# Strong Stability of a Nonlinear Difference System

**G Naga Jyothi, T S Rao , G Suresh Kumar ,T Nageswara Rao**

***Abstract:*** ***The object of this work, is to give ufficient conditions for strong stability of the zero solution of difference system***

$$y(n+1) = A(n)y(n) + \sum_{0}^{n} F(n,t,y(t)) --(1)$$ **As perturbed equation of**

$$y(n+1) = A(n)y(n) --(2)$$

***Keywords:*** ***Difference equations, stability, strong stability and Fundamental matrix.***

## I. INTRODUCTION

Difference equations plays critical role in many areas such as finite element methods, control systems, numerical methods, non continuous mathematical structures and many areas of numerical modeling. The theory of difference equations can explain better the construction of discrete mathematical models, when compared to continuous models.Previous works for stability of solutions of differential system have been studied by many authors like Itoh, Yoneyama and Hara[2], Avramescu [3]. Coppel's paper [6, Chapter III, Theorem 12], [4] deal with the conditional and instability asymptotic stability of the solutions of a systems of differential systems. More results on stability of solutions of differential system was studied by Lakshmikantham and Rama Mohana Rao [5]. Further results on Volterra integro-differential systems was studied by Mahfoud [6] and others. Results on the conditional stability of solutions of systems of differential equations was studied by Spath's paper [7] and Weyl's paper [8] Further, Ψ-asymptotic stability of Non linear difference system and Ψ-asymptotic stability of Non homogeneous matrix difference system and bounded ness was studied by T S Rao, GSK and Murthy M S N[14, 15, 16, 17, 18].The strong stability for nonlinear difference equations are not yet studied. With the motivation of the above works, here we got the only if conditions for the strong stable for the zero output of nonlinear difference equations.

## II DEFINATIONS AND NOTATIONS

Let $R^m$ denotes the Euclidean *m*-space. For $y \in R^m$ , ||y|| be the norm of y. For an $m \times m$ square matrix *A*, we consider the norm |*A*| of *A* by

$$|A| = \sup_{|x|\le 1} \|Ay\|$$

In equation (1) we consider that *A* is a m×m matrix on *N* and *F* is a vector valued function of order *m* on N

A. *Definition[1]* : If the vector valued function u(n) in $R^m$ satisfies the difference equation (1), then u(n) is called a solution(output) of (1). It is clear that u(n)=0 (zero vector) is always solution of (1) and is called zero solution or trivial solution of (1). And also u (n) is trivial solution of (2).
B. *Definition* [1] : Any *m X m* matrix Z (n) whose columns are linear independent solutions of difference equation (1) is called a fundamental matrix or origin matrix of (1).
C. *Definition* [1] : **If** for each $n_0$ in *N* and every ε> 0, there exists δ= δ(ε, $n_0$) > 0 such that any solution y(n) of (1) which satisfies the inequality $\|y(n_0)\|<\delta$, also exists and the inequality $\|y(n)\| < \varepsilon$ is true for all $n \ge n_0$, then zero output of (1) is called to be stable on *N.*
D. *Definition [1]* If for each ε> 0, we have δ=δ(ε) > 0 , any output $\bar{y}(n)$ of (1) satisfies the inequality $\|\bar{y}(n) - y(n_0)\| < \delta$ for some $n_0 \ge 0$,exists and the inequality $\|\bar{y}(n) - y(n)\| < \varepsilon$ is true for each and every $n \ge 0$, then output y(n) of (1) is said to be strongly stable on N

**Note:** Clearly of other types of stability are not implies to strong stability.

## III. MAIN RESULTS

**Theorem 3.1**: Let Z(n) be a origin matrix for (2). The necessary and sufficient condition for zero solution of (2) strongly stable on N is, there exists a positive constant m satisfies

$$|Z(n)Z^{-1}(t)| \le k \text{ for all } 0 \le t, n < \infty$$

$$\text{or } |Z(n)| \le m \text{ and } |Z^{-1}(n)| \le m \text{ for all } n \ge 0$$

Proof: Proof is clear, from the definition of strong stability of zero solution of (2).

