Unscented Kalman Filter for GPS Based Positioning and Tracking Services

Ashok Kumar.N, G. Sasibhushana Rao

Abstract*: In the present world, GPS requirement is increasing in almost every field for different applications. One of the major applications is GPS receiver positioning in defense and civilian applications like online transportation tracking status, tracking location of senior citizens, school kids and in different types of surveying applications etc. GPS positioning is a nonlinear process. Received GPS signal is corrupted by noise and hence the extraction of original signal from corrupted signal is the main task. The implementation of adaptive filters for such type of non linear estimation problems provides better results. In this paper, position of a GPS receiver is estimated by Unscented Kalman Filter (UKF) and Extended Kalman Filter (EKF). Required data is taken from GPS navigation and observation Files. The estimated position is compared with surveyed receiver position coordinates, along with receiver clock error. The results show that Unscented Kalman Filter (UKF) provides better positioning with less time of convergence compared to EKF. Thus, Unscented Kalman Filter (UKF) is fast and accurate for estimation of GPS receiver position and also for nonlinear system applications.

***Index Terms*: GPS, Kalman Filter, Unscented, Clock error.**

I. INTRODUCTION

Global Positioning System (GPS) is a satellite based navigational system, which offers navigation, positioning, and timing services. GPS receiver's position is estimated in three dimensional space by using trilateration principle. Satellites location in the orbit and distance between each satellite to GPS receiver are required to estimate the location of a GPS receiver. To compute GPS receiver position, four or more satellites are required. In this paper the ephemeris data has been taken from the GPS navigation and observation files. Adaptive filters UKF [1]-[3] and EKF [4], [5] are implemented to estimate GPS receiver position.

In section-II, GPS and basic pseudorange equation are explained. In the remaining sections, Section-III covers flaws in Extended Kalman filter and as a rectification to these flaws Unscented Transform is applied in Kalman Filter which creates a new filter named as Unscented Kalman Filter is also discussed. In section-IV, implementation of EKF and UKF algorithms to estimate GPS receiver position is presented; comparative results are given in tabular forms. The paper concludes the discussion in section-V.

II. GLOBAL POSITIONING SYSTEM (GPS)

Worldwide used real time radio navigation system is GPS [6], [7], which provides users to have accurate three dimensional position of any GPS receiver. Position estimation of a GPS receiver in 3-Dimension using four satellites is as shown in Fig. 1.

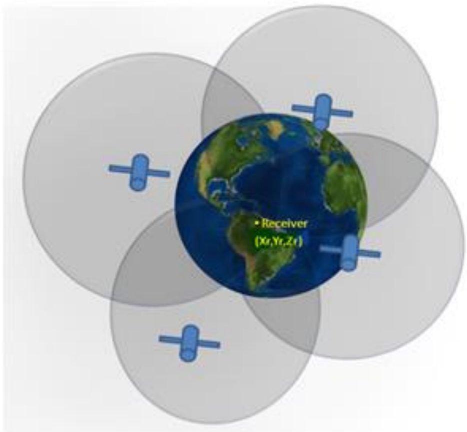

Figure-1. 3D Position of a GPS receiver using 4 satellites

The basic pseudorange equation (P) is given by,

$$P = \rho_r^{sv} + c(dt - dt^{sv}) + dl_{ion} + dl_{trop} + \varepsilon_{mr} \qquad (1)$$

Where dt and dt^{sv} are receiver clock and satellite clock offsets respectively. dl_{ion} is the error due to ionosphere, dl_{trop} is the error due to troposphere. ε_{mr} is receiver measurement noise and the effects of multipath. From ephemeris data, the position of satellites (x^{sv}, y^{sv}, z^{sv}) can be computed. The geometrical range (ρ_r^{sv}) between a satellite and GPS receiver is given by,

$$(\rho_r^{sv})^2 = (x_{sv} - x_u)^2 + (y_{sv} - y_u)^2 + (z_{sv} - z_u)^2 \qquad (2)$$

