Abstract: In the present world, GPS requirement is increasing in almost every field for different applications. One of the major applications is GPS receiver positioning in defense and civilian applications like online transportation tracking status, tracking location of senior citizens, school kids and in different types of surveying applications etc. GPS positioning is a nonlinear process. Received GPS signal is corrupted by noise and hence the extraction of original signal from corrupted signal is the main task. The implementation of adaptive filters for such type of non-linear estimation problems provides better results. In this paper, position of a GPS receiver is estimated by Unscented Kalman Filter (UKF) and Extended Kalman Filter (EKF). Required data is taken from GPS navigation and observation files. The estimated position is compared with surveyed receiver position coordinates, along with receiver clock error. The results show that Unscented Kalman Filter (UKF) provides better positioning with less time of convergence compared to EKF. Thus, Unscented Kalman Filter (UKF) is fast and accurate for estimation of GPS receiver position and also for nonlinear system applications.

Index Terms: GPS, Kalman Filter, Unscented, Clock error.

I. INTRODUCTION

Global Positioning System (GPS) is a satellite based navigational system, which offers navigation, positioning, and timing services. GPS receiver’s position is estimated in three dimensional space by using trilateration principle. Satellites location in the orbit and distance between each satellite to GPS receiver are required to estimate the location of a GPS receiver. To compute GPS receiver position, four or more satellites are required. In this paper the ephemeris data has been taken from the GPS navigation and observation files. Adaptive filters UKF [1]-[3] and EKF [4], [5] are implemented to estimate GPS receiver position.

In section-II, GPS and basic pseudorange equation are explained. In the remaining sections, Section-III covers flaws in Extended Kalman filter and as a rectification to these flaws Unscented Transform is applied in Kalman Filter which creates a new filter named as Unscented Kalman Filter is also discussed. In section-IV, implementation of EKF and UKF algorithms to estimate GPS receiver position is presented; comparative results are given in tabular forms. The paper concludes the discussion in section-V.

II. GLOBAL POSITIONING SYSTEM (GPS)

Worldwide used real time radio navigation system is GPS [6], [7], which provides users to have accurate three dimensional position of any GPS receiver. Position estimation of a GPS receiver in 3-Dimension using four satellites is as shown in Fig. 1.

Figure-1. 3D Position of a GPS receiver using 4 satellites

The basic pseudorange equation (P) is given by,

\[ P = \rho_r + c(dt - dt^w) + dl_{ion} + dl_{trop} + \varepsilon_{mr} \]  

(1)

Where \( dt \) and \( dt^w \) are receiver clock and satellite clock offsets respectively. \( dl_{ion} \) is the error due to ionosphere, \( dl_{trop} \) is the error due to troposphere. \( \varepsilon_{mr} \) is receiver measurement noise and the effects of multipath. From ephemeris data, the position of satellites \((x^o, y^o, z^o)\) can be computed. The geometrical range \( \rho_r \) between a satellite and GPS receiver is given by,

\[ \rho_r^2 = (x_r - x_o)^2 + (y_r - y_o)^2 + (z_r - z_o)^2 \]  

(2)

Where, \((x_r, y_r, z_r)\) is the position of user/ receiver. To simplify the pseudorange equation, ionosphere error, troposphere error, multipath error, and satellite clock offsets in (1) are ignored. Thus, (1) is modified as,

\[ P = \rho_r^w + c.dt \]  

(3)

Where, \( c.dt \) is the receiver clock bias. Let us
consider $b_u = c. dt$. Therefore, (3) can be written as,

$$P = \rho^x_u + b_u \tag{4}$$

In (4), there are four unknowns $x_u, y_u, z_u$ and $b_u$. To estimate these unknowns, requires four such range expressions from 4 different satellites. Therefore,

$$P_1 = \left((x_{n1} - x_u)^2 + (y_{n1} - y_u)^2 + (z_{n1} - z_u)^2 \right)^{1/2} + b_u$$
$$P_2 = \left((x_{n2} - x_u)^2 + (y_{n2} - y_u)^2 + (z_{n2} - z_u)^2 \right)^{1/2} + b_u$$
$$P_3 = \left((x_{n3} - x_u)^2 + (y_{n3} - y_u)^2 + (z_{n3} - z_u)^2 \right)^{1/2} + b_u$$
$$P_4 = \left((x_{n4} - x_u)^2 + (y_{n4} - y_u)^2 + (z_{n4} - z_u)^2 \right)^{1/2} + b_u \tag{5}$$

Above equations in (5) are non linear simultaneous equations, to solve for four unknowns first linearize the non linear equations. Then, the above equations can be written in matrix form as,

$$\begin{bmatrix} \Delta\mathbf{P}_1 \\ \Delta\mathbf{P}_2 \\ \Delta\mathbf{P}_3 \\ \Delta\mathbf{P}_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ a_{31} & a_{32} & a_{33} & 1 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta y \\ \Delta u \\ \Delta u \end{bmatrix} \tag{6}$$