**Note 3.1**: Let Z(n) be a origin matrix for (2). Now consider the following statements

$L_1$: The function $f : N \to \infty$ and the constants $q_1 \ge 1$, $M_1 > 0$ exists for

$$\sum_{t=0}^{t=n} (f(t)|Z(n)Z^{-1}(t)|)^{q_1} \le M_1 \text{ for all } n \ge 0$$

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$L_2$: The function $f: N \to \infty$ and the constants $q_2 \geq 1$, $M_2 > 0$ exists for

$$\sum_{t=0}^{n} (f(t)\left|Z^{-1}(n)Z(t)\right|)^{q_2} \leq M_2 \text{ for all } n \geq 0$$

$L_3$: The function $f: N \to \infty$ and the constants $q_3 \geq 1$, $M_3 > 0$ exists for

$$\sum_{t=0}^{n} (f(t)\left|Z^{-1}(n)Z(t)\right|)^{q_3} \leq M_3 \text{ for all } n \geq 0$$

$L_4$: The function $f: N \to \infty$ and the constants $q_4 \geq 1$, $M_4 > 0$ exists for

$$\sum_{t=0}^{n} (f(t)\left|Z^{-1}(n)Z(t)\right|)^{q_4} \leq M_4 \text{ for all } n \geq 0$$

**Theorem 3.2**. Assume that the following rules are satisfied by the origin matrix Z (n) of (2)

R1: $L_1$ and $L_2$ are true.

R2: $L_1$ and $L_4$ are true.

R3: $L_2$ and $L_3$ are true.

R4: $L_3$ and $L_4$ are true.

Then, the zero solution of (2) is strongly stable on N.

Proof. First our aim is Z (n) and $Z^{-1}(n)$ are bounded on N. First, we assume the case R2. For that, we show that Z (n) is bounded on N.

Let r (n) = $f^{q_1}$ (t) $|Z(n)|^{-q_1}$ for $n \geq 0$. From the equality

$$(\sum_0^n r(t))Z(n) = \sum_0^t (f(t)Z(n)Y^{-1}(t))$$
$$(r(t)f(t))^{-1} Z(t) \text{ for } n \geq 0$$

it gives that

$$(\sum_0^n r(t))\,|Z(n)| \leq$$
$$\sum_0^n (f(t)\,|Z(n)Z^{-1}(t)|)(r(t)f(t))^{-1}\,|Z(t)|$$
$$\text{for } n \geq 0 \text{-----(3)}$$

For $q_1=1$, we got that $r(t)(f(t)^{-1}\left|Z(t)\right| = 1$. From (3) and the given statement $L_1$ it gives that

$$(\sum_0^n r(t))\,|Z(n)| \;\leq \sum_0^n f(t)\,|Z(n)Z^{-1}(t)| \leq M_1 \quad \text{for}$$

n>0

For $q_1 > 1$, we got that $r(t)(f(t))^{-1}\left|Z(t)\right| = (r(t))^{\frac{1}{r_1}}$,

$$\frac{1}{q_1} + \frac{1}{r_1} = 1.$$

By using (3), it gives that

$$(\sum_0^n r(t)) f(n)\,|r(n)|^{\frac{-1}{q_1}} \leq$$
$$\sum_0^n f(t)\left|Z(n)Z^{-1}(t)\right|\left|(r(t))^{\frac{-1}{r_1}} \text{ for } n \geq 0\right.$$

$$(\sum_0^n r(t)) f(n)\,|r(n)|^{\frac{-1}{q_1}} \leq$$
$$(\sum_0^n f(t)\left|Z(n)Z^{-1}(t)\right|^{q_1})^{\frac{1}{q1}} (\sum_0^t r(t))^{\frac{1}{r_1}} \text{ for } n \geq 0$$

From Holder inequality, we got

From L1, we obtain that,

$$\left(\sum_0^t r(t)\right)^{\frac{1}{q1}} f(n)(r(n))^{\frac{-1}{q_1}} \leq M_1^{\frac{1}{q_1}} \quad n \geq 0$$

Thus, for , $q_1 \geq 1$ the function |Z (n)| satisfies the inequality

$$\left|Z(n)\right| \leq k_1^{\frac{1}{q_1}} \left(\sum_0^n r(t)\right)^{\frac{-1}{q_1}}, \ t \geq 0$$

Represent $R(n) = \sum_0^n r(t) \text{ for } n \geq 0.$

$$|Z(n)| \leq M_1^{\frac{1}{q_1}} (R(n))^{\frac{-1}{q_1}}$$

for n≥0

Since

$R_{n+1}(n) = r(n) \geq M_1^{-1}(f(n))^{q_1}$ Q(n) for n≥0

We got $$R(n) \geq Q(1)e^{M_1^{-1}\sum_{s=1}^{n} f^{q1(s)}} \quad \text{for } n \geq 1$$

It gives that

$$\left|z(n)\right| \leq M_1^{\frac{1}{q1}} (R(1))^{\frac{-1}{q1_1}} e^{-(q1M1)^{-1}\sum_1^n f^{q1}(t)},$$
$$\text{for } n \geq 1$$

since |Z (n)| is a mapping on [0, 1], it gives that we have a positive constant $N_1$ satisfies $|Z(n)| \leq N_1$ for $n \geq 0$.

