Performance Comparison of Downlink Channel Estimation in FDD Massive MIMO using CS-Aided and Bayesian Compressed Sensing Methods for 5G Systems

T.Ravibabu and C. Dharmaraj

Abstract: Future mobile communications involves high data rates across large coverage area, latency, reliability and large number of devices in small area. To achieve this, the systems are categorized based on their abilities and their potential at work such as higher Mobile Bandwidth, Ultra-Reliable Low Latency Communication systems employed with reduced latency. These connections require efficient resources based on time and frequency or Frequency-division duplexing (FDD) systems, and many antennas imply high pilot overhead. For eliminating this problem, compressed sensing based channel estimation provides a suitable. Moreover, Bayesian method gives overtime in matter of estimation of channel performance for attaining desired achievable rates. The results of simulation have proved the effectiveness of proposed Bayesian compressed sensing based estimation of channel having minimum pilot overhead. Comparison of various techniques compressed with sensing based and traditional LS methods have been presented.

Index Terms: Bayesian Learning, Compressed sensing, Channel estimation, Frequency Division Duplex, Gaussian Mixture, Massive MIMO.

I. INTRODUCTION

The huge demand of high data rates in 5G technology, the usage of conventional MIMO antenna system is needed to extend massive MIMO to increase the potential support of spectral efficiency, reliability & overall capacity in TDD/FDD cellular systems, detailed identification in [1]. The subsequent work analyzed on massive MIMO systems, more details in [2], to provide higher spectral efficiency, resultant using simple techniques of transmission and reception. Because of more number of transmitters and receivers are using at massive MIMO systems, the CSIT acquisition resembles as a most challenging problem detailed explanation in [18]. The base station receives pilot symbols from different antennas in the downlink channel, moreover to move backward to the BS with channel state information to analyze pre-coding coefficients. However, to calibrate for studying quantitatively the channel reciprocity for obtaining CSIT by training sequence in uplink having length independent of downlink antennas of transmission, many researchers presented that full duplex types is helped by time division duplexing (TDD) detailed research work in [4, 5]. Therefore, the precise accomplishment of CSIT at FDD massive MIMO systems over downlink training and the feed backed signal from the receiver is of great importance. However, as the work of calibration is relatively complex in TDD, the current cellular system prefers FDD for 5G systems.

In FDD systems no channel reciprocity attained due to usage different frequencies in uplink and downlink transmission. Usually, estimating pictures of the channel requires a specific sequence to train provided that information of the previous outputs received in course of training on standard procedure is available. Additional feedback is required for getting the statistics of non-stationary MIMO. To circumvent this situation, a novel technique of CSI determination along with feedback strategy giving accurate reliable CSIT having reduced complexity as well as overhead is desirable. Compressed sensing is offers suitable method for estimating short sequence type of sparse with unknown statistics identified in [11, 12, 23, 24]. In this paper, combined of LS and CS techniques are used to obtain estimation in FDD [15, 16, 24] having various of sparse and dense vectors. Due to improved recovery performance methods in [3, 17, 25], Bayesian estimation scores over other equivalent methods; it can increase monitoring of fractional space of the channel by reducing pilot overhead.

The remaining part of this paper is divided as per the following. Second Section discusses downlink and estimation models of noise. Third section discusses training sequence design and principle of estimation to existing technique, forth section discusses OMP and Bayesian approaches, fifth section practical issues and finally sixth section concludes.

II. SYSTEM MODEL

Let $h_1, h_2 \in \mathbb{C}^M$ channel response between a Base station and two different user terminals. The fading model is given by $h_0 = h(i-i)L + l$, $l = 1, 2, 3, ..., L$. Here $h_0$ and $h_i$ represents the vectors of channel having symbols respectively. These are orthogonal $E[h_0^* h_1] = 0$. The separated signals $x_1, x_2$ transmitted by the users, so that $h_0^* y = h_0^* h_1 x_1 + h_0^* h_2 x_2$, due to the orthogonality and non-zero vectors, the inter-user interference disappeared, so that $h_0^* h_1 = 0$.