Where, (x_u, y_u, z_u) is the position of user/ receiver. To simplify the pseudorange equation, ionosphere error, troposphere error, multipath error, and satellite clock offsets in (1) are ignored. Thus, (1) is modified as,

$$P = \rho_r^{sv} + c.dt \qquad (3)$$

Where, $c.dt$ is the receiver clock bias. Let us

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consider $b_u = c.dt$. Therefore, (3) can be written as,

$$P = \rho_r^{sv} + b_u \tag{4}$$

In (4), there are four unknowns x_u, y_u, z_u and b_u . To estimate these unknowns, requires four such range expressions from 4 different satellites. Therefore,

$$\begin{aligned}
P_1 &= \left((x_{sv1} - x_u)^2 + (y_{sv1} - y_u)^2 + (z_{sv1} - z_u)^2\right)^{1/2} + b_u \\
P_2 &= \left((x_{sv2} - x_u)^2 + (y_{sv2} - y_u)^2 + (z_{sv2} - z_u)^2\right)^{1/2} + b_u \\
P_3 &= \left((x_{sv3} - x_u)^2 + (y_{sv3} - y_u)^2 + (z_{sv3} - z_u)^2\right)^{1/2} + b_u \\
P_4 &= \left((x_{sv4} - x_u)^2 + (y_{sv4} - y_u)^2 + (z_{sv4} - z_u)^2\right)^{1/2} + b_u
\end{aligned} \tag{5}$$

Above equations in (5) are non linear simultaneous equations, to solve for four unknowns first linearize the non linear equations. Then, the above equations can be written in matrix form as,

$$\begin{bmatrix} \delta P_1 \\ \delta P_2 \\ \delta P_3 \\ \delta P_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ a_{31} & a_{32} & a_{33} & 1 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} \tag{6}$$

The (6) can be expressed as,

$$\Delta Z = H * \Delta X \tag{7}$$

Where,

$$\Delta Z = \begin{bmatrix} \delta P_1 \\ \delta P_2 \\ \delta P_3 \\ \delta P_4 \end{bmatrix}, H = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ a_{31} & a_{32} & a_{33} & 1 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix}, \Delta X = \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} \tag{8}$$

$$a_{i1} = -\frac{x_i - x_u}{P_i - b_u}, a_{i2} = -\frac{y_i - y_u}{P_i - b_u}, a_{i3} = -\frac{z_i - z_u}{P_i - b_u} \tag{9}$$

Using the expression given in (7), the GPS receiver/user position can be estimated [6].

III. UNSCENTED KALAMN FILTER

For many years, EKF was used for recursive nonlinear estimation as a standard technique. Later, a new algorithm was proposed by Julior et al [1], [2], called as Unscented Kalman Filter (UKF), and further it was developed by Wan and van der merwe [8] - [11].The basic difference between UKF and EKF is in the way of propagating Gaussian random variables through system dynamics.

The state distribution of EKF is approximated by Gaussian random variables. Then, it is propagated through the first order linearization of the nonlinear system. Large errors can be introduced in the true posterior covariance and mean of transformed Gaussian random variable, which sometimes diverge the filter.

UKF represents a derivative free filtering procedure which would be an alternative to EKF. The state distribution of UKF is approximated by a Gaussian random variable is represented using a set of chosen sample points. These sample points captures true covariance and mean of the distribution when propagated through the true nonlinear system, accurately to the 2nd order for any non linearity.

UKF is the application of Unscented Transform [12] in Kalman Filter for solving nonlinear estimation problems. The following sub topics will give brief idea about Unscented Transform and its application in Kalman Filter.

Unscented Transform

Unscented Transform (UT) is founded based on an intuition that 'it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function or transformation' [13], [14]. Unscented Transform uses exact nonlinear function applied to approximating probability distribution, but in EKF approximation is applied to the known nonlinear function rather than partial probability distribution.