Now we claim that $Z^{-1}(n)$ is bound on N.

Let r (n) = $f^{q_4}(n) \ |Z^{-1}(n)|^{-q_4}$ for $n \geq 0$

From the equality

$$(\sum_{t=0}^{n} r(t))\,|Z^{-1}(n)| =$$
$$\sum_0^n \left[r(t)[\ \ f(t)\right]^{-1} Z^{-1}(t)\ \ ](f(t)Z(t)Z^{-1}(n)) \quad \text{, for } n \geq 0$$

it gives that

$$(\sum_0^n r(t))\left|Z^{-1}(n)\right| \leq$$
$$\sum_0^n (r(t)f(t))^{-1} \left|Z^{-1}(t)\right|(f(t)\,|Z(t)Z^{-1}(n)|) \text{--}$$
$$\text{for } n \geq 0 \text{----(4)}$$

For $q_4$=1, it gives that $(r(t)(f(t))^{-1}\left|Z^{-1}(t)\right|=1$

For $q_4>1$, we got that

$$r(t)(f(t))^{-1}|Z^{-1}(t)|=((r(t))^{1/q_4},$$

where $\frac{1}{q_4}+\frac{1}{r_4}=1$

From (4) it gives that

$$(\sum_0^n r(t))\left|Z^{-1}(n)\right|\le\sum_0^n r^{\frac{1}{r_4(t)}}(f(t)\left|Z(t)Z^{-1}(n)\right|$$

for all n≥0

From Holder inequality, we got that

$$\sum_0^n r(t)\left|Z^{-1}(n)\right|$$

$$\le(\sum_0^n(f(t)\left|Z(t)Z^{-1}(n)\right|)^{q_4})^{1/q_4}(\sum_0^n r(t))^{\frac{1}{r_4}}$$

n≥0

From $L_4$, we got

$$\left(\sum_0^n r(t)\right)\left|Z^{-1}(n)\right|\le\left(\sum_0^n r(t)\right)^{\frac{1}{r_4}}M_4^{\frac{1}{q_4}}\quad n\ge0$$

$$\left(\sum_0^n r(t)\right)^{\frac{1}{q_4}}\left|Z^{-1}(n)\right|\le M_4^{\frac{1}{q_4}}\qquad n\ge0$$

Thus, for $q_4\ge1$, the function $|Z^{-1}(n)|$ gives the inequality

$$\left|Z^{-1}(n)\right|\le M_4^{\frac{1}{q_4}}(\sum_0^n r(t))^{\frac{-1}{q_4}}\qquad \text{for}$$

n≥0

Take $R(n)=\sum_0^n r(t)$ for n ≥ 0. Thus, we have

$$|Z^{-1}(n)|\le M_4^{\frac{1}{p_4}}\left(R(n)\right)^{\frac{-1}{q_4}}\ \text{for } n\ge0$$

Since

$$R(n+1)=r(n)\ge f^{q_4}(n)M_4^{-1}R(n)$$

for n≥0

We have

$$R(n)\ge R(1)e^{-M_4^{-1}\sum_1^n f^{q3(t)}}$$

It gives that

$$\left|Z^{-1}(n)\right|\le M_4^{\frac{1}{q_4}}\left(R(1)\right)^{\frac{-1}{q_4}}e^{-(q_4M_4)^{-1}\sum_1^n f^{q_4(t)}}$$

for n≥1

Since $\left|Z^{-1}(n)\right|$ is mapping on [0, 1], it gives that we have a positive constant $N_2$ that $\left|Z^{-1}(n)\right|\le N_2$ for $n\ge n_0$ .

So, the final output gives immediately from Theorem 3.1.
Lastly, in the rules R1, R3 or R4, the proof is same.
Hence the proof.

**Theorem 3.3.** If

1. The origin matrix Z (n) of the (2) satisfies $|Z(n)Z^{-1}(n)|<M$ for every 0<t, n<∞, where M is Fixed value

2. The mapping G satisfies the condition

$$||G(n,t,a)-G(n,t,b)||\le g(n,t)||a-b||$$

for $0\le t\le n<\infty$, and for each and every a, b∈ $R^n$ here g is a positive mapping on D satisfies

$$N=\sum_0^\infty\sum_0^n g(n,t)<M^{-1}$$

Then, for each $n_0$>0, $y_0\in R^n$ and $\eta>0$, we have one and only solution of (1) on N such that $y(n_0)=y_0$ and $||y(n)||\le\eta$ for all n ∈$[0,n_0]$ , if $||x_0||$ is small as we need.