The signal corresponds to n-th symbol is

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\[ y[n] = \sqrt{\rho} h^u[n] x[n] + z[n] \]  

(2)

Here, \( h[n] \) and \( x[n] \) are the vectors for channel and the case corresponding to \( n \)-th transmission symbol time with

\[ E\left[|x[n]|^2\right] = 1, \quad \rho \] is given SNR in presence of white noise. If \( h_i \in C^M \) and \( h_j \in C^M \) have independent random vectors with zero mean, such that

\[ \frac{h_i^H h_j}{M} \to 0 \]

(3)

This shows that the inner product between \( h_i \) and \( h_j \). The multivariate circularly symmetric complex Gaussian distribution \( h_i, h_j \sim C^N(0, \mathbf{I}_M) \), where \( \mathbf{I}_M \) is the \( M \times M \) identity matrix [1, 2]. Hence, variance of the inner product (3) 1/M decreases linearly with the number of antennas. The inner product \( \langle h_i^H h_j \rangle / M \to 0 \), where as \( M \to \infty \), does not give that \( h_i^H h_j = 0 \). The inner product \( h_i^H h_j \) increases with \( \sqrt{M} \) towards Rayleigh cases.

A. Downlink Channel estimation

Coherent transmission using CSIT is desirable. If the uplink feedback delay is neglected, data transmission is given by \( L-T_p \), where \( L \) is coherence block of length \( L \) and \( T_p \) is channel training period. The block length is imperfect due to channel. The training time \( T_p \) increases, while available channel is used for downlink transmission is decreases. Good channel training having small \( T_p \) to desirable.

The received signal is given by

\[ y_{i,t}^r = \left[ y[i-1]L+1], [y[i-1]L+2], ..., [y[i-1]L+T_p] \right] = \sqrt{\rho} h_i^H X_{i,t} + w_{i,t} \]

(4)

where \( X_{i,t} = [X_i[1], ..., X_i[T_p]] \in C^{M \times T_p} \) this is pilot sequence having \( \text{tr}(X_i^H X_i) = T_p \).

B. LS Channel estimation

For LS channel coefficients are expressed as

\[ \hat{h}_i^{LS} = \frac{1}{\sqrt{\rho} \Omega_i} (y_{i,t}^r X_{i,t}^H) \]

(5)

Where \( X_{i,t} = X_i^H (X_i X_i)^{-1} \) give inverse and pseudo-inverse of \( X_{i,t} \). \( T_p \geq M \) requiring large amount of downlink resources.

The channel estimation can be done by short sequence. The length of the training sequence, \( T_p \) satisfying \( T_p \geq k \) is sufficient \( s_{i,j} \). For training the \( k \) non-zero coefficients constructed as \( X_r \in C^{k \times T_p} \) and \( M \times k \) matrix obtained from column vectors.

III. CS-BASED CHANNEL ESTIMATION

Simultaneous operation of LS and CS approaches is necessary for reducing training overhead. As the support of transformed channel vector changes sluggishly, this scheme can be based on two vectors considering previous support [6-9, 19].

A. Training Sequence Design

A fraction of total dimension \( k \) is used for determining the values \( s_{i,j} = (\mathbf{I}_M)^{T_i} s_i \). The other part of length \( T_p-k \) is used to estimate \( s_{i,j} = (\mathbf{I}_M)^{T_i} s_i \). The pilot is given by

\[ X_{i,t} = [X_{i,t} X_{i,t}] \]

(6)

For estimating \( s_{i,j} \) the \( X_{i,t} \) can be given by

\[ X_{i,t} = \psi_{\Omega_i} \Phi^H \]

(7)

Here, \( X_{i,t} \in C^{1 \times k} \) satisfies condition of orthogonality \( X_{i,t}^H X_{i,j} = \mathbf{I}_k \), with \( k \) can be obtained by LS filter. The \( X_{i,t} \) is used for the estimation of determine \( s_{i,j} \). Since \( s_{i,j} \) is sparse. This can be calculated using CS method by this condition \( T_p-k < \Omega_{\Omega_i} \). It can be restricted with a compressed number.

\[ X_{i,t} = \psi_{\Omega_i}, \Phi^H \]

(8)

Here pertains to the estimate measured in CS method.