UT is a mathematical function, often used to estimate the result by applying given nonlinear transformation to probability distribution that is characterized only in terms of a finite set of statistics. In another words, it is a method for calculating the statistics of a random variable which under goes a nonlinear transformation [15]. Fig. 2 and Fig. 3 illustrate the approach. A set of points, called sigma points, are chosen and their mean and covariance are x_m , σ_x respectively. The non linear function is applied to each point which yields a set of transformed points and then the statistics of these points are y_m and σ_y .

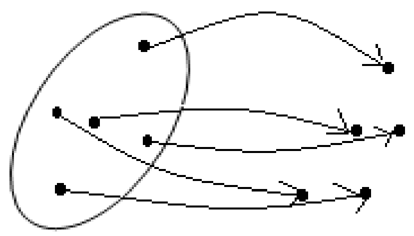

Figure-2. Principle of Unscented Transform

Unscented Transform procedure includes, computing a set of sigma points and weights as each sigma point has a weight. Transform these points through the non linear function. Then compute a Gaussian from weighted points and avoid linearize around the mean as Taylor expansion.

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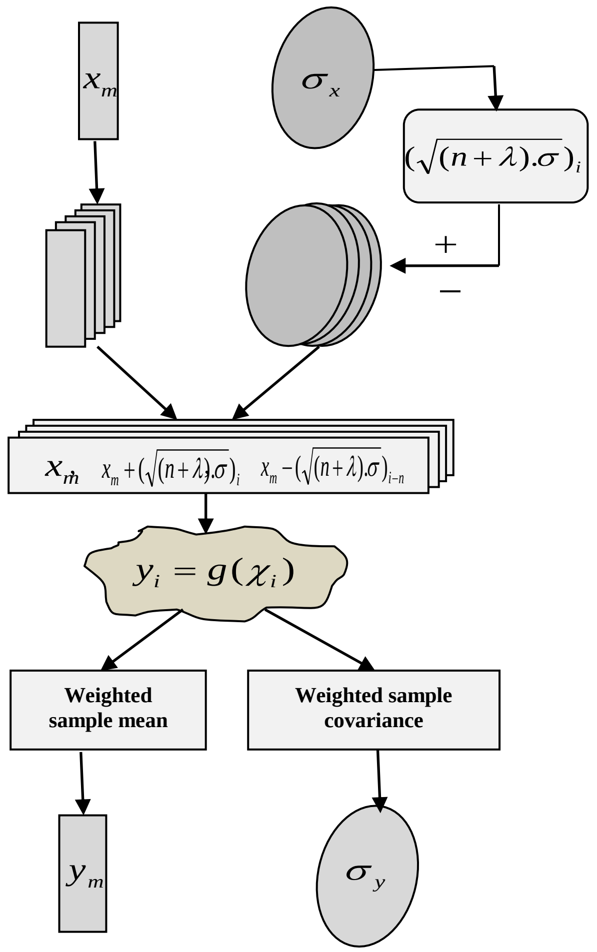

Figure-3. Block diagram of unscented transform

Sigma Points

Sigma points are computed as follows:

$$\begin{aligned} \xi^{[0]} &= x_m \\ \xi^{[i]} &= x_m + (\sqrt{(n+\lambda).\sigma})_i \quad for\ i=1,2...n \\ \xi^{[i]} &= x_m - (\sqrt{(n+\lambda).\sigma})_{i-n} \quad for\ i=n+1,...2n \end{aligned} \tag{10}$$

Where ξ is the sigma point, here observed that always the first sigma point is mean. 'n' indicates dimensionality, λ is a scaling parameter, *(i-n)* indicates column vector of covariance matrix to be taken and 'σ'is covariance.