Proof. By known statement

$$y(n+1)\ =A(n)y$$

$$+\sum_0^n G(n,t,y(t))\quad ,x(n_0)=x_0$$

can be changed by means of named technique, variation of constants to the nonlinear difference system

$$y(n)=Z(n)Z^{-1}(n_0)y_0$$

$$+\sum_{n_0}^n Z(n)Z^{-1}(t)\sum_0^t G(t,u,y(u))$$

n≥0

We take the named Frechet space $F_C$ of all continuous mappings from N → $R^n$ with the semi norms $\left\|y\ \right\|_\tau=\sup\left\|y(n)\right\|,\ \lambda\ge0.$

Thus, approaches in Fc are nothing but the usual approaches on all compact intervals of N.

For $n_0$>0 and $\eta$ > 0, let $y_0\in R^n$ be such that $||y_0||<\eta(1-MN)M^{-1}$.

Let $Y_\eta$ be the set

$$Y_\eta=\begin{Bmatrix}y\in F_C:||y||_{t_0}\le\eta,\ ||y||_\tau\\ \le\eta e^{MN}\ \text{for }\lambda>n_0\end{Bmatrix}$$

We introduce operator O from $S_\eta$ in to $F_C$ :

$$(Oy)(n) = Z(n)Z^{-1}(n_0)y_0 + \sum_{n_0}^{n} Z(n)Z^{-1}(n)\sum_{0}^{t} G(t,u,y(u)) \qquad n\geq 0.$$

For $y \in S_\rho$ and $n \in [0, n_0]$ we got

$$\| (Oy)(n) \| \leq M \| y_0 \| + M\sum_{n}^{n_0}\sum_{0}^{t} g(t,u) \| y(u) \| \leq M\eta(1-MN)M^{-1} + M\eta N = \eta$$

Let a, b ∈ $Y_\eta$ For n in [0, $n_0$], we have

$$\|(Oa)(n)-(Ob)(n)\| = \left\| \sum_{n_0}^{n} Z(n)Z^{-1}(t)\sum_{0}^{s}(G(t,u,a(u))-G(t,u,b(u))) \right\| \leq MN\|a-b\|_{t_0}$$

Then $\|Oa - Ob\|_{n_0} \leq MN\,|a-b|_{t_0}$

Similarly, for $\lambda > n_0$, , we got

$$\|Oa - Ob\|_\tau \leq MN\|a-b\|_{t_0}$$

So, O is a contraction. From the known named theorem i.e.Banach's Theorem for Frechet spaces [4], unique fixed point $\bar{a} = O\bar{a}$ contained by $S_\eta$ i. e., the difference system (1) has one and only one output $\bar{y}$ (n) on N such that $\bar{y}(n_0) = y_0$ and

$\|\bar{y}(n)\| < \eta$ for all $n \in [0, n_0]$ and

$\|\bar{y}(n)\| \leq \eta e^{MN}$ for all $n \geq 0$, if $\|y_0\|$ is sufficiently small

Now, we think that y(n) is a output in $F_c$ of (5) such that $\|y(n)\| \leq \eta$ for $n \in [0, n_0]$ and $\|y_0\| \leq \eta(1-MN)M^{-1}$. For $n \geq n_0$ we got

$$\| y(n) \| = \| Z(n)Z^{-1}(n_0)y_0 + \sum_{n_0}^{n} Z(n)Z^{-1}(t)\sum_{0}^{t} T(t,u,y(u))$$
$$\leq M \| y_0 \| + M\sum_{n_0}^{n}\sum_{0}^{t} f(t,u) \| y(u) \|$$
$$= M \| y_0 \| + M\sum_{n_0}^{n}\sum_{0}^{n_0} f(t,u) \| y(u) \| + M\sum_{n_0}^{n}\sum_{n_0}^{t} f(t,u) \| y(u) \|$$
$$\leq M \| y_0 \| + M\eta\sum_{n_0}^{n}\sum_{0}^{n_0} f(t,u) + M\sum_{n_0}^{n}\sum_{n_0}^{t} f(t,u)\|y(u)\|$$
$$= \eta + M\sum_{n_0}^{n}\sum_{n_0}^{t} f(t,u)\|y(u)\|$$

Now $R(n) = \sum_{n_0}^{n}\sum_{n_0}^{t} f(t,u)\| y(u) \|$ is increasing on [$n_0$, ∞).