B. Principle of Estimation

At the receiver the value is determined by

\[ y_{i,t}^r = [y_{i,t}^r y_{i,t}^r] \]

where

\[ y_{i,t}^r = \sqrt{\rho} h_i^H X_{i,t} + z_{i,t} \]

(9)

\[ = \sqrt{\rho} h_i^H (\mathbf{I}_M)^{T_i} X_{i,t} + z_{i,t} \]

and

\[ y_{i,t}^r = \sqrt{\rho} h_i^H X_{i,t} + z_{i,t} \]

(10)

The matrix vector \( s_{i,j} \) is obtained from \( y_{i,t} \). Under the LS channel estimation approach, we can express the mean value of \( s_{i,j} \) gives of \( y_{i,t} \) in place of \( y_{i,t} \), the estimate of \( s_{i,j} \), comes from \( y_{i,t} \) with condition of sparse algorithm.

IV. SPARSE RECOVERY ALGORITHMS

The CS channel strategy is employed by the OMP algorithm is presented in [8, 9, 13]. The OMP algorithm is one of the greedy methods, based on residual error and errors differences.

A. Orthogonal Matching Pursuit (OMP) Algorithm

Starts with empty set of uncovered channel paths, \( z \) becomes measurement iteration is used.

\[ \tilde{e}_i = z - \mathbf{O} \hat{h}_i \]

(11)

Where \( h_i \) is the sparse vector impulse of \( i^{th} \) case. Fundamentally, two strategies are involved on the every iteration. Firstly, single delay of transmitting includes all the delays. This is sum of accumulated delay. Secondly to select as many uncovered paths is required to estimate gains.

The OMP Algorithm procedure as follows:

1) Actuate the remaining error \( \tilde{e}_0 = y_{i,t}^r \), with index

\[ \Lambda_0 = \Phi \quad \text{and} \quad i = 1. \]

2) observe column matrix, \( \Phi \) that possess the substantial amount

\[ \tilde{e}_0 = y_{i,t}^r \]

(11)
of the residual error $e_{i,j}$.  
3) $\Lambda_i = \Lambda_{i-1} \cup \{j\}$ is updated, where 
$$j = \arg\max \left\{ j \in \{1, 2, ..., M-k\} \right\} \left\| \phi_j^T e_{i-1} \right\|$$ and $\phi_j$ is $j^{th}$ column of matrix $\Phi$.

4) Setting $u_j = (I_{M-k}\Lambda_i)(\Phi_{\Lambda_i}^T \Phi_{\Lambda_i})^{-1} \Phi_{\Lambda_i}^T y_{i,j}$.

5) If $i = k$, stop and evaluate $\hat{s}_{i,j} = \frac{1}{\sqrt{\rho_i}} u_j$, else need of continuation is there.

With the estimates $\hat{s}_{i,j}$ and $\hat{s}_{i-1}$, a primary estimate can be obtained as
$$\hat{s}_i = (I_{M-k}\Lambda_{i-1})\hat{s}_{i-1} + (I_{M-k}\Lambda_i)\hat{s}_{i,j}.$$ (12)

When OMP is used for process of getting better, due to noise, closing of rule 5 covers residual error. So, if $i = k$, or $\left\| \hat{y}_i - \Phi \hat{h}_i \right\| < \eta$, stop is applied $\eta$ is threshold depending on noise. In proposed scheme the iteration extends up to $i = k$.

The support can be chopped after results are considered. For the CS channel estimation [20-22], the pilot is designed when $S_i$ is sparse ($k < M$)
$$X_{CS} = \Psi \Phi_{CS}^H.$$ (13)

Here $\Phi_{CS}$ is a compressive measurement matrix. At this stage combination of the techniques mentioned in [15, 16] is necessary.

