A Face Change of Monophonic Dominating Sets by Non-adjacency in Graphs

A. Sadiquali , P. Arul Paul Sudhahar

Abstract:The concept of an independent domination number (id-number in short) of a graph is related to the movements of a chessboard. In this paper, we extend the notion of id-number into an independent monophonic domination number of graphs(abridged as imd-number) by introducing chordless paths and non-adjacency property among vertices. The imd-number can be used to optimize the number of mutually non-attacking queens in the play of Chessboard. Discussed the face changing process of monophonic dominating sets by non-adjacency property and some of its properties are studied. It is shown that for $d_m, k, p \in Z^+$ with $3 \le d_m \le k$ andp $\ge k + 1 + \lceil \frac{2d_m}{3} \rceil$ there exists connected graphsGsuch that$|V| = p$, $diam(G) = d_m$ (monophonic diameter of G) and imd- number $i_m(G) = k$. Also for $k, l \in Z^+$with $2 \le k \le l \le p$, there exists connected graphsG $\in \xi(G)$ such that $m(G) = k, \gamma(G) = k$ and $i_m(G) = l$. The imd-number of certain common class of graphs are determined.

Keywords:Monophonic Domination Number, Independence Number, Independent Monophonic Domination Number and Independent Monophonic Dominating sets.

I. INTRODUCTION

In 1962 both Berge [4] and O. Ore [17] formalized the concept and theory of independent domination in graphs and related parameter known as independent domination number (id-number in short) of a graph G. The notation $i(G)$ for id-number was introduced by Cockayne and Hedetniemi [7,8]. In the year 1944, the notion Independent dominating sets (id-sets in short) were presented into the platform of game theory by Neumann and Margenstern, see [18]. Here, we introduce the concept of independent monophonic dominating sets in graphs (imd-set in short) by using chordless path or induced paths and non-adjacency property among vertices.The term dominating set was first coined by O. Ore and introduced the parameter domination number of a graph in his book [17].Inthis paper, all the graphs are finite, undirected, connected and simple. For the basic graph theoretic terminology and notations,refer to [3,5,11]. By a graph G we mean an ordered pair of the vertex set V and edge set E .The order and size of G are denoted by $p = |V|$ and $q = |E|$ respectively.

T.W. Hayness,et.al, established the domination parameters through his work entitled Fundamentals of Domination in Graphs[15].

The dominating set and domination number of a graph G are abbreviated by d-set and d-number respectively. As cited by [15], for $k \in Z^+$, we say that $D \subseteq V$ isa k-dominating set of G if $|N(v) \cap D| \ge k$for all $v \in V$.Also $\gamma^k(G) = \min\{|D|: D, \text{ a k} - \text{dominating set}\}$is termed as k-domination number of G. F.Buckley and F.Harary[3] was introduced the notion namely monophonic number (m-number) of a graph in his work entitled Distance in Graphs. and was studied by Ignacio M. Pelayo et al [14]. A chord of a path P in G is an edge when it is the join of two non-adjacent vertices of G. A $u - v$ path P is said to be a monophonic path (m-path or an induced path), if it is a chordless path, that is a path P from u to v is an m-path when there is no edge joining non-consecutive vertices in P. The monophonic distance$d_m(u, v)$ from the vertex u to v in G is defined to be the length of the longest $u - v$ m-path in G. For convenience, we use some notationslike $e_m(v)$, d_m, $r_m Ext(G)$ and $Pend(G)$ as in [20,21].

As cited by [23], we say that $M \subseteq V$ is m- set (monophonic set) of G if every vertex of G is contained in a m-path of some pair of vertices of M and the m-number $m(G) = min\{|M|: M, \text{a m} - \text{set of G}\}$. Further references, see[9,23,24] and read [6,12,13,14] for the similar concepts like geodetic sets

Definition 1(Goddard, Wayne et.al [10]) A set M of vertices in G is said to be independent if no two of its vertices are adjacent in G. The largest number of vertices in such a set is called the independent number of G and it is denoted by $\alpha(G)$

Definition 2(Goddard, Wayne et.al [10])A set D of vertices in graphs$G$ is called an independent dominating set (id-set in short) of G if D is both an independent and dominating set of G. The minimum of the cardinalities of an id-sets is called independent domination number (id-number) of G.

An id-set is also called a maximal independent set so that symbol $i(G)$ denotes an id- number of G or called Stable number of the graph. Further refer to [1,10,19].