*For* $n \geq n_0$, we have

$$R(n+1) = \sum_{n_0}^{n} f(n,u)\| y(u) \|$$
$$\leq \sum_{n_0}^{n} f(n,u)(\eta + MR(u))$$
$$= \eta\sum_{n_0}^{n} f(n,u) + M\sum_{n_0}^{n} f(n,u)R(u)$$

Then,

$$\left[ R(n)e^{-M\sum_{nn}^{n}\sum_{n_0}^{t} f(t,u)} \right] =$$
$$e^{-K\sum_{n_0}^{n}\sum_{tn}^{t} f(t,u)}\left[ R(n+1)(t) - MR(n)\sum_{n_0}^{n} f(n,u) \right]$$
$$\leq e^{-M\sum_{n_0}^{n}\sum_{n_0}^{t} f(t,u)}\left[ \eta\sum_{n_0}^{n} f(n,u) \right]$$
$$= \left[ -\eta M^{-1} e^{-M\sum_{n}^{n}\sum_{n_0}^{t} f(t,u)} \right]$$

By consider the summation from $n_0$ to $n \geq n_0$, we have

$$R(n)e^{-M\sum_{tn}^{n}\sum_{n0}^{t} f(t,u)} - R(n_0)$$
$$\leq -\eta M^{-1} e^{-K\sum_{t_0}^{n}\sum_{tn}^{t} f(t,u)} + \eta M^{-1}$$

We got that

$$\|y(n)\| \leq \eta + MR(n) \text{ for } n \geq n_0$$

*This implies* $\|y(n)\| \le \eta e^{kM}$ for $n \ge n_0$

This proves that $y \in S_\eta$ and then $y = \bar{y}$ Thus, for all $n_0 \ge 0, y_0 \in R^n$ *and* $\eta > 0$, we have a one and only one solution of (1) on N such that y ($n_0$) = $y_0$ and $\|y(n)\| \le \eta$ for every $n \in [0,\ n_0]$, if $\|y_0\|$ is very and very small. Hence the claim was over.

**Theorem 3.4**. The zero solution of (1) is strongly stable on N, when the theorem 3.3 is satisfied.

Proof. Assume $\alpha > 0$ be any positive number

Let $\delta(\alpha) = \alpha(1 - MN)M^{-1}e^{-MN}$, $n_0 \ge 0$
and let $y \in R^n$ *satisfies* $\|y_0\| \le \delta(\alpha)$

By Theorem 3.3, we got that one and only one solution y(n) on N of (1) with y($n_0$ ) =$y_0$ such that $y \in S_{\alpha e^{-MN}}$ i.e. $\|y(n)\| \le \varepsilon$ for $n \ge 0$

Now the above part gives the zero solution of (1) satisfies the condition of strongly stable on N.

Hence our claim is over.

**Result Analysis:** From, above part we completed the proofs of only if part for strong stability of nonlinear difference system. This plays the important role in signal proceing units and stability analysis. The following example gives the proofs of main results of strong stability on linear and nonlinear difference system (1) and (2).

**Example1**: Take the difference system (1.2) with $A(n) = \begin{bmatrix} 1 & \frac{(n+1)^2}{n+3} \\ 0 & \frac{(n+1)^2}{(n+3)^3} \end{bmatrix}$, satisfies all conditions of Main Theorem (3.1), so zero solution of (2) satisfies the condition of strong stable on N.

**Example2**: Assume the difference equation (1.2) with

$A(n) = \begin{bmatrix} 0 & n+1 \\ 0 & n^2 \end{bmatrix}$ and

$F(n, y(n)) = \begin{bmatrix} n3^n \sin(y_1(n)) \\ n^2 5^n y_2(n) \end{bmatrix}$, satisfies all conditions of Main Theorem (3.3), so zero solution of (1) is satisfies the condition of strong stable on N.

**Discussion**: For continuous functions, Differential equations place important role. Similarly, for discrete functions, difference equations place very important role. Previously the concept of stability for continuous functions studied by many authors. But concept of stability is not studied completely for discrete systems. So, that reason we discussed strong stability concept for non-linear difference system with particular examples.

## IV. CONCLUSIONS:

Here, we derived the sufficient conditions for strong stability of nonlinear difference system. These stability analysis place very important role in different engineering fields, in particular control theory. We can extended the same concepts to derived the strong stability for nonlinear matrix difference system, Ψ-conditional strong stability and Ψ-conditional asymptotic strong stability for linear and nonlinear difference system, as well as linear and nonlinear matrix difference system.

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