknowns and which has the unique solution. Final exact solutions for velocity profiles u_{c_0} and u_{p_0} for core plasma regions obtain by putting the values of C_1, C_3 and C_4 in and respectively.

The velocity profile of the two phase blood flow in the stenosed artery is

$$U = u_p + u_c \qquad (38)$$

The total volumetric flow rate of the blood flow in the artery is calculated as

$$Q = 2\pi R_0^2 u_0 \int_0^R urdr \qquad (39)$$

The shear stress at the interface wall of core and the plasma region is obtained as

$$\tau = \left(-\mu\frac{\partial U}{\partial r}\right)_{r=R} \qquad (40)$$

IV. RESULT AND DISCUSSION

For having adequate insight into the two-phase flow behavior of blood flow through a vertical stenosed arterial segment flow resistive impedance, volumetric flow rate, wall shear stress and velocity profile have been estimated assuming pulsatile, unsteady and Newtonian nature of the blood flow for both core and plasma regions. A computational study has been carried out to graphically show the effects of RBC-depleted plasma layer on blood flow with the variation of different quantities of interest.

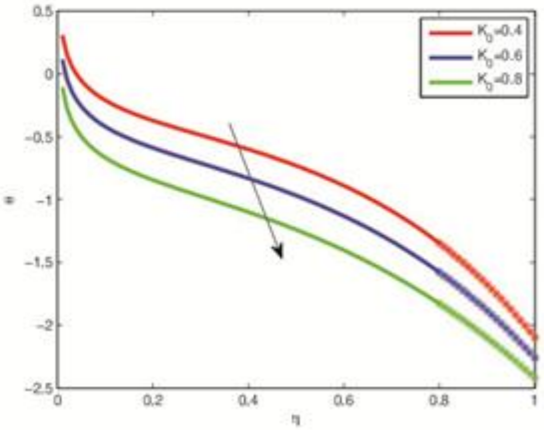

Fig. 2 Variation of temperature profile for two phasetemperature model of blood flow with (K0)

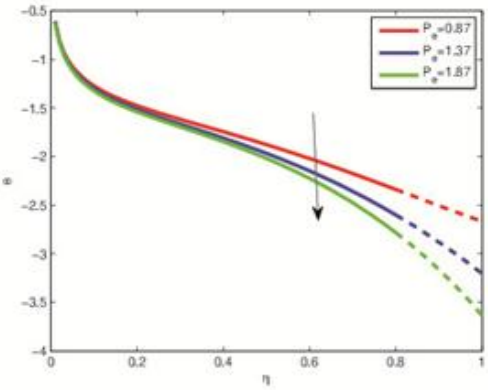

Fig. 3 Effects of Peclet number (P_e) on profile for two phase model

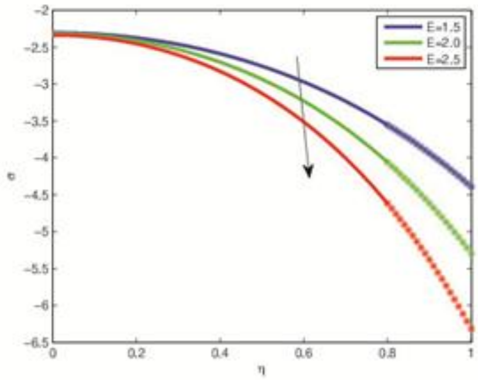

Fig. 4 Effect of chemical reaction parameter (E) onconcentration profile for two phase model

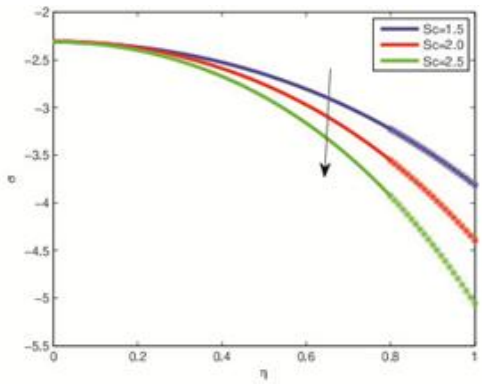

Fig. 5 Effect of Schmidt number (S_c) on concentration profile for two phase model