The (6) can be expressed as,

$$\Delta\mathbf{Z} = \mathbf{H} * \Delta\mathbf{X} \tag{7}$$

Where,

$$\Delta\mathbf{Z} = \begin{bmatrix} \Delta\mathbf{P}_1 \\ \Delta\mathbf{P}_2 \\ \Delta\mathbf{P}_3 \\ \Delta\mathbf{P}_4 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ a_{31} & a_{32} & a_{33} & 1 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix}, \quad \Delta\mathbf{X} = \begin{bmatrix} \Delta u \\ \Delta y \\ \Delta u \\ \Delta u \end{bmatrix} \tag{8}$$

$$a_{11} = -\frac{y - y_u}{\sigma_y^2}, \quad a_{12} = -\frac{z - z_u}{\sigma_z^2}, \quad a_{13} = -\frac{u - u_u}{\sigma_u^2} \tag{9}$$

Using the expression given in (7), the GPS receiver/user position can be estimated [6].

### III. UNSCENTED KALMAN FILTER

For many years, EKF was used for recursive nonlinear estimation as a standard technique. Later, a new algorithm was proposed by Julier et al [1], [2], called as Unscented Kalman Filter (UKF), and further it was developed by Wan and van der Merwe [8] - [11]. The basic difference between UKF and EKF is in the way of propagating Gaussian random variables through system dynamics.

The state distribution of EKF is approximated by Gaussian random variables. Then, it is propagated through the first order linearization of the nonlinear system. Large errors can be introduced in the true posterior covariance and mean of transformed Gaussian random variable, which sometimes diverge the filter.

UKF represents a derivative free filtering procedure which would be an alternative to EKF. The state distribution of UKF is approximated by a Gaussian random variable is represented using a set of chosen sample points. These sample points captures true covariance and mean of the distribution when propagated through the true nonlinear system, accurately to the 2nd order for any non linearity.

UKF is the application of Unscented Transform [12] in Kalman Filter for solving nonlinear estimation problems. The following sub topics will give brief idea about Unscented Transform and its application in Kalman Filter.

#### Unscented Transform

Unscented Transform (UT) is founded based on an intuition that “it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function or transformation” [13], [14]. Unscented Transform uses exact nonlinear function applied to approximating probability distribution, but in EKF approximation is applied to the known nonlinear function rather than partial probability distribution.

UT is a mathematical function, often used to estimate the result by applying given nonlinear transformation to probability distribution that is characterized only in terms of a finite set of statistics. In another words, it is a method for calculating the statistics of a random variable which under goes a nonlinear transformation [15]. Fig. 2 and Fig. 3 illustrate the approach. A set of points, called sigma points, are chosen and their mean and covariance are $X_m$, $\sigma_x$ respectively. The non linear function is applied to each point which yields a set of transformed points and then the statistics of these points are $Y_m$ and $\sigma_y$.

**Figure-2. Principle of Unscented Transform**

Unscented Transform procedure includes, computing a set of sigma points and weights as each sigma point has a weight. Transform these points through the non linear function. Then compute a Gaussian from weighted points and avoid linearize around the mean as Taylor expansion.
The unscented transform suggests taking parameter values as $\alpha \in (0,1], \ k \geq 0, \ \beta = 2, \ \lambda = \alpha^2 (n+k) - n$.

**Unscented Kalman Filter**

Application of unscented transform in Kalman filter for estimation of nonlinear functions yields Unscented Kalman Filter [10, 16]. Steps to be followed in UKF Algorithm are given below:

**Step-1:**
In the prediction part of the filter, first form sigma points.

$$\xi_{i}^{[0]} = m_{i-1}, \quad \xi_{i}^{[1]} = m_{i-1} + (\sqrt{(n+\lambda)} \sigma), \quad \text{for } i = 1,2 \ldots n$$

$$\xi_{i}^{[i]} = m_{i-1} - (\sqrt{(n+\lambda)} \sigma), \quad \text{for } i = n+1, \ldots 2n$$

**Step-2:**
Propagate sigma points through system dynamic model.

$$\hat{\xi}_{i} = g(\xi_{i}^{[i]}), \quad \text{for } i = 0,1,2 \ldots 2n$$

**Step-3:**
Compute predicted mean and covariance, $m_{i}^{-}$ and $\sigma_{i}^{-}$

$$m_{i}^{-} = \sum_{i=0}^{2n} W_{i}^{[m]} \xi_{i}^{[i]}, \quad \sigma_{i}^{-} = \sum_{i=0}^{2n} W_{i}^{[m]} (\xi_{i}^{[i]} - m_{i}^{-}) (\xi_{i}^{[i]} - m_{i}^{-})^{T} + Q_{i-1}$$

