Effect of Volume Discounts on Optimal Transportation Cost

D.R.S. Narsingh Rao , G. Sreeram Reddy, V.V. Satyanarayana, J. Jagadesh Kumar

Abstract: ***The transportation of goods from source to destination is a critical issue in the light of supply chain management. The pricing of products depends not only on basic costs like material and manufacturing, but also on the transportation cost. The variables affecting the transportation involve unit transportation cost, port selection, time, weather conditions and also on volume of items. As many of the variables commonly exist in all situations and the unique factor that distinguishes from the remaining is by its volume of products transported and subsequently the discounts offered by the transporter. In the current investigation various optimal solutions have been worked out under different conditions of discount and the total transportation cost which help in decision making.***

Keywords: ***Transportation, discount, genetic algorithm, optimal cost, supply chain management, Rim conditions.***

NOMENCLATURE

C_{ijk} - Costs
X_{ijk} - Quantity
$a;b_j$ - Rim Conditions
p_{ijk} - Discount
i – 1,2,----m sources
j- 1,2,-----n departments
k- 1,2,-----r price breaks
u_i, v_j – dual variables

I. INTRODUCTION

The transportation of goods and materials involves the factors like port selection and in land movement apart from the allied issues like the deployment of material handling equipment, unitization, environment effects while in transit etc. All these involve cost and effective planning methods are essentially to be prepared in order to curb the burden of higher pricing of the products. The transportation cost reduction is planned in many organizations by various strategies like fewer carriers, consolidating the shipments, single sourcing, best inventory quantity, transshipment etc. The discounts offered towards shipping cost are to be investigated vis-a-vis with the volume of items to be transported with a view to reduce overall cost.

It is found that the feasible region of equilibrium with convex cost is larger than with concave costs of transportation in the linear method [1]. For a multi-objective transportation a nonlinear function is formulated for each fuzzy goal and worked for its minimization [2]. Fuzzy programming techniques with linear and nonlinear cost functions are employed to minimize cost of a multi objective capacitated transportation problem [3]. The optimization of vehicle routing with multiple depots, customer and products along with traveling time by genetic algorithm was studied [4]. Various pairings are considered in evaluating the costs of transportation [5] and also the attempts of time-cost trade off have given out the variants in transportation [6].Analysis of various factors and subsequently providing solution to reduce transportation cost such that the contribution margin is increased [7]. Also using of graph theory allows the transportation flow distribution is more compact and the cost computation is in line with traditional approach values [8]. The unitization and the discounts are studied together to reduce the cost employing optimality principles [9].

II. METHODOLOGY

In a conventional transportation problem the unit cost of transportation is constant irrespective of quantity transported between source and destination and such problems are solved by linear programming techniques. But in actual practice the unit cost of transportation is not linear due to volume discounts, transportation during emergency periods like calamity, wars [10]. In the current investigation various kinds of solution procedure are undertaken for a set of "n" sources with known supply capacity and a set of "m" destinations with known demand transportation matrix.

2.1 Fixed charge linear transportation

It is concerned with distribution of goods and services from several supply locations with limited supply, to several locations with a specified demand with an objective of minimizing total cost, while the unit cost is assumed to be constant and linear without any dependence on volume of quantity supplied. In analyzing this transportation model and devising on optimization algorithm, total supply and total demand are assumed to be equal and charge is fixed. The Vogel approach method takes the trial solution and improvement is performed iteratively by modified distribution method such that convergence occurs for optimality.

$$\text{Minimize } Z=\sum_{j=1}^{n}\sum_{i=1}^{m}C_{ij}X_{ij}$$
$$\sum_{i=1}^{m}X_i=a_i$$
$$\sum_{j=1}^{n}X_j=b_j$$

2.2 Incremental Quantity Discount Scheme (IQDS)

The features of linear transportation are applied along with the discounts offered to each unit transported from the source to destination. In this lower unit cost feature is available to units transported above a specified quantity only [10, 11]. This is formulated and solved as a generalized fixed charge transportation problem.

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D.R.S.Narsing Rao, Associate Professor, Vidya Jyothi Institute of Technology, Hyderabad, India.
G.Sreeram Reddy, Professor, Vidya Jyothi Institute of Technology, Hyderabad, India.
V.V.Satyanarayana, Professor, Vidya Jyothi Institute of Technology, Hyderabad, India.
J.Jagadesh Kumar, Associate Professor, Vidya Jyothi Institute of Technology, Hyderabad, India.

Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} C_{ijk} X_{ijk}$

$$\sum_{j=1}^{n}\sum_{k=1}^{r} X_{ijk}=a_i$$
$$\sum_{i=1}^{m}\sum_{k=1}^{r} = b_j$$
$$X_{ijk} \le (q_{ijk}-1).Y_{ijk}$$
$$X_{ijk}\ge (q_{ijk}-q_{ijk-1})Y_{ijk-1}$$
$$Y_{ijk}= 0 \text{ or } 1$$

2.3 All Quantity Discount Scheme (AQDS)

In this scheme the discount is allowed to each item transported irrespective of the quantity. The transportation in all quantity discount scheme results in lowest cost for the entire lot. The scheme is built upon conventional transportation problem with discounts riding over it [10, 11]. The method is built on branch and bound algorithm with the deletion of infeasible solutions.

Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} C_{ijk} X_{ijk}$

Subjected to $\sum_{j=1}^{n}\sum_{k=1}^{r} X_{ijk} =a_i$

$$\sum_{i=1}^{m}\sum_{k=1}^{r} = b_j$$
$$\sum_{k=1}^{r} y_{irk} \le 1$$

2.4 Non linear transportation scheme

The total cost function is governed by an additional constraint where in the discount is taken such that the resultant is nonlinear function in terms of quantity. In this method the discount is applied to quantity and the total cost function formulated is subjected to Karishkun Takah condition KKT [11]. After the initial feasible point, employ a step length for a new point and test the objective function for convergence.

Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

$$A X_{ij} = b_{ij}$$

The cell cost is affected with discount;

$$CijXij = C_{ij}X_{ij}-P_{ij}[X_{ij}]^2 \text{ while}$$

$\frac{\partial(cx)}{\partial xij}$ - $(u_i,v_j) = 0$ is the necessary condition for optimality.

2.5 Genetic Algorithm for transportation problem

The transportation problem characterized by mixed continuous discrete variables and discontinuous convex or concave feasible space can be effectively searched for near optimal solutions by Genetic algorithms. The constrained problem is transformed to unconstrained by the concept of penalty function. In this an initial basic feasible solution is selected and crossover is performed from it. Mutation alters one or more gene values in a chromosome from the initial state and results in a changed solution. After several iterations the result is taken at the converged point [9]. The cross over aims at minimizing the transportation cost while also addressing the feasibility constraints [12, 13].

III. ILLUSTRATION

A Company with two resources having a capacity of 45 each getting the goods transported to two depots with a requirement of (60, 30) while the unit transportation cost matrix is given in Table 1.

5	8
5	15

Table 1. Transportation cost matrix

It is proposed by the transporter to offer discount to a tune of 20% in the unit cost depends on the volume transported. The items transported and the respective costs are computed for various schemes propounded for the illustration undertaken.

IV. RESULTS

The linear transportation with Vogel's approximation as basic feasible solution and optimal solution computed by modified distribution method is given in Table 2 with the total cost is $675.

15	30
45	-----

Table 2. Linear transportation solution

The incremental quantity discount scheme has been applied to the given transportation matrix. The optimal solution obtained is given in Table 3 and the total cost is found to be $536. The cost discount would be 20% on the existing value with a quantity break of 20% being transported satisfying the demand requirement.

45	----
15	30

Table 3. Incremental quantity discount solution

All quantity discount scheme with a discount of 20% on the items transported without any quantity breaks is undertaken. The optimal solution obtained is given in the Table 3. The total cost of transportation is computed to be $475.

40	5
20	25

Table 4. All quantity discount solution

In the non linear transportation the cell cost is affected with a discount on the volume transported in a non linear equation and the optimal solution is obliging the Karish Kun Takah condition.

Minimize $Z=5X_{11} +8X_{12} +8X_{21} +15X_{22}$

$$X_{11} +X_{11} = 45$$
$$X_{21} +X_{22} = 45$$
$$X_{11} +X_{21} = 60$$
$$X_{12} +X_{22} = 30$$

Where $C_{11}X_{11} = 5x_{11}-0.2\ [X_{11}]^2$

$$C_{12}X_{12} = 8X_{12}-0.2\ [X_{12}]^2$$
$$C_{21}X_{21} = 8X_{21}-0.2\ [X_{21}]^2$$
$$C_{22}X_{22} = 15X_{22}-0.2\ [X_{22}]^2$$

The initial basic feasible solution is X :($XB_{11} = 15$, $XB_{12} = 30$, $XB_{21} = 45$, $XB_{22} = 0$):

The KKT optimal condition employed to improve solution and iterated three times for the convergence. The optimal solution thus resulted is given in Table 5 and the cost is $480.

39	6
21	24

Table 5. Non linear transportation solution

By adopting the genetic approach, with an initial solution taken from ordinary linear transportation scheme, it has realized a solution as given in Table 6. Uniform distribution is employed with rim conditions as the upper bound values in the random generation of demand and supply units. Coupling to that a change in transportation cost in each is also affected by imposing 20% discount.

30	15
30	15

Table 6. Genetic Algorithm solution

V. DISCUSSION

The optimal transportation cost obtained from various schemes has been compiled in Table7. The linear transportation solution obtained was highest while the solution from the all quantity discount scheme was lowest; It is due to the discount not apportioned in the former case while allowed to each item transported in the later case. The incremental cost stands in between these two because the differential discount implemented based on batches lead to higher cost than the all quantity discount scheme. The solution obtained by genetic algorithm can be improved further by having more iterations and also by selecting rational mathematical functions and by other mathematical techniques like response surface methodology [14]. The linking of unitizing the load based on the shape and size is assumed to be constant while transporting in various modes; otherwise the cost would be subjected to further perturbation.

S.No.	Solution method	Optimal transportation cost ($)
1.	Linear transportation solution	675
2.	Incremental quantity discount solution	536
3.	All quantity discount solution	475
4.	Nonlinear solution	480
5.	Genetic algorithm solution	645

Table. 7. Optimal Solutions

VI. CONCLUSIONS

The cost of transportation would be computed by Vogel's trial solution and improved iteratively by modified distribution method without taking any discount factor on the unit transportation cost. The discounts offered in the transportation based on the quantity transported from source to destination eventually reduce the optimal cost.

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FIRST AUTHOR PROFILE

Mr. D.R.S. Narsingh Rao works as Associate Professor in the department of mechanical engineering, Vidya Jyothi Institute of Technology, Hyderabad, India. His areas of interest include Operations Research, Optimization Techniques, Surface Integrity, Design of Experiments etc.