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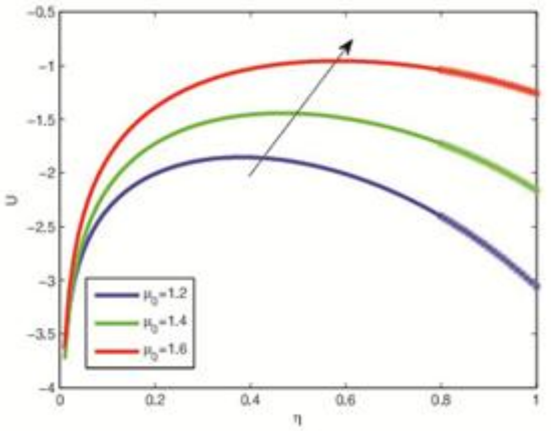

Fig. 6 Variation of velocity profile for two phase model with the ratio of viscosity in plasma and core regions

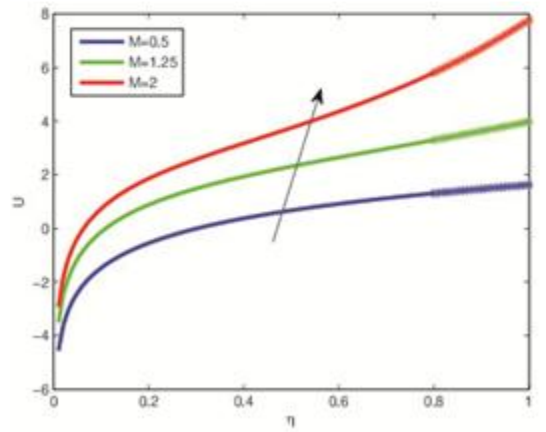

Fig. 7 Effect of magnetic field parameter (M) on velocity profile for two phase model for plasma and core regions

Figure 2 and Figure 3 illustrate the effects on the temperature profile of two phase blood flow with increase in the values of (K_0) and (P_e) parameters respectively. Continuous lines in the plot show the variation of the temperature profile in the core region and dotted lines display the variations of the temperature profile in the plasma region. As it could be seen from these figures that as the ratio of the thermal conductivity of plasma and core regions (K_0)and value of the Peclet number (P_e) increase, the temperature profile of the two-phase blood flow decrease in both core and plasma regions respectively. Figure 4 and Figure 5 reveal that under the purview of the present computational study, concentration profiles for both core and plasma regions in two phase blood flow decrease as the values of the chemical reaction parameter and Schmidt number increase respectively. Now, the following figures are plotted to analyze the effects of various dimensionless parameters on velocity, temperature and concentration profiles. Figure 6 displays the variation of velocity profile for different values of (μ_0) which is the ratio of the viscosity in plasma region to the core region. In figure continuous and dotted lines show the velocity variations in core and plasma regions respectively, in which plasma region varies from 0.8 to 1 radius of the artery.From Figure 6 it is clear that as the value of viscosity ratio increases, the value of velocity profiles also increase for both core and plasma regions. Figure 7 shows the effect of varying applied magnetic field on velocity profiles as continuous lines in the figure show the magnetic field effect on velocity profile of core region and dotted lines display the effect of varying magneticfield on velocity profile of plasma region. It is clear from Figure 7 that as the value of uniform applied magnetic field increases velocity profiles for both core and plasma regions increase respectively. This happens because the mature red blood cells contain high concentration hemoglobin molecules in its content which are oxides of iron. So, when MHD blood flows under the influence of uniform applied magnetic field erythrocytes orient with their own disk plane parallel to the direction of applied magnetic field. This action forms red blood cellsand magnetic particles more suspended in blood plasma, so increased concentration of magnetic particles causes an increase in the internal blood viscosity.

V. CONCLUSION

It is analyzed through our results that as the value of both thermal Radiation and chemical reaction parameter increase as the temperature and concentration profiles in both core and plasma regions decrease respectively. To show the effectiveness of the two-phase model we carried out the comparative study between the single-phase and two-phase model of the blood flow and found that the two-phase model fits with the experimental data more accurately than the single phase model of blood flow. To get the proper understanding of the effects of two-phase blood flow on stenosed artery of the graphs of the variations of wall shear stress, resistive impedance, volumetric flow rate and velocity profile have been plotted which show their sinusoidal behavior with time. The model predicts increase in wall shear stress with peripheral layer viscosity. Predicted trends are found to exist in artery and hence validate the model. More experimental results are required for further development from clinical point of view.

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