Erudition on Ladder Network and Prolonged Ladder Network - Semigraph Approach

Thiagarajan K, Mansoor P

Abstract: *Let $u_1, u_2, \dots, u_m$ and $v_1, v_2, \dots, v_m$ be two paths on m nodes. For $i = 1,2,\dots,m$ if we link the nodes u_i and v_i by an edge e_i, then the new graph obtained is called a Ladder network which denoted by $L_{2m}, m \geq 1$. A ladder network can be expanded by introducing semi nodes to each of its edges. This paper introduces an expression for the domination number of a ladder network in terms of the number of internal steps of the ladder and attempts to study some properties of expanded ladder networks obtained by introducing semi nodes to its steps, that is, the edges connecting the nodes of the paths.*

Index Terms: *Ladder network, Domination number, Expanded ladder network, Roman domination number.*

I. INTRODUCTION

A graph G comprises of a non-empty set V of points, called nodes, and a set E of two point subsets of V, called edges connecting pairs of nodes. A path in a graph G is a sequence of nodes and edges in which all nodes except the first and last are distinct. A path on m vertices is denoted by P_m.A graph G is said to be connected if there exists a path between every pairs of nodes of G. A graph G is said to be complete if each node in G is adjacent to the other nodes of G. A complete graph on n nodes is denoted by K_n which has $\frac{n(n-1)}{2}$ edges. A graph G is said to be k- regular if all node of G has of degree k. A cycle is a graph having equal number of nodes and edges whose nodes can be arranged around a circle so that two nodes are adjacent if and only if they appear consecutively along the circle.

Let G = (V, E) be a graph and let D ⊆ S. If every node in V−D is adjacent to a minimum of one node in D, then D is said to be a dominating set for G. A dominating set with minimum number of nodes is known as a minimal dominating set for G and the number of nodes in a minimal dominating set is known as the domination number of the graph G which is denoted by $\gamma(G)$. A node v in a graph G dominates itself and each of its adjacent nodes. A Roman dominating function φ is an assignment of colors for the nodes of a graph G with the colors 0, 1 and 2 in such a way that each node u, say, colored 0 is adjacent to a minimum of one node v, say, colored 2. The sum of the colors assigned for the nodes is known as the weight of the corresponding Roman dominating function. The minimum weight of all possible Roman dominating function of a graph G is known as the Roman domination number which is denoted by $\gamma_R(G)$.

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A proper coloring of a graph G is a labeling of nodes of G such that no two adjacent nodes receive the same colour. A coloring of a graph G using at most k colors is called a k-coloring. The minimum number of colors required for a proper coloring of a graph G is known as the chromatic number which is denoted by $\chi(G)$.

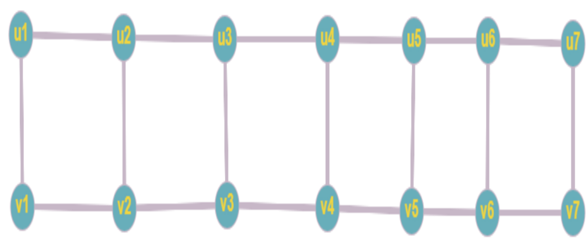
Let $u_1, u_2, \dots, u_m$ and $v_1, v_2, \dots, v_m$ be two paths on m nodes. For $i = 1,2,\dots,m$ if we link the nodes u_i and v_i by an edge e_i, then the new graph obtained is called a Ladder network which denoted by $L_{2m}, m \geq 1$.

A Ladder network graph $L_{2m}, m \geq 1$ has $2m$ nodes and $3m - 2$ edges, i.e., steps. It is a planar graph with maximum degree $\Delta(L_{2m}) = 3$ and minimum degree $\delta(L_{2m}) = 2$.

II. THE DOMINATION NUMBER OF A LADDER NETWORK

Example : 1

Consider the following illustration: A ladder network L_{14} on 14 nodes is as follows:



Let $u_1, u_2, \dots, u_7$ and $v_1, v_2, \dots, v_7$ be the nodes of L_{14}. Note that the node u_1 is adjacent to two nodes u_2 and v_1, the node v_3 is adjacent to three nodes v_2, v_4 and u_3, the node u_5 is adjacent to three nodes u_4, u_6, v_5, and the node v_7 is adjacent to v_6 and u_7. Therefore D = {u_1, v_3, u_5, v_7} constitute a dominating set forL_{14}, since all nodes not in D are adjacent to some nodes in D. The elements in D are the nodes taken from alternate steps of the ladder L_{14}. Any two consecutive nodes in D are connected in such a way that the first node lies on one path P_7 and the next node lies on the other path P_7 alternatively.