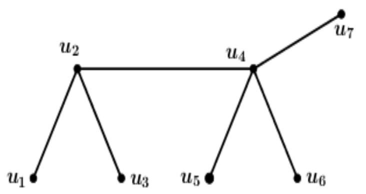

Figure 1 . A graph G with $\alpha(G) = 5, \gamma(G) = 2$

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Example 1 Consider the graph G given in the Figure 1, $M_1 = \{u_2, u_4\}$ is the minimum d- set of G but it is not independent so that $\gamma(G) = 2$. The vertices of the set $M_2 = \{u_1, u_3, u_5, u_6, u_7\}$are independent and maximum.Note that $M_2 = End(G)$ Clearly, it is an id-set of G and so that $\alpha(G) = 5$ .Thus $i(G) = 5$ is the smallest size of a maximal independent set in G.

Note that maximum size of a minimal d-set is denoted by $\gamma^+(G)$ known as upper d-number of G. Hence for any connected graph G with vertex count p,we have $1 \le \gamma(G) \le i(G) \le \alpha(G) \le \gamma^+(G) \le pand1 \le \alpha(G) \le p$. The notion of the monophonic domination number (shortened as md-number) of a graph was presented by J. John et al [16] in the year 2012. As referred by [16],monophonic dominating set (md-set in sort) can be defined as follows.

Definition 3(J.John et.al [16]) A subset M of V is said to be a md-set of a graph G if M is both monophonic and a d-set of G. The minimum of the cardinalities of the md-sets in G is called the md-number of G, and is denoted by $\gamma_m(G)$

By definition 3, $\gamma(G) \le \gamma_m(G) \le i(G) \le \alpha(G)$and $2 \le \gamma_m(G) \le p$. For the detailed works on md-sets,see [2, 20, 21,22]. A md-set of any connected graph G may or may not be an independent set also its converse need not be true in general. This has motivated us to study those subsets of the vertex set of graphs that are both monophonic dominating and independent.

II. MONOPHONIC DOMINATION IN GRAPHS BY NON-ADJACENCY

In this section, we observe the non-adjacency of vertices in the different md-sets of graphs. It lights up into a new variable, called as independent monophonic domination number (imd-number in short) of G, which is associated to an imd-set of graphs.

Definition 4. A subset M of V is an independent monophonic dominating set (imd-set in short) of G if M is both md-set and an independent set of G. The minimum of the cardinalities of the imd-sets of G is termed as imd-number, and is denoted by$i_m(G)$ and

$i_m(G) = min\{|M|: \text{M, imd} - \text{set ofG}\}$.

The md-number $\gamma_m(G)$ is different from imd-number $i_m(G)$. We establish this by an example. Is all the graphs are possess an imd-sets?answer will be more useful to this study.

Example 2.Consider the graph G given in the Figure 2, $M_1 = \{v_1, v_2, v_5\}$and$M_2 = \{v_1, v_4, v_5\}$are the only 3-element md-set of G. So$\gamma_m(G) = 3$. But it does not induce an independent vertex set. If we add more independent vertices by replacing v_2 and v_4 from the md-sets M_1 and M_2 of G, then we have$M_3 = \{v_1, v_5, v_6, v_7, v_8, v_9\}$ which is the minimum imd - set of G. It follows that imd- number $i_m(G) = 6$ and note that $\alpha(G) = 7$.

The variables $\gamma_m(G)$ and $i_m(G)$ may not be equal, when $G = C_3$, Clearly $i_m(C_3) = 0$,that is, cycle graph C_3 has no imd-number but $\gamma_m(C_3) = 3$

Notation: $\xi(G)$ denotes the collection of all connected graphsGwith

$i_m(G) \ge 1\xi(G) =$

$\{G: G \text{ a connected graph with} i_m(G) \ge 1\}$

For unexplained terms and symbols, see [3,5]. We cite a few preliminary observations which are to be used in the sequel.

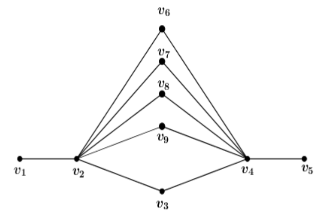

Figure 2 . A graph G with $i_m(G) = 6, \alpha(G) = 7, \gamma_m(G) = 3$

III. PRELIMINARY OBSERVATIONS

Observation1. For connected graphs$G$ with vertex count p, $i_m(G) \in [0, p]$

Observation 2.For graphs$G \in \xi(G)$with vertex count p, $i_m(G) \in [0, p]$and $2 \le \gamma_m(G) \le i_m(G) \le \alpha(G)$

Observation 3.For every imd-set Min G, $Pend(G) \subseteq M$

Observation 4.Every imd-sets of G is a md-set. But not conversely.