**Step-4:**
In the correction part of filter, first update the sigma points.

$$\hat{\xi}_{i}^{[0]} = m_{i}^{-}, \quad \hat{\xi}_{i}^{[1]} = m_{i}^{-} + (\sqrt{(n+\lambda)} \sigma), \quad \text{for } i = 1,2 \ldots n$$

$$\hat{\xi}_{i}^{[i]} = m_{i}^{-} - (\sqrt{(n+\lambda)} \sigma), \quad \text{for } i = n+1, \ldots 2n$$

**Step-5:**
Propagate sigma points through measurement model.

$$\gamma_{i}^{[i]} = h(\hat{\xi}_{i}^{[i]}), \quad \text{for } i = 0,1,2 \ldots 2n$$

**Step-6:**
Compute the predicted covariance of measurement ‘ $S_{i}$’, cross covariance ‘ $C_{i}$’, and mean ‘ $\mu_{i}$’.

$$K_{i} = C_{i} S_{i}^{-1}$$

$$m_{i} = m_{i}^{-} + K_{i} (y_{i} - \mu_{i})$$

$$\sigma_{i}^{-} = \sigma_{i}^{-} - K_{i} C_{i} K_{i}^{T}$$

In (17) mean and covariance are the estimated results. Practical implementation of this filter is discussed in the following section.

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**Sigma Points**

Sigma points are computed as follows:

$$\hat{\xi}_{i}^{[0]} = x_{m}, \quad \hat{\xi}_{i}^{[1]} = x_{m} + (\sqrt{(n+\lambda)} \sigma), \quad \text{for } i = 1,2 \ldots n$$

$$\hat{\xi}_{i}^{[i]} = x_{m} - (\sqrt{(n+\lambda)} \sigma), \quad \text{for } i = n+1, \ldots 2n$$

Where $\hat{\xi}$ is the sigma point, here observed that always the first sigma point is mean. ‘$n$’ indicates dimensionality, $\lambda$ is a scaling parameter, (i-n) indicates column vector of covariance matrix to be taken and ‘$\sigma$’ is covariance.

**Weights**

Weights are computed as follows:

$$W_{i}^{[0]} = \frac{\lambda}{n+\lambda}$$

$$W_{i}^{[1]} = W_{i}^{[0]} + (1 - \alpha^2 + \beta)$$

$$W_{i}^{[i]} = W_{i}^{[1]} = \frac{1}{2(n+\lambda)} \quad \text{for } i = 1, \ldots 2n$$

Where $W_{i}^{[0]}$, $W_{i}^{[1]}$ weights are for computing mean and $W_{i}^{[0]}$, $W_{i}^{[1]}$ weights are for computing covariance. Scaled
IV. RESULTS AND DISCUSSION

GPS ephemeris information is taken from a GPS receiver (dual frequency receiver) installed at Andhra University (Dept of ECE), Visakhapatnam (17.73° N / 83.31° E). More than four satellites are available in each epoch (Let us say ‘m’ satellites). Hence, equation (6) can be modified as,

\[
\begin{bmatrix}
\delta P_1 \\
\delta P_2 \\
\vdots \\
\delta P_m
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & 1 \\
a_{21} & a_{22} & a_{23} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
a_{m1} & a_{m2} & a_{m3} & 1
\end{bmatrix}
\begin{bmatrix}
\delta x \\
\delta y \\
\delta z
\end{bmatrix}
\]

(18)

Consider (0 0 0 0) as the initial receiver location for the implementation of UKF and EKF. The resultant estimated coordinates and surveyed receiver position are compared and it is presented in Table-1.

