Domination Parameters of Middle Graph of Sunlet Graph

B.Logapriya, K.Pandiyan

Abstract - If all the vertex of G is dominated by one vertex of S atleast, then a set S in vertex of G is a dominating set of G. The middle graph of an undirected graph G is another graph $M(G)$ that represents adjacencies between vertex and edges of G. In this paper, we obtain the domination parameters of middle graph of Sunlet graph S_n denoted by $M(S_n)$ and its structural properties.

Keywords - Sunlet graph, Middle graph, Domination paramters

I. INTRODUCTION

Domination in graphs helps in finding the shortest and the longest route. In graph theory, it is the fastest growing area which came as a result of study of games such as game of chess where the goal is to dominate the various squares of a chessboard. In graph, the concept of domination number was defined by Berge but it was introduced by De Jaenisch in 1862. In this paper, the finite, simple and un-directed graphs are considered to determine the domination number of middle graphs of n-sunlet graph. From the definition of domination, every vertex of graph must be protected by its neighbourhood, so the domination number is found by considering the neighbourhood of vertices.

II. PRELIMINARIES

G is a non-trivial connected graph with set of vertices $V(G)$ and set of edges $E(G)$.The notation $M(G)$ denotes the middle graph of G.

Definition 2.1: If every vertices of G is adjacent to one vertex of S atleast then V(G) is said to be dominating set of G.

Definition 2.2: The cardinality of dominating set is minimum dominating set. It is notated as $\gamma(G)$.

Definition 2.3: The dominating set with minimum cardinality among G is called domination number of G.

Definition 2.4: The attachment of n-pendent edges to the cycle C_n results in n-sunlet graph with 2n vertices and it is denoted by S_n.

Definition 2.5: $M(G)$ is the middle graph with the vertex set $V(G) \cup E(G)$ .Two vertices in the vertex set $M(G)$ are adjacent, if any one of the following conditions holds:

Revised Manuscript Received on June 07, 2019.

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(i) x, y are in $E(G)$ and x, y is adjacent in G.

(ii) x in $V(G), y$ is in $E(G)$ and x, y are incident in G.

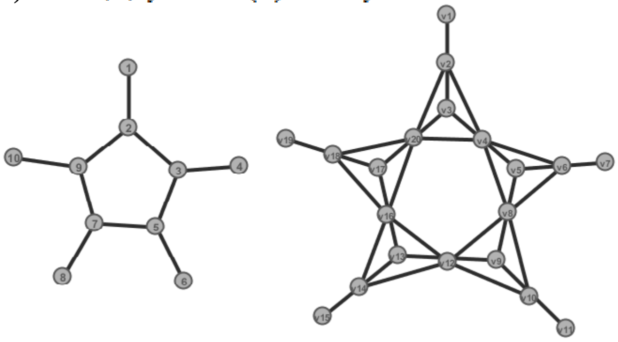

Fig 1 : 5-Sunlet graph and its Middle graph

Definition 2.6: The dominating set S is said to be split dominating set of G if induced sub graph $\langle V - S \rangle$ is disconnected. The split dominating set with minimum cardinality is the split domination number $\gamma_s(G)$ of G.

III. DOMINATION PARAMETERS OF MIDDLE GRAPH OF N-SUNLET GRAPHS

Propositions 3.1:

The domination number of middle graph of 3-sunlet graph, $\gamma[M(S_3)] = 3$.

Proof:

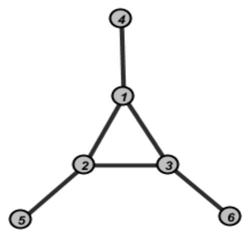

Fig 2 : 3-Sunlet graph

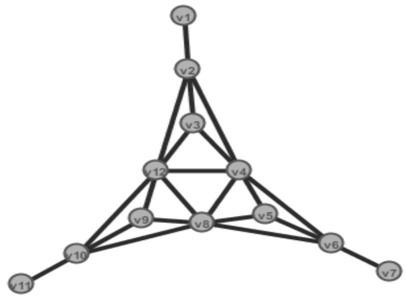

Fig 3: Middle graph of 3-Sunlet graph

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The figure 2 is the 3-sunlet graph obtained by attaching 3 pendant edges on 6 vertices to a cycle c_3 and the figure 3 is the middle graph of 3 sunlet graph with vertices $V(G) = \{V_1 \dots \dots V_{12}\}$ and edges $E(G) = \{e_1, \dots \dots . e_{21}\}$.