Weights

Weights are computed as follows:

$$\begin{aligned} w_m^{[0]} &= \frac{\lambda}{n+\lambda} \\ w_c^{[0]} &= w_m^{[0]} + (1-\alpha^2+\beta) \\ w_m^{[i]} &= w_c^{[i]} = \frac{1}{2(n+\lambda)} \quad for\ \ i=1,...2n \end{aligned} \tag{11}$$

Where $w_m^{[0]}$, $w_m^{[i]}$ weights are for computing mean and $w_c^{[0]}, w_c^{[i]}$ weights are for computing covariance. Scaled unscented transform suggests taking parameter values as $\alpha \in (0,1],\ \ k \geq 0,\ \ \ \beta = 2,\ \ \lambda = \alpha^2(n+k) - n$.

Unscented Kalman Filter

Application of unscented transform in Kalman filter for estimation of nonlinear functions yields Unscented Kalman Filter [10, 16]. Steps to be followed in UKF Algorithm are given below:

Step-1:
In the prediction part of the filter, first form sigma points.

$$\begin{aligned} \xi_{t-1}^{[0]} &= m_{t-1} \\ \xi_{t-1}^{[i]} &= m_{t-1} + (\sqrt{(n+\lambda).\sigma_{t-1}})_i \quad for\ i=1,2...n \\ \xi_{t-1}^{[i+n]} &= m_{t-1} - (\sqrt{(n+\lambda).\sigma_{t-1}})_{i-n} \quad for\ i=n+1,...2n \end{aligned} \tag{12}$$

Step-2:
Propagate sigma points through system dynamic model.

$$\hat{\xi}_t^i = g(\hat{\xi}_{t-1}^i) \qquad for \quad i=0,1,2...2n \tag{13}$$

Step-3:
Compute predicted mean and covariance, m_t^- and σ_t^-

$$\begin{aligned} m_t^- &= \sum_{i=0}^{2n} w_i^{[m]} \hat{\xi}_t^i \\ \sigma_t^- &= \sum_{i=0}^{2n} w_i^{[c]} [\hat{\xi}_t^i - m_t^-][\hat{\xi}_t^i - m_t^-]^T + Q_{t-1} \end{aligned} \tag{14}$$

Step-4:
In the correction part of filter, first update the sigma points.

$$\begin{aligned} \xi_t^{-[0]} &= m_t^- \\ \xi_t^{-[i]} &= m_t^- + (\sqrt{(n+\lambda).\sigma_t^-})_i \quad for\ i=1,2...n \\ \xi_t^{-[i+n]} &= m_t^- - (\sqrt{(n+\lambda).\sigma_t^-})_{i-n} \quad for\ i=n+1,...2n \end{aligned} \tag{15}$$

Step-5:
Propagate sigma points through the measurement model.

$$\hat{y}_t^{(i)} = h(\xi_t^{-(i)}) \qquad for \ \ i=0,1,2...2n \tag{16}$$

Step-6:
Compute the predicted covariance of measurement 'S_t', cross covariance'C_t', and mean 'μ_t'.

$$\begin{aligned} K_t &= C_t S_t^{-1} \\ m_t &= m_t^- + K_t(y_t - \mu_t) \\ \sigma_t &= \sigma_t^- - K_t S_t K_t^T \end{aligned} \tag{17}$$

In (17) mean and covariance are the estimated results. Practical implementation of this filter is discussed in the following section.

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IV. RESULTS AND DISCUSSION

GPS ephemeris information is taken from a GPS receiver (dual frequency receiver) installed at Andhra University (Dept of ECE), Visakhapatnam (17.73^0 N / 83.31^0 E). More than four satellites are available in each epoch (Let us say 'm' satellites). Hence, equation (6) can be modified as,

$$\begin{bmatrix} \delta P_1 \\ \delta P_2 \\ \vdots \\ \delta P_m \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b u \end{bmatrix} \quad (18)$$

Consider (0 0 0 0) as the initial receiver location for the implementation of UKF and EKF. The resultant estimated coordinates and surveyed receiver position are compared and it is presented in Table-1.