Observation 5. For every imd-set $M$of ,$Ext(G) \subseteq M$. If $M = Ext(G)$,then M is unique and $i_m(G) = |M|$.

Note that if a graph G contains a clique, then any md-set of G contains at most one vertex of the clique. It means that, if G contains a clique, then at most one vertex of the clique is an element of Ext (G). Also vertex of the clique belonging to any imd-set of G need not be an element of Ext(G) .

IV. IMD- NUMBER OF STANDARD GRAPHS

This section gives the imd-number of standard graphs.

Theorem 1.Let K_p be a complete graph on p vertices. Then$i_m(K_p) = 0$, that is imd-number does not exists in K_p

Proof.We know thatvertex set V is theunique md-set of K_p. Since every pair of vertices of K_p is adjacent, K_p hasno independent set. Hence $i_m(K_p) = 0$.

Remark 1. If we delete one edge $e = \{v_1, v_2\}$fromK_p, we obtained a new graph $G = K_p - \{e\}$. Clearly $M = \{v_1, v_2\}$ is an unique md-set of G and so $\gamma_m(G) = 2$ also M is independent. Hence$i_m(G) = 2$.

Theorem 2 Let P_p be a path graph on p vertices.Then $i_m(P_p) = \lceil\frac{p+2}{3}\rceil$ where$p \ge 3$.

Proof. Let $V(P_p) = \{x_1, x_2, \dots x_p\}$ where $p \ge 3$. Then we consider the following cases based on p

Case 1:- $p \equiv 0(mod 3)$.

Here, we choose some non-adjacent vertices from the path graph to form imd-set. Take $M = \{x_1, x_4, \dots x_{p-2}, x_p\}$ Clearly M isa minimum md-set in P_p and M is independent. Thus, $i_m(P_p) \le \gamma_m(P_p)$. Henceby the Observation 2, $i_m(P_p) = \gamma_m(P_p) = \lceil\frac{p+2}{3}\rceil$.

Case2:-$p \equiv 1(mod 3)$.

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Choose $M = \{ x_1, x_4, \dots x_{p-3}, x_p\}$. Clearly M is aminimum md-set in P_p. By the Observation 2, imd-number coincide with md-numberof P_p. Finally

Case3:- $p \equiv 2(mod 3)$.

Choose $M = \{ x_1, x_4, \dots x_{p-4} x_{p-2}, x_p\}$.Clearly M isboth independent and md-set of P_p. Hence result is true in this case also.

Theorem 3.Let C_p be a cycle graph on p vertices.Then$i_m(C_p) = \lceil\frac{p}{3}\rceil$ where $p \geq 4$ but $p \neq 5$.

Proof.Let $V(C_p) = \{x_1, x_2, \dots x_p\}$where $p \geq 4$ but $p \neq 5$. Then consider the following cases based on p

Case 1:- $p \equiv 0(mod 3)$.

Here, we choose some non-adjacent vertices from the cycle graph to form imd-set.Take $M = \{ x_1, x_4, \dots x_{\{p-2\}}\}$ Clearly M is a minimum md-set in C_p and M is independent. Thus, $i_m(C_p) \leq \gamma_m(C_p)$.Henceby the Observation2, $i_m(C_p) = \gamma_m(C_p) = \lceil\frac{p}{3}\rceil$.

Case2:-$p \equiv 1(mod 3)$.

Choose $M = \{ x_1, x_4, \dots x_{p-3} x_{p-1}\}$. Clearly M is aminimum md-set in C_p. By the Observation 2, imd-number coincide with md-numberof C_p. Finally

Case3:- $p \equiv 2(mod 3)$.

Choose $M = \{ x_1, x_4, \dots x_{p-4} x_{p-1}\}$.Clearly M is both independent and md-set of C_p. Hence the result is true in this case also.

Theorem 4. For the complete bipartite graphs$G = K_{pq}$, $i_m(G) = min\{p, q\}$ where $p, q \geq 2$.