Here the distance between two consecutive nodes in D is 3. Clearly D is a minimal dominating set, otherwise, the elements in D are at a distance one, two or more than three from each node already chosen, which in turn increases the cardinality of the dominating set D.

Example : 2

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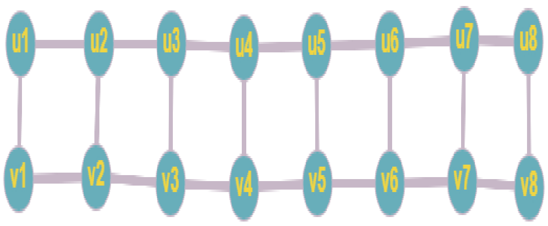
Consider the ladder network L_{16} on 16 nodes:



In this network u_1 is adjacent to u_2 and v_1, v_3 is adjacent to v_2, v_4, and u_3, u_5 is adjacent to u_4, $u_6$and v_5, v_7 is adjacent to v_6, v_8 and u_7. Therefore D={ u_1, v_3, u_5, v_7, u_8} constitute a dominating set for L_{16}, since all nodes not in D are adjacent to some nodes in D. As in the previous example, the distance between two consecutive nodes in D is 3 except the last one, for which it is 2. Clearly D is a minimal dominating set, otherwise the elements in D except the last one are at a distance one or two or more than three from each node already chosen, which in turn increases the cardinality of the dominating set D.

Example : 3

For the ladder network L_{14},

$$\gamma(L_{14}) = \left[\frac{\frac{14}{2}-2}{2}\right] + 2 = \left[\frac{7-2}{2}\right] + 2 = \left[\frac{5}{2}\right] + 2 = 4.$$

Example: 4

For the ladder network L_{16},

$$\gamma(L_{16}) = \left[\frac{\frac{16}{2}-2}{2}\right] + 2 = \left[\frac{8-2}{2}\right] + 2 = \left[\frac{6}{2}\right] + 2 = 5.$$

III. RESULTS AND OBSERVATIONS ON DOMINATION NUMBER OF LADDER NET WORK

Observation: 1

From the above two illustrations, the domination number of L_{14} is $\gamma(L_{14}) = 4$ and that of L_{16} is $\gamma(L_{16}) = 5$. .

In a similar manner, we can find the domination number of all ladder networks.

Note that, if n is odd, we can choose steps of the ladder alternatively from the first step to the last without remaining any step. But if n is even, one step will remain after the alternative selection of steps. In such case one node from this step should be chosen for a minimal dominating set.Let the initial and terminal steps be considered as the edges with end nodes (u_1, v_1) and (u_m, v_m), respectively, of the ladder graph $L_{2m}, m \geq 1$. The remaining $m-2$ steps are considered as the internal steps of the ladder graph $L_{2m}, m \geq 1$. Then the domination number for $L_{2m}, m \geq 1$ can be obtained by the following expression:

Proposition: 1

For a ladder network$L_{2m}, m \geq 1$, the domination number is

$$\gamma(L_{2m}) = \left[\frac{\frac{2m}{2}-2}{2}\right] + 2 = \left[\frac{m-2}{2}\right] + 2,$$

where $\left[\frac{m-2}{2}\right]$ denotes the greatest integer less than or equal to $\frac{m-2}{2}$.

For all ladder networks, since the minimal dominating set contains the nodes of the first and last steps, the constant '2' appears in the above formula.

Observation: 2

Since the two paths in a ladder network can be coloured with two colours, a ladder network $L_{2m}, m \geq 1$ can be coloured properly with two colours. Therefore, the chromatic number of a ladder network $L_{2m}, m \geq 1$ is $\chi(L_{2m}) = 2$.

IV. EXPANDED LADDER NETWORK

An expanded ladder network $L'_{2ms}, m \geq 1$, can be obtained from the ladder network $L_{2m}, m \geq 1$ by introducing semi nodes to each steps, i.e., the edges connecting the two paths.

If we introduce semi nodes on the alternate steps starting from the first step, then the new expanded ladder network is denoted by L'_{2ms1} . It has $2m + \left[\frac{m}{2}\right]$ nodes and $3m - 2 + 2\left[\frac{m}{2}\right]$ edges. The maximum degree is 4 and the minimum degree is 2.

The minimal dominating set S for L_{2m} can be chosen in such a way that the newly introduced semi nodes will become adjacent to the nodes in the minimal dominating set S. Thus, we have the following observation.

