Solving a Particular Form in Laplace and Z Transform

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***Abstract:** In this paper we will discuss on two types of transforms. One is Laplace transform and the other one is Z transform. Evaluating inverse transform for a particular form in both the transforms by different methods. A different solutions are evaluated and analyzed. Applying the transform techniques in the form $L^{-1}\left[\frac{1}{(S-P)(S-Q)}\right]$ and $Z^{-1}\left[\frac{1}{(Z-P)(Z-Q)}\right]$ are discussed. Here I have used a particular problem and solving in both the transform techniques and discussed briefly.*

Index Terms: Partial fraction, Residue, convolution.

I. INTRODUCTION

A transformation is an operation which converts a mathematical expression to a different but equivalent form. For Example: $\frac{d}{dx}(\sin x) = \cos x$, $\int cosx\ dx = sinx + c$

Laplace transform is used to solve ordinary differential equations and partial differential equations. Laplace transform helps in solving differential equation with initial value problems directly and boundary value problems indirectly. But Z-Transform is used to solve difference equations. Z transform helps in digital filters, digital control system can be analyzed and designed. The Laplace transform does for continuous system and Z-Transform plays the role for discrete systems. The need of Laplace transform and Z Transform increases as we always work in Frequency domain of signals and Control systems. Laplace transforms transform differential equation to algebraic equations and thus help in finding out the output when input to a system is given. Z transform is used to convert discrete time domain signal into discrete frequency domain signal. The Laplace transform has a number of properties that make it useful for analyzing linear dynamical systems. The transform turns integral equations and differential equations to polynomial equations, which are much easier to solve. Once solved, use of the inverse Laplace transform reverts to the original domain.

II. PRELIMINIERIES

DEFINITION OF LAPLACE TRANSFORM

Let $h(\emptyset)$ be a function of $\emptyset$ defined for $\emptyset > 0$ then the Laplace Transform of $h(\emptyset)$ is defined as $L[h(\emptyset)] = \int_0^\infty e^{-\alpha\emptyset}h(\emptyset)d\emptyset = F(\alpha)$ Provided the integral exists. Here α is the parameter which may be real or complex. L is called the Laplace transform operator.

DEFINITION OF Z TRANSFORM

If $\{h(\emptyset)\}$ is a discrete sequence defined for $\emptyset = 0,1,2$ .... then Z transform of $f(n)$ is defined as $Z\{h(\emptyset)\} == \sum_{\emptyset=0}^{\infty} h(\emptyset))z^{-\emptyset} = H(z)$ where $\emptyset$ is an integer and z is an arbitrary complex number. Z is called the Z transform operator.

Convolution definition in Laplace Transform

If $p(n)$ and $q(n)$ are two sequences then the convolution of $p(n)$ and $q(n)$ denoted by $p(n) * q(n)$ and is defined as $p(n) * q(n) = \int_0^t p(u)q(t-u)du$

Convolution theorem

The Laplace transform of convolution of two functions is the product of their Laplace transforms.
i.e. $L[p(n) * q(n)] = L[p(n)]L[q(n)]$

Residue method in Z Trasnsform

For a simple pole, the residue of $[z^{n-1}P(z)]$ at $z = a$

1. $Res[z^{n-1}P(z)]_{z=a} = \lim_{z\to a}(z-a)z^{n-1}P(z)$
2. $Res[z^{n-1}P(z)]$ at a pole of order 'm' is given by

$$Res[z^{n-1}P(z)]_{z=a} = \lim_{z\to a}\frac{1}{(m-1)!} * \frac{d^{m-1}}{dz^{m-1}}[(z-a)^m z^{n-1}P(z)]$$

Therefore,
$Z^{-1}[P(z)] = sum\ of\ the\ residues\ of\ z^{n-1}P(z)$ at the isolated singularities.

III. PROBLEM SOLVING

a. Solution of $L^{-1}\left[\frac{1}{(S-P)(S-Q)}\right]$ by using Partial Fraction method:

Let $\left[\frac{1}{(S-P)(S-Q)}\right] = \frac{A}{S-P} + \frac{B}{S-Q}$

$$\frac{1}{(S-P)(S-Q)} = \frac{A(S-Q)+B(S-P)}{(S-P)(S-Q)}$$

$$1 = A(s-Q) + B(S-P)$$

Put S = P

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$$A = \frac{1}{P-Q}$$

Put $S = Q$

$$B = \frac{1}{Q-P}$$

Taking L^{-1} on both sides,

$$L^{-1}\left[\frac{1}{(S-P)(S-Q)}\right] = \frac{e^{Pt}-e^{Qt}}{P-Q}$$

b. Solution of $Z^{-1}\left[\frac{1}{(z-p)(z-q)}\right]$ by using Residue method:

Let $F(z) = \left[\frac{1}{(z-p)(z-q)}\right]$

$$z^{n-1}F(z) = \left[\frac{z^{n-1}}{(z-p)(z-q)}\right]$$

Then, $z = p, q$ are simple poles.

$$Res[z^{n-1}F(z)]_{z=p} = \lim_{z\to p}(z-p)\frac{z^{n-1}}{(z-p)(z-q)} = \frac{p^{n-1}}{p-q}$$

$$Res[z^{n-1}F(z)]_{z=q} = \lim_{z\to q}(z-q)\frac{z^{n-1}}{(z-p)(z-q)} = \frac{q^{n-1}}{q-p}$$

Therefore,

$$Z^{-1}\left[\frac{1}{(z-p)(z-q)}\right] = \frac{p^{n-1}-q^{n-1}}{p-q}$$

IV. CONCLUSION

In this paper, we have discussed two different solutions of evaluating a particular form in both the transform techniques. Since the problem form was standard, but we got different solutions. The concepts of Laplace transform are very similar to the Fourier transform. The Fourier transform is called the frequency domain representation of the original signal. Z-transforms are very similar to Laplace transform but are discrete time-interval conversions, closer for digital implementations. The solution of the form $\frac{1}{(S-P)(S-Q)}$ by two different methods for two different transform have been determined.

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