Proof. Consider the vertex partitions $V_1$and V_2 of $G = K_{pq}$with vertex counts$p$ and q respectively. Take $V_1 = \{x_1, x_2, \dots x_p\}andV_2 = \{y_1, y_2, \dots y_q\}$, clearly $V_1$and V_2 are independent sets in G. If weassume M is a imd-set of G then we can find, either $M = V_1$ or $M = V_2$. For, if$M$is a proper subset of V_1 or V_2 , then the vertices of $V_1 - M$or $V_2 - M$ are not adjacent to any vertex of M. Therefore, $i_m(G) = min\{|V_1|, |V_2|\} = min\{ p, q\}$.

V. ON THE GENERAL BOUNDS OF IMD-NUMBER

This section gives the general bounds of $i_m(G)$of G.

Theorem 5. For any graph $G \in \xi(G)$ with vertex count p, $i_m(G) \leq p - \gamma(G)$

Proof.Let M be a minimum imd- set of G. Then M and $V - M$ are dominating sets of$G$. It follows that

$$\gamma(G) \leq |V - M| \Rightarrow \gamma(G) \leq |V| - |M|$$
$$\Rightarrow \gamma(G) \leq p - i_m(G).$$

Hence $i_m(G) \leq p - \gamma(G)$.

Theorem 6.Let $G \in \xi(G)$ with vertex count p and let M be an imd-set of G.Then $V - M$ is a k-dominating set of G and $i_m(G) \leq p - \gamma^k(G)$, where $\delta(G) \geq k$ and $\gamma^k(G)$ is the k-domination number of G.

Proof.Since M is an independent set of vertices, no two of its members are adjacent in G. If $v \in M$, then v is adjacent to at least k-vertices of $V - M$ where $k \leq \delta(G)$. It follows that $V - M$ is a k-dominating set of G. Also

$$\gamma^k(G) \leq |V - M| = |V| - |M|$$
$$\Rightarrow \gamma^k(G) \leq p - i_m(G).$$

Hence the required bound attained.

Next result gives the uniqueness of $\gamma_m(G)$ and $i_m(G)$

Theorem 7.Let $G \in \xi(G)$ with vertices $p \geq 3$.Then$i_m(G) = 2 \Leftrightarrow \gamma_m(G) = 2$

Proof. Assume that $i_m(G) = 2$. By the Observation2 ,$2 \leq \gamma_m(G) \leq 2$. It follows that $\gamma_m(G) = 2$. Conversely assume that $\gamma_m(G) = 2$,then there exists a minimum imd-set M such that $|M| = 2$. Take $M = \{ x, y\}$. Clearly we can see that $|V - M|$ is non-empty. It follows that every $x - y$ m-path contains one more vertex and $d_m(x, y) \geq 2$. Hence M is an imd- set of G and $i_m(G) = |M| = 2$.

Theorem 8.For any $G \in \xi(G)$ with vertex count p,

$$\left\lceil\frac{p}{1+\Delta}\right\rceil \leq i_m(G) \leq p$$

whereΔ is the maximum vertex degree of G

Proof. Let $G \in \xi(G)$ be any connected graph with vertex count $p = |V|$. Then V may or may not be an independent set in G. Therefore, by the Observation1, $i_m(G) \leq p$. Without loss of generality we assume that $i_m(G) = k$ where $k \leq p$. Then there exists an imd-set $M = \{v_1, v_2, \dots v_k\}$ such that $|M| = k$. For $i \in \{1,2,3, \dots, k\}$, $deg(v_i) \leq \Delta$. we can write $\sum_{i=1}^{k}(1 + \deg(v_i)) \geq p$, that is

$$p \leq \sum_{i=1}^{k}(1 + \deg(v_i)) \leq \sum_{i=1}^{k}(1 + \Delta) \leq k(1 + \Delta).$$

It follows that$\left\lceil\frac{p}{1+\Delta}\right\rceil \leq k$. Hence we conclude that $\left\lceil\frac{p}{1+\Delta}\right\rceil \leq i_m(G) \leq p$

Corollary 9.Let $G \in \xi(G)$ with vertex count p and edge count q

$$\left\lceil\frac{p}{1+\Delta}\right\rceil \leq i_m(G) \leq 2q - p + 2$$

whereΔ is the maximum vertex degree of G

Proof.Forthe graphs$G$,we have $q \geq p - 1$. By the Observation1, we see that $i_m(G) \leq p = 2(p - 1) - p + 2$

$$\Rightarrow i_m(G) \leq 2q - p + 2 .$$

Hence by the Theorem8,$\left\lceil\frac{p}{1+\Delta}\right\rceil \leq i_m(G) \leq 2q - p + 2$

Corollary 10. Let $G \in \xi(G)$ with vertex count p. Then

$$\left\lceil\frac{p}{1+\Delta}\right\rceil \leq i_m(G) \leq p - \gamma(G)$$

Proof. It clearly follows from Theorem 5, 8

Remark 2.The lower bound in the Theorem 8 is sharp even for regular graphs.