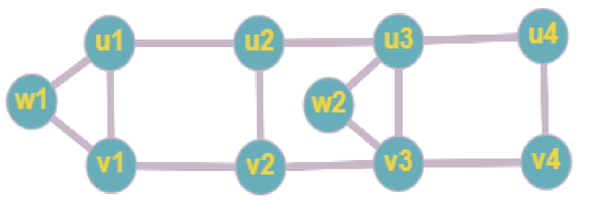
Observation: 3

The domination number of L'_{2ms1} is same as the domination number of L_{2m},

i.e., $\gamma'(L'_{2ms1}) = \gamma(L_{2m})$,

Example: 5

The expanded ladder network $L'_{8s1}$is as follows:



Here w_1 and w_2 are the newly introduced semi nodes on the alternate steps from the first step. If we introduce semi nodes on the alternate steps starting from the second step, then the new expanded ladder network is denoted by L'_{2ms2}. It has $2m + \left[\frac{m}{2}\right]$ nodes and $3m - 2 + 2\left[\frac{m}{2}\right]$ edges. The maximum degree is 4 and the minimum degree is 2.

Example: 6

The expanded ladder network L'_{8s2} is as follows

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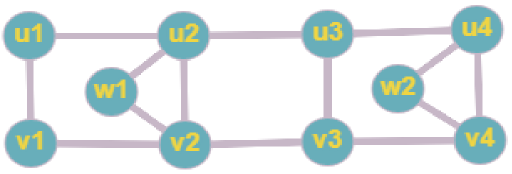
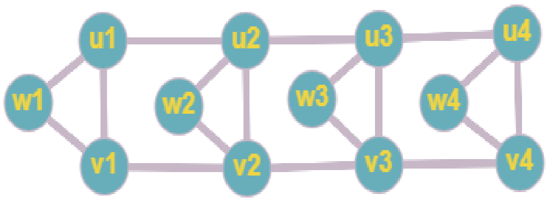


Here w_1 and w_2 are the newly introduced semi nodes on the alternate steps from the second step. If we introduce semi nodes on each steps, then the new expanded ladder network is denoted by L'_{2ms}. It has $3m$ nodes and $5m-2$ edges. The maximum degree is 4 and the minimum degree is 2. The domination number is $\gamma'(L'_{2ms}) = m$.

Example: 7

The expanded ladder network L'_{8s} is as follows:



Here w_1, w_2, w_3 and w_4 are the newly introduced semi nodes on the steps. After introducing semi nodes to each of the steps of a ladder graph, it can be seen that there exists a cycle of length 3 corresponding to each of the steps. Since three colours are needed for a proper colouring of a cycle of length 3, it can be concluded that three colours are needed for a proper colouring of an expanded ladder graph L'_{2ms}. Thus the chromatic number of an expanded ladder graph L'_{2ms} is $\chi(L'_{2ms}) = 3$.

V. ROMAN DOMINATION NUMBER

In [7], the Roman domination number of a graph G is defined and the authors proved the following two propositions on Roman domination number of a graph G.

Proposition 2: [7]

Let $G = (V, E)$ be a connected graph on $|V|$ nodes. Then $\gamma_R(G) = \gamma(G) + 1$ if and only if G contains a node of degree $|V| - \gamma(G)$.

Proposition 3: [7]

Let $G = (V, E)$ be a connected graph on $|V|$ nodes. Then $\gamma_R(G) = \gamma(G) + 2$ if and only if

a) G does not contain a node of degree $|V| - \gamma(G)$.

b) Either G contains a node of degree $|V| - \gamma(G) - 1$ or G contains two nodes u_1 and u_2 for which $|N[u_1] \cup N[u_2]| = |V| - \gamma(G) + 2$. From these two propositions we observed the following:

VI. RESULTS AND OBSERVATIONS ON ROMAN DOMINATION NUMBER EXPANDED NETWORKS

Observation 4:

For the ladder network L_4, $|V| = 4$, $\gamma(L_4) = 2$. Now, $|V| - \gamma = 4 - 2 = 2$.

Clearly L_4 contains a node of degree 2. Therefore by the proposition 2, the Roman domination of the ladder network L_4 is $\gamma_R(L_4) = \gamma(L_4) + 1 = 2 + 1 = 3$.

Observation 5:

For the expanded ladder network L'_{4s}, $|V| = 6$, $\gamma(L'_{4s}) = 2$.

Now, $|V| - \gamma - 1 = 6 - 2 - 1 = 3$.

Note that L'_{4s} contains a node of degree 3, but not contain a node of degree $|V| - \gamma = 6 - 2 = 4$. By the proposition 3, the Roman domination of the ladder network L'_{4s} is $\gamma_R(L'_{4s}) = \gamma(L'_{4s}) + 2 = 2 + 2 = 4$.

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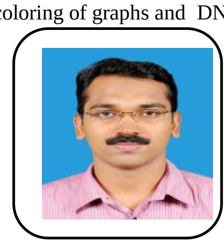

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