Table-1. Comparisons of Position Error Estimation

GPS time in Hours of the day	← UKF →				← EKF →			
	Position error in meters			Rclk (nano second)	Position error in meters			Rclk (nano second)
	X_e	Y_e	Z_e		X_e	Y_e	Z_e	
00 - 02	50.91	105.61	39.84	58	50.98	105.89	39.93	58
02 - 04	49.50	108.16	33.22	75	49.59	108.63	33.37	74
04 - 06	42.12	97.88	30.69	119	42.20	98.29	30.79	118
06 - 08	41.73	83.18	31.88	151	41.83	83.59	32.06	150
08 - 10	45.73	83.04	33.92	152	45.82	83.37	34.02	152
10 - 12	51.01	97.63	39.57	121	51.16	98.10	39.71	120
12 - 14	50.81	102.71	35.42	106	50.90	103.07	35.50	105
14 - 16	51.27	107.19	34.01	96	51.37	107.50	34.08	95
16 - 18	47.24	106.09	35.58	91	47.39	106.48	35.72	90
18 - 20	51.58	107.46	34.42	84	51.69	107.76	34.51	83
20 - 22	48.72	101.13	31.67	90	48.84	101.60	31.83	89
22 - 24	50.94	105.45	34.96	80	50.99	105.76	35.05	79
Mean Position Error	**48.3**	**100.2**	**34.5**		**48.5**	**100.6**	**34.6**	

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Table-2. Comparisons of Variance Estimation

GPS time in Hours of the day	← UKF →				← EKF →			
	σ_x^2	σ_y^2	σ_z^2	σ_{clk}^2	σ_x^2	σ_y^2	σ_z^2	σ_{clk}^2
00 - 02	0.039	0.151	0.034	0.043	0.130	0.188	0.126	0.127
02 - 04	0.036	0.205	0.046	0.069	0.128	0.211	0.133	0.139
04 - 06	0.035	0.187	0.040	0.061	0.127	0.204	0.130	0.136
06 - 08	0.037	0.175	0.058	0.065	0.128	0.198	0.139	0.138
08 - 10	0.032	0.162	0.046	0.046	0.124	0.192	0.134	0.129
10 - 12	0.047	0.187	0.044	0.054	0.133	0.204	0.132	0.132
12 - 14	0.041	0.163	0.039	0.053	0.131	0.193	0.129	0.132
14 - 16	0.045	0.150	0.034	0.050	0.133	0.186	0.127	0.131
16 - 18	0.056	0.167	0.042	0.062	0.139	0.193	0.130	0.136
18 - 20	0.048	0.146	0.035	0.044	0.135	0.185	0.127	0.128
20 - 22	0.046	0.190	0.047	0.076	0.134	0.203	0.133	0.143
22 - 24	0.040	0.156	0.039	0.058	0.131	0.189	0.130	0.135

In Table-1, results for total 24 hours data is given as mean values of every two hours. In the Table-1, the position errors in x-, y-, and z- directions are X_e, Y_e , and Z_e in meters respectively along with receiver clock error (Rclk) in nanoseconds. Table -2 presents variance in x-, y-, and z- directions (σ_x^2, σ_y^2, and σ_z^2) and receiver clock variance (σ_{clk}^2).

From the results (for 24 hours data), the mean position errors in x-, y-, and z-directions for the Unscented Kalman Filter (UKF) are 48.3m, 100.2m and 34.5m respectively. Whereas for the Extended Kalman Filter (EKF) mean position errors are 48.5m, 100.6m and 34.6m respectively. From the observation, UKF offers slightly good accuracy compared to EKF. Because, the UKF linearizes Gaussian distribution instead of linearize the non linear function. Even though, both Extended and Unscented Kalman Filters use the process of linearization; EKF uses more number of iterations to converge compared to UKF. If iteration procedure is not used, then the EKF converges after 30 epochs where as UKF converges after 5 epochs.

From Table-2, variance provided by UKF is less compared to EKF. From the overall results, it is observed that the UKF is better for the estimation of GPS receiver position as it is fast and accurate.