VI. REALIZATION THEOREMS

This section gives some realization problems.

Theorem 11For $k, l \in Z^+$with $2 \leq k < l \leq p$, there exists connected graphs $G \in \xi(G)$ such that

1). $m(G) = k$
2). $\gamma(G) = k$
3). $i_m(G) = l$

Proof.Consider a path graph: $P_p : [v_1, v_2, \dots, v_p]$. Construct a graph G from P_p by adding $l - k$ pendent vertices $\{u_1, u_2, \dots, u_{l-k}\}$ to vertex v_2.we obtain a new graph G, which is given in the Figure 3.

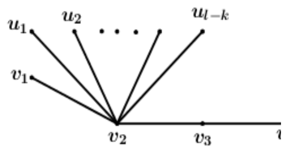

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Figure 3. A graph G with $\mathbf{m(G) = k}$, $\boldsymbol{\gamma}\mathbf{(G) = k}$ & $\mathbf{i_m(G) = l}$

Let $M_1 = \{v_2, v_5, \dots, v_{3p-1}, \dots, v_{p-2}, v_p\}$ be a subset of V. clearly M_1 is a dominating set of G and we have

$$\gamma(G) = |M_1| = \frac{(p-2)-2}{3} + 1 + 1 = \frac{p+2}{3}$$

To establish this result, we take $p = 3k-2$.Thus set M_1 is minimum and so $\gamma(G) = |M_1| = k$. Next, we consider set $M_2 = \{v_1, v_4, \dots, v_{p-3}, v_p, u_1, u_2, \dots, u_{l-k}\}$. Clearly it is a minimum md-set of G and all vertices in M_2 are non-adjacent. Thus M_2 is a minimum imd-set of G.It follows that $i_m(G) = |M_2| = \frac{p+2}{3} + (l-k)$. To establish this result, again we take $p = 3k-2$.Thus $i_m(G) = k + (l-k) = l$.

Suppose that $p = 3(l-k) + 4$ and we replace the vertex count $(l-k)$ by $(k-2)$. Let $M_3 = \{u_1, u_2, u_3, \dots, u_{k-2}, v_1, v_p\}$. Clearly M_3 is a minimum m-set and $m(G) = |M_3| = k-2+2 = k$. Hence result follows.

Theorem 12.For d_m, k, p $\in Z^+$ with $3 \le d_m \le k$ and $p \ge k + 1 + \lfloor \frac{2d_m}{3} \rfloor$, there exists connected graphsG such that

1). $|V| = p$
2). $diam(G) = d_m$
3). $i_m(G) = k$

Proof. Consider the following cases based on the positive integer triplets(d_m, k, p).

Case (1): If $d_m = 3 \le k$, then $p \ge k+3$. Let A and B be the vertex partite sets of a complete bipartite graph $K_{\{2,t\}}$ where $A = \{x, y\}, B = \{u_1, u_2, \dots, u_t\}$ and choose $t = p-k-1$. Construct a graph G by adding some pendent vertices $\{v_1, v_2, \dots, v_{k-1}\}$ to y in $K_{\{2,t\}}$, we obtain a new graph G,which is given in the Figure 4.

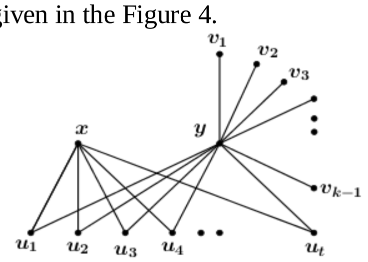

Figure 4. A graph G with $\mathbf{i_m(G) = k}$, $\mathbf{diam(G) = d_m = 3}$

Let $M = \{v_1, v_2, \dots, v_{k-1}\}$. Then by the Observation 5, M is contained in every imd-sets of G so that $i_{md}(G) \ge k$. Itfollows that $M_1 = M \cup \{x\}$ is a md-set of Galso vertices are not adjacent. Hence $i_m(G) = k$, monophonicdiameter $diam(G) = d_m = 3$ and $|V(G)| = k+t+1 = p$.