However, the OMP solves the optimization problem: $\min_h \left\| y_i - \Phi h_i \right\|$, such that $\left\| y_i - \Phi h_i \right\| \leq \lambda_{OMP}$. The parameter $\lambda_{OMP}$ is dependent on $h_i$ and variance. Compared to LS on OMP, the BCS Algorithm does not need a prior knowledge statistics.

**B. Bayesian Compressed Sensing (BCS) Algorithm**

Bayesian channel estimation improves effectively by allocating the same pilots to spatially separating users in different cells. Acquiring to the culmination it requires some overhead of estimating the covariance matrices and computation complexity.

The distribution of $h_i$ should be known. For this the following two conditions are, the important. Firstly, it each element of $h_i$ consists of few random variable elements of $h_i$ need substantially diverse variances, more specifically, some instances are very small however a few are large. After the observations, the elements are modeled as $h_i \sim [h_i]$ Gaussian-mixture (GM) distribution is being a mean.

$$P(h_{i,j} ; \rho_i, \sigma_i^2) = \sum_{l=1}^{L} \rho_{l,i} N(h_{i,j}; 0, \sigma_i^2),$$ (14)

Where $N(h_{i,j}; 0, \sigma_i^2)$ is the PDF and gives mixing probability [3]. The parameter $\sigma_i^2$ is minimized. The rest of the GM components $\{\rho_{l,i}, \sigma_i^2\}_{l=1}^{L}$ give a part of greater variances. Value is decided by $L$ gives the number of different variances in $h_i$. Finally, it is assumed that the $BK$-dimensional ($BK=T$) $h_{i}$ gives this

For omitting $\eta_i$ from $P(h_{i,j}; \eta_i)$ and index $i$ from $\rho_{l,i}, \sigma_i^2, \rho_i, \sigma_i, \eta_i$. The parameters $\{\rho_{l,i}, \sigma_i^2\}_{l=1}^{L}$ are a learning algorithm. There are 2 algorithms (i) AMP algorithm is presented in [8, 9] (ii) Bayesian algorithm is presented in [5, 14, 25].Elements of $h_i$ follow the Gaussian Mixture pattern for earlier values of $\eta_i$. To obtain the most profitable values of $\eta_i$, $h_{i,j}$ is termed as a output of AMP. From Bayes theorem, the $h_{i,j}$ is calculated from the output as

$$\lambda_{AMP}(h_{i,j}) = \hat{P}(h_{i,j} | \text{AMP}) = \int N(h_{i,j}; a_{i,j}, v_{i,j})P(h_{i,j}; \eta_i) \phi_{h_{i,j}}.$$ (16)

For $a_{i,j}$ and $v_{i,j}$ change with iteration $i$ with $\lambda_{AMP}(h_{i,j})$ plugging the GM prior (15) into (16) then we get

$$\lambda_{AMP}(h_{i,j}) = \sum_{i=1}^{N} Q_{i,k}(h_{i,j}; \gamma_{i,k}, \xi_{i,k})$$ (17)

Where
$$Q_{i,k}(h_{i,j}; a_{i,j}, v_{i,j}) = \frac{\rho_{i,j} N_k(a_{i,j}, \sigma_i^2, v_{i,j})}{\sum_{j=1}^{L} \rho_{i,j} N_k(a_{i,j}, \sigma_i^2, v_{i,j})},$$ (18a)

$$\gamma_{i,k} = \frac{\sigma_i^2}{\sigma_{i,j} + v_{i,j}},$$ (18b)

$$\xi_{i,k} = \frac{\sigma_i^2 v_{i,j}}{\sigma_{i,j} + v_{i,j}}.$$ (18c)

In [10] using EM technique and Bayesian Channel estimation [25] a proper expression can be made for estimation of $H$.

**Algorithm 1 BCS-Based Algorithm**

For $\Phi$ based inputs and Observation matrix $y$.