Let $D = \{V_2, V_6, V_{10}\}$, where V_2 dominates $\{V_1, V_3, V_{4,} V_{12}\}$, V_6 dominates $\{V_5, V_7\}$ and V_{10} dominates $\{V_9, V_8, V_{11}\}$.Therefore D is the dominating set of middle graph of 3-sunlet graph. On removal of any one vertex from D i.e., on removal of V_6 from D, V_6, V_5 *and* V_7 are not dominated, on removal of V_2 from D, V_1, V_3, V_4 *and* V_{12} are not dominated and on removal of V_{10} from D, V_9, V_8, V_{11} are not dominated. Hence, on removal any one vertex from D, it cannot be the dominating set. So, D is can be said as minimal dominating set with the domination number 3.i.e., $\gamma[M(S_3)] = 3$

Propositions 3.2:

The domination number of middle graph of 4-sunlet graph, $\gamma[M(S_4)] = 4$.

Proof:

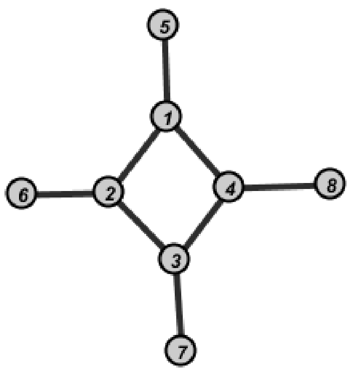

Fig 4 : 4-Sunlet graph

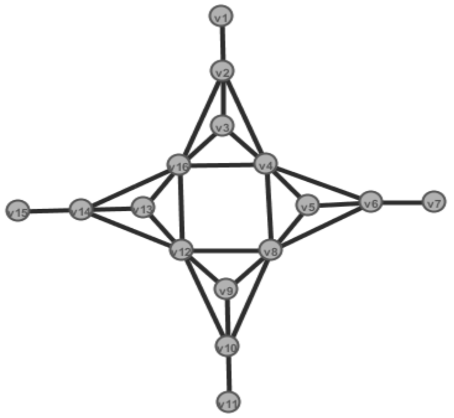

Fig 5: Middle graph of 4-Sunlet graph

The figure 4 is the 4-sunlet graph on 8 vertices with 4 pendant edges to cycle c_4, and by definition figure 5 is the middle graph with the vertices $V_1, V_2, \dots \dots . V_{16}$ and with the edges $e_1, \dots \dots . e_{28}$.

$D = \{V_2, V_6, V_{10}, V_{14}\}$ is the minimal dominating set, since each and every vertex of D dominates minimum of two to maximum of six vertices in S_4 so on removal of any one vertex from D, some neighbourhood vertices remains un dominated, therefore D is the minimal dominating set with the domination number 4.i.e.,$\gamma[M(S_4)] = 4$.

Theorem 3.3:

The domination number of middle graph of n-sunlet graph, $\gamma[M(S_n)] = n$.

Proof:

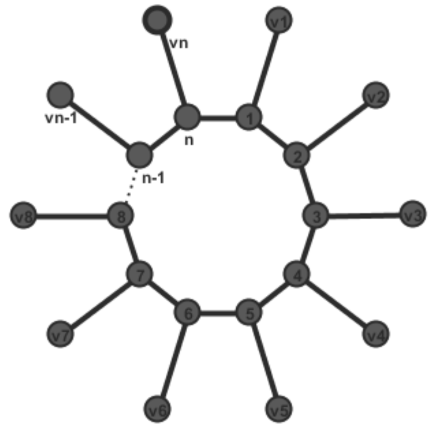

Fig 6 : n-Sunlet graph

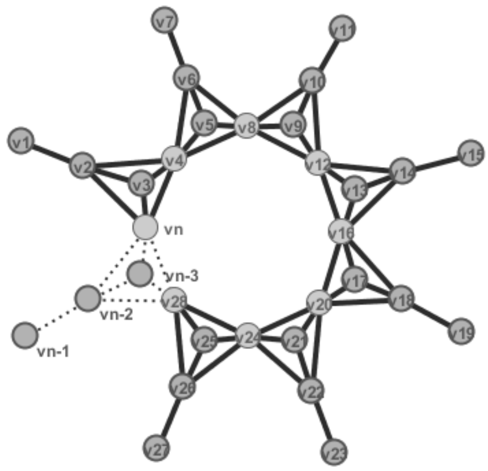

Fig 7 : Middle graph of n-Sunlet graph

By attaching n pendant edges to an cycle c_n the n-sunlet graph figure 6 is obtained on 2n vertices.
Let $\{V_1, V_2, \dots . . V_{2n}\}$ be the vertices and $\{e_1, \dots \dots . . e_{2n}\}$ be the edges of n-sunlet graph.