V.CONCLUSIONS

EKF linearizes the non-linear function, whereas UKF approximates Gaussian distribution instead of approximating non linear function. Therefore, the problem of linearizing the non-linear function does not exist in UKF and it also helps to increase the accuracy of results. The mean position errors offered by UKF are 48.3m, 100.2m and 34.5m, whereas EKF offers 48.5m, 100.6m and 34.6m in x-, y-, and z- directions respectively. Hence, UKF is a better optimization method for the estimation of GPS receiver position which offers less variance, good estimation accuracy of position, less number of iterations, and converges early compared to EKF.

REFERENCES

1. Julier,S.J., Uhlmann, J.K., Durrant-Whyte, H., (1995) "A new approach for filtering nonlinear systems", In Proceedings of the American Control Conference, 1995, pp. 1628–1632
2. Julier, S.J., Uhlmann, J.K., (1996) "A general method for approximating nonlinear transformations of probability distributions", Technical Report, RRG, Department of Engineering Science, University of Oxford, 1996.
3. Dan J.Simon, (2006) "Optimal State Estimation", John Wiley and Sons, Inc., first edition , 2006.
4. Neil Gordan, Michael Pitt., (1998) "A comparison of sample based filters and the Extended Kalman filter for the bearings only tracking problem", In signal processing conference, EUSIPO, 1998.

5. Ashok Kumar, N., Suresh, Ch., Sasibhushana Rao, G., (2018) "Extended Kalman Filter for GPS Receiver Position Estimation", Intelligent Engineering Informatics, Advances in Intelligent Systems and Computing, Vol. 695, 2018, pp. 481-488. doi: 10.1007/978-981-10-7566-7_47
6. G S Rao, (2010) "Global Navigation Satellite Systems", McGraw Hill Education Private limited, 2010, pp. 49-54
7. Teunissen, Montenbruck, "Springer Handbook of Global Navigational Satellite Systems", 2017, pp.561-563. doi: 10.1007/978-3-319-42928-1
8. Wan, E.A., van der Merwe, R., Nelson, A.T.,(2000) "Dual estimation and the unscented transformation", In S.A. Solla, T.K. Leen, and K.R. Muller (Eds.), Advances in Neural Information Processing Systems 12, Cambridge, MA: MIT Press, 2000, pp. 666–672
9. Wan, E.A., van der Merwe, R., (2000) "The Unscented Kalman Filter for nonlinear estimation", In Proceedings of Symposium 2000 on Adaptive Systems for Signal Processing, Communication and Control (AS-SPCC), IEEE, Lake Louise, Alberta, Canada, 2000.
10. Van der Merwe,R., de Freitas, J.F.G., Doucet, D., Wan, E.A., (2000) "The unscented particle filter", Technical Report CUED/F F-INFENG/TR 380, Cambridge University Engineering Department, 2000.
11. Van der Merwe, R., Wan, E.A., (2001) "Efficient derivative-free Kalman filters for online learning", In Proceedings of European Symposium on Artificial Neural Networks (ESANN), Bruges, Belgium, 2001.
12. Uhlmann, Jeffrey, (1995) "Dynamic Map Building and localization: New Theoretical Foundations", (Ph.D) thesis. University of oxford, 1995.
13. Uhlmann, J.K., (1994) "Simultaneous map building and localization for real time applications", Technical report, University of Oxford, Transfer thesis, 1994.
14. Ross, M., Proulx, R.J., Karpenko, M., (2015) "Unscented Guidance", American Control Conference, 2015, pp.5605-5610. doi: 10.1109/ACC.2015.7172217
15. Julier, S.J., Uhlmann, J.K.,(1997) "A new extension of the Kalman filter to nonlinear systems", In Proceedings of AeroSense: The 11th International Symposium on Aerospace Defence Sensing, Simulation and Controls, 1997.
16. Binqi Zheng, Pengcheng Fu, Baoqing Li, Xiaobing Yuan., (2018) " A Robust Adaptive Unscented Kalman Filter for Nonlinear Estimation with Uncertain Noise Covariance", Sensors, 18, 808 (2018). doi:10.3390/s18030808

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