1. The first formulations $L$ and $\hat{h}_{i,j}^0$ are selected:
2. For $i = 1$ to $N$ do
3. $t \leftarrow 1$;
4. Repeat
5. AMP Algorithm:
Generate $Q(h_{i,j}) \cap N(h_{i,j}; a_{i,j}^t, v_{i,j}^t)$ with $\hat{h}_{i,j}^{t-1}$ from AMP Algorithm;
6. Parameter learning:
7. Learn the noise Variance;
8. Update the GM priors $\hat{h}_{i,j}$;
9. $t \leftarrow t + 1$;
10. Until $\left\| \hat{h}_{i,j}^t - a_{i,j}^{t-1} \right\| < \varepsilon$;
11. Compute $(\hat{h}_{i,j}^t, \Delta_i^t) \leftarrow (\hat{h}_{i,j}^t, \Delta_i)$

Outputs: Return the Estimated Channel $H$.

The Steps 7–8 are used for earlier values including noise for obtaining convergence. In Step 11, the learned parameter of estimating $h_i$ is used with the initial guess. From numerical results, it is evident that such initialization is better than
estimation performance additionally improves convergence.

V. DISCUSSIONS AND SIMULATION RESULTS

The performance of Bayesian Gaussian mixture needs to be compared with other estimation schemes. OMP is used as sparse recovery algorithms. The baseline schemes can be summed up as Genie-aided LS: For this, value of \( \Omega \) is considered in subspace. LS estimate is then carried out. For Static LS, the value of first block is considered. In Random LS, orthonormal sequence is used for trained LS. In OMP, random Gaussian matrix \( M \times T \) is selected. M-SP algorithm gets back the channel initiated with the previous information, similar to CS-aided scheme.

Fig.1.Normalized gain vs training sequence with SNR = 20 dB.

The Fig 1 illustrates beam forming for gain various schemes in which training is done at \( pH = 20 \text{dB}, (k \text{ to } M) \). It cannot be utilized by random orthogonal pilot. OMP plus CS based estimator’s fails because of reduced additional parameters. It can be inferred that sparsity cannot be determined by Compressing Sensing.

Fig.2.Normalized mean square errors the mismatched parameter with SNR = 20 dB.

Moreover, as support where is sluggish so that a static method can give wrong results compared with random variables best methods. Because the coefficients for \( k \), an estimate of supporting parameters cannot be done by static methods, M-SP is better than traditional OMP. Basically construction can estimate sparse vector which is similar to the inverted value of the earlier structure by including shorter program for training.

Fig 2 displays the degradation of performance of estimation schemes such as M-SP and CS-aided with prior support information with parameter mismatch. The gradation of CS-aided is low whereas M-SP is likely to have more mismatched. Bayesian CS-aided channel estimation outscores the CS-aided and is of their order of generalized scheme. The results show the efficiencies of separation are clear from the results shown.

Fig.3 NMSE vs SNR of estimation schemes with \( M=100 \text{ and } k=40 \).

Fig. 3 illustrates NMSE - SNR for the case of changing SNR. It is formed that CS algorithms is basically noise sensitive. All the estimation schemes connected to CS shows poor result in low SNR portion. In comparison to this, LS based schemes have less error. For \( \text{SNR} > 15 \text{dB} \) is, Bayesian scheme is better than Static LS and as reasonably good performance. Overall, the performed Bayesian method has less error than conventional CS techniques.

VI. CONCLUSION

The Bayesian estimation scheme discussed in this paper, the channel component is used with GM distribution to acquire parameters of prior’s. CS-aided scheme based on LS and CS is used for FDD type of MIMO in which channel can be divided depending on sparsity. By using the sparse vector we can diminish the overhead when adequate sparsity does not exist are when CS is not applicable. From simulation figures, it is seen that GM is better than GB distribution in CS. Additionally, for many Bayesian estimation matching, AMP algorithm can be employed having less computational complexity by suitable pilot design is presented. The Bayesian Gaussian mixture, as proposed, can work without covariance considerations of the channel, related noise level, or the coordination amongst the cells. From the simulation results it is seen that a considerable improvement over pilot overhead based estimators.
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