By the definition, figure 7 is the middle graph of n-sunlet graph where each edges are subdivided by the vertices. Therefore the vertex set $V[M(S_n)] = \{V_1, V_2, \dots \dots . V_{2n}\} \cup \{e_1, e_2, \dots \dots e_{2n}\}$ where v is the vertex and e is the edges.

Let $D = \{V_2, V_6, V_8, \dots . . V_{2n-2}\}$, since the maximum degree is n.This set D dominates atleast two vertices, on removal of any one vertex makes D a non dominating set. Therefore D is a minimal dominating set of the middle graph of n-sunlet graph with the domination number n. $i.e., \gamma[M(S_n)] = n$.

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Proposition 3.4:

The split domination number of middle graph of 3 sunlet graph $\gamma_s[M(S_3)] = 3$.

Proof:
Let the dominating set be $D = \{V_2, V_6, V_{10}\}$. By the proposition 4.1, as D is the dominating set it dominates all the vertices of S_3. D cannot be the dominating set on removal any one vertex from D, so D is the minimal dominating set. On removal of D from S_3 the graph is disconnected as the vertex V_1, V_7, V_{11} becomes isolated. The dominating set S_3 is split dominating set of G as induced sub graph $\langle V - D \rangle$ is disconnected. The split domination number of M(G) of 3 sunlet graph is 3.i.e., $\gamma_s[M(S_3)] = 3$.

Lemma 3.5:

By proposition 3.4, we have the split domination number of M(G) of 4-sunlet graph $\gamma_s[M(S_4)] = 4$ since all the vertices of S_4 is adjacent to atleast one vertex of S.

Theorem 3.6:

The split domination number of middle graph of n sunlet graph $\gamma_s[M(S_n)] = n$.

Proof:
Let $D = \{V_2, V_6, V_8, \ldots . . V_{2n-2}\}$ be the dominating set. By the theorem 4.3, as D is the dominating set it dominates all the vertices of S_n. On removal any one vertex from D, D cannot be the dominating set. So D is the minimal dominating set. The graph is disconnected as the vertex $V_1, V_7, V_{11}, ..$ becomes isolated on removal of D from V. The dominating set S_n is split dominating set of G as induced sub graph $\langle V - D \rangle$ is disconnected. The split domination number of middle graph of n sunlet graph is n. i.e., $\gamma_s[M(S_n)] = n$.

IV. RESULTS

Domination number of middle graph of 3-sunlet graph is 3 as 3 vertices dominates the remaining vertices of middle graph of 3-sunlet graph. Similarly, Domination number of middle graph of 4-sunlet graph is 4 and therefore domination number of middle graph of n-sunlet graph is $\gamma[M(S_n)] = n$. At the same time, middle graph of 3-sunlet graph remains disconnected on removal of 3 vertices of dominating set so that the split domination number of middle graph is 3 and therefore the split domination number of n-sunlet graph is $\gamma_s[M(S_n)] = n$.

V. DISCUSSION

From the graph, we have certain points under discussion.

For $n \geq 3$, in n-sunlet graph

Number of vertices and edges in 3-sunlet graph is 6, 4-sunlet graph is 8. Therefore, in n-sunlet graph number of vertices and edges will be $2n$. Also we can understand that the Maximum degree of n-sunlet graph is 3 and Minimum degree is 1.

For $n \geq 3$, in middle graph of n-sunlet graph

Number of vertices in middle graph of 3-sunlet graph is 12, 4-sunlet graph is 16. Therefore, in middle graph of n-sunlet graph number of vertices will be $4n$. Number of edges in middle graph of 3-sunlet graph is 21, 4-sunlet graph is 28. Therefore, in middle graph of n-sunlet graph number of vertices will be $7n$.The Maximum degree of middle graph of sunlet graph is 6 and Minimum degree is 1.

VI. CONCLUSION

In this paper, we have identified the domination number and split domination number of middle graph of n-sunlet graph.

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