Case (2): If $d_m = 4 \le k$ then $p \ge k+4$. Let A and B be the vertex partite sets of a complete bipartite graph $K_{\{2,t\}}$ where $A = \{x, y\}, B = \{u_1, u_2, \dots, u_t\}$ and choose $t = p-k-2$. Form a graph G by adding some pendent vertices $\{v_1, v_2, \dots, v_{k-2}\}$ to vertex y in $K_{\{2,t\}}$ and join a path $P: [x, u, v, y]$,we obtain a new graph G, which is given in the Figure 5.

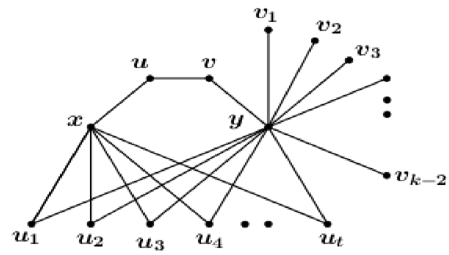

Figure 5.A graph G with $\mathbf{i_m(G) = k}$, $\mathbf{diam(G) = d_m = 4}$

In $G, \{x, y\}$ is the minimumdominating set of G. Let $M = \{x, v, v_1, v_2, \dots, v_{k-2}\}$. Clearly M is both monophonic and id-set of G Hence M is an imd-set so that imd-number $i_m(G) \le k$. For $j = 1,2, \dots, t$ we have

$$|M \cap \{x, u, v, y, \quad u_j\}| \ge 2\,.$$

It follows that $|M| \ge k \Rightarrow i_m(G) \ge k$. Hence $i_m(G) = k$, $diam(G) = d_m = 4$ and $|V(G)| = k+t+2 = p$.

Case(3): Suppose that $5 \le d_m \le k$, If we choose $r = k - \lfloor \frac{d_m}{3} \rfloor$, and $t = p - d_m - r \ge 1$ then we obtain a graph G,which is given in the Figure 6.

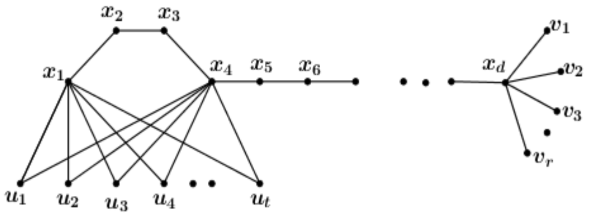

Figure 6 . A graph G with $\mathbf{i_m(G) = k}$, $\mathbf{diam(G) = d_m \ge 5}$

Let $P_{\{d_m\}+1}$: $[x_1, x_2, \dots, x_d, x_{\{d_m\}+1} = v_j]$ be the monophonic diametral path for any $j = 1,2, \dots$. Then $A = \{x_1, x_4, \dots, x_{\{3\lceil d_m \rceil\}-2}\}$is not an imd-set, but $A$is contained in every imd-set of G

If we add the vertices $v_1, v_2, \dots, v_r$to A,we get $M = A \cup \{v_1, v_2, \dots, v_r\}$. It follows that, M is both monophonic and an id-set of G. Thus $i_m(G) \le |M| = k$. Let M be the minimum imd-set of G. Then we have

$$|M \cap \{x_1, x_2, x_3, x_4, u_1, u_2, u_3, \dots, u_t\}| \ge 2$$

Also, for any $j = 1,2,3, \dots r$. Consider a path $P_1 = [x_4, x_5, \dots, x_d, x_{d+1} = v_j]$. This shows that $|M \cap V(P_1)| \ge \lceil \frac{d_m}{3} \rceil$, and conclude from the Observation5,

$$i_m(G) \ge 1 + \lceil \frac{d_m}{3} \rceil + r - 1 = k.$$

Hence$|V(G)| = d_m + 1 + r - 1 + t = p, diam(G) = d_m$, monophonic diameter of G and $i_m(G) = k$.

VI. CONCLUSION

The id- number unified to an 8X8 size chessboard is the smallest number of reciprocally non-offensive queens that spell all the squares of a chessboard. If we follow a path related attack (known as 'chordless attack '), we can easily optimize the number of mutually non-offensive queens. Exactly this particular parameter is an imd- number of the graph, which is designed from an 8X8 size chessboard by taking the squares as the vertices with two vertices adjacent if a queen situated on one square attacks the other square. Throughout the study, we identified face change of md-sets when non-adjacency occurred in graphs. This work can be extended to find an upper monophonic domination and connected monophonic domination in various graphs.

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