### Table-1. Comparisons of Position Error Estimation

<table>
<thead>
<tr>
<th>GPS time in Hours of the day</th>
<th>Position error in meters</th>
<th>Rclk (nano second)</th>
<th>Position error in meters</th>
<th>Rclk (nano second)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(x_e)</td>
<td>(y_e)</td>
<td>(z_e)</td>
<td>58</td>
</tr>
<tr>
<td>00 - 02</td>
<td>50.91</td>
<td>105.61</td>
<td>39.84</td>
<td>58</td>
</tr>
<tr>
<td>02 - 04</td>
<td>49.50</td>
<td>108.16</td>
<td>33.22</td>
<td>75</td>
</tr>
<tr>
<td>04 - 06</td>
<td>42.12</td>
<td>97.88</td>
<td>30.69</td>
<td>119</td>
</tr>
<tr>
<td>06 - 08</td>
<td>41.73</td>
<td>83.18</td>
<td>31.88</td>
<td>151</td>
</tr>
<tr>
<td>08 - 10</td>
<td>45.73</td>
<td>83.04</td>
<td>33.92</td>
<td>152</td>
</tr>
<tr>
<td>10 - 12</td>
<td>51.01</td>
<td>97.63</td>
<td>39.57</td>
<td>121</td>
</tr>
<tr>
<td>12 - 14</td>
<td>50.81</td>
<td>102.71</td>
<td>35.42</td>
<td>106</td>
</tr>
<tr>
<td>14 - 16</td>
<td>51.27</td>
<td>107.19</td>
<td>34.01</td>
<td>96</td>
</tr>
<tr>
<td>16 - 18</td>
<td>47.24</td>
<td>106.09</td>
<td>35.58</td>
<td>91</td>
</tr>
<tr>
<td>18 - 20</td>
<td>51.58</td>
<td>107.46</td>
<td>34.42</td>
<td>84</td>
</tr>
<tr>
<td>20 - 22</td>
<td>48.72</td>
<td>101.13</td>
<td>31.67</td>
<td>90</td>
</tr>
<tr>
<td>22 - 24</td>
<td>50.94</td>
<td>105.45</td>
<td>34.96</td>
<td>80</td>
</tr>
<tr>
<td>Mean Position Error</td>
<td><strong>48.3</strong></td>
<td><strong>100.2</strong></td>
<td><strong>34.5</strong></td>
<td></td>
</tr>
</tbody>
</table>
In Table-1, results for total 24 hours data is given as mean values of every two hours. The position errors in x-, y-, and z-directions are $X$, $Y$, and $Z$ in meters respectively along with receiver clock error ($R_{clk}$) in nanoseconds. Table-2 presents variance in x-, y-, and z-directions ($\sigma_x^2$, $\sigma_y^2$, and $\sigma_z^2$) and receiver clock variance ($\sigma_{clk}^2$).

From the results (for 24 hours data), the mean position errors in x-, y-, and z-directions for the Unscented Kalman Filter (UKF) are 48.3m, 100.2m and 34.5m respectively. Whereas for the Extended Kalman Filter (EKF) mean position errors are 48.5m, 100.6m and 34.6m respectively. From the observation, UKF offers slightly good accuracy compared to EKF. Because, the UKF linearizes Gaussian distribution instead of linearize the non linear function. Even though, both Extended and Unscented Kalman Filters use the process of linearization; EKF uses more number of iterations to converge compared to UKF. If iteration procedure is not used, then the EKF converges after 30 epochs where as UKF converges after 5 epochs.

From Table-2, variance provided by UKF is less compared to EKF. From the overall results, it is observed that the UKF is better for the estimation of GPS receiver position as it is fast and accurate.

V. CONCLUSIONS

EKF linearizes the non-linear function, whereas UKF approximates Gaussian distribution instead of approximating non linear function. Therefore, the problem of linearizing the non-linear function does not exist in UKF and it also helps to increase the accuracy of results. The mean position errors offered by UKF are 48.3m, 100.2m and 34.5m, whereas EKF offers 48.5m, 100.6m and 34.6m in x-, y-, and z- directions respectively. Hence, UKF is a better optimization method for the estimation of GPS receiver position which offers less variance, good estimation accuracy of position, less number of iterations, and converges early compared to EKF.

REFERENCES


AUTHORS PROFILE

Mr. Ashok Kumar, N is a Senior Research Fellow (SRF) in Andhra University. He received his B.Tech Degree in Electronics and Communication Engineering from JNTU Hyderabad, Andhra Pradesh, India in 2008 and M.Tech Degree in Radar and Microwave Engineering from Andhra University, Andhra Pradesh, India in 2013. Now, As SRF submitted his Ph.D in the area of Global Positioning System in the Department of Electronics and Communication Engineering, Andhra University, Andhra Pradesh, India. His research interests include Global Positioning System, Satellite Signal Processing, Artificial intelligence and SONAR signal Processing.

Dr. G. Sasibhushana Rao received his B.E. Degree in Electronics and Communication Engineering from Andhra University, Andhra Pradesh, India, and M. Tech. Degree in Electronics and Communication Engineering from JNTU University, Andhra Pradesh, India. He obtained his Ph.D degree in Global Positioning System from the Osmania University, Andhra Pradesh, India. He is currently working as Professor in Department of Electronics and Communication Engineering, Andhra University. He is the author of about 517 scientific publications in journals, international and National conferences. His research interests include Global Positioning Systems, Wireless communication, Signal Processing. He is an author of the 4 text books “Global Navigation Satellite System” (published by McGraw-Hill Education), “Mobile Cellular Communication” (Published by Pearson Education), “Electromagnetic Filed Theory and Transmission Lines” (Published by John Wiley & Sons), “Microwave and Radar Engineering” (Published by Pearson Education).