# Application of queuing theory in Hospital Management

**R. Prabakaran , K. Kumar**

***Abstract In this paper finds an applications of queuing theory in hospital management .The goal of this paper explains the problem in their urgency of medical cases with respect to allocation problem of the patients and can be categorized. We assume first come first served queue discipline, Poisson arrival of the patients and exponential service times first come first served queue discipline.***

***KEYWORDS: Queuing system, FCFS, Poisson arrival***

## I. INTRODUCTION

In a hospital system, increasing resource utilization to reduce costs and decreasing patients' waiting time to provide timely care and improve patient satisfaction are important but conflicting goals. Queueing models can provide reasonably accurate evaluations of system performance and are popular among researchers and system designers because of their analytical nature and their ability to provide quick solutions for "what-if" analyses. There is a considerable amount of published research on using queueing to analyze and design hospital facilities. We review and categorize this literature in an attempt to motivate further research in applying queueing models in the healthcare domain[5].

Planning is the most important aspect of establishing a hospital. If the plan is good all may go well. If the plan is not thought out carefully the work may never be completed. Planning a new hospital starts with setting goals for the hospital, without which the organization cannot have a definite direction or focus. This is followed by the study of the external environment of the organization, and the internal and external resources with which the goals set are to be achieved. This exercise facilitates selection of the means by which to achieve goals within a reasonable cost.[6]

Queuing models have been extensively used to model and analyze different healthcare systems such as hospitals (Cochran and Roche, 2009), pharmaceutical industry (Viswanadham and Narahari, 2001), and organ transplant (Zenios, 1999). Nearly a decade ago, Preater (2001) compiled a bibliography of queuing applications in healthcare. This paper reviews and categorizes applications of queuing models in hospital Management. The proper allocation of beds in different wards of a hospital has been discussed. The volume of demand in the system is first estimated. The estimated demand is divided into two components termed controllable and uncontrollable. The controllable component of demand is defined as the patients who need advance appointment for physical care. The latter category includes emergency cases.

**Revised Manuscript Received on June 05, 2019**

**R. Prabakaran** , Department of Mathematics , Coimbatore Institute of Technology, Coimbatore , India.

**K. Kumar** , Department of Science and Humanities , Sri Krishna College of Engineering and Technology , Coimbatore , India.

For each case values of various queuing characteristics are calculated and finally conclusion regarding bed allocation is discussed.

## II. PROBLEM ALLOCATION:

The hospital management is faced with the problem of allocating beds in various wards of the hospital together with allotment of duties to doctors in different wards and outdoors. Rao (1972) dealt with different aspects of management and hospital for treatment[1].

The problem that are not alike as regards their urgency of medical cases with respect to allocation problem of the patients and can be categorized as follows:

1. The patients who need only medical advice and outdoor treatment.
2. The patient who needs admission in hospital for thorough examination as well as time to time check up by doctors
3. The patients who need emergency service Eg: Intensive care unit, maternity, accident etc

In the first two cases patients can be treated by the appointment as a general patient may wait without causing excessive worry either to the patient or the hospital management .But in the third case the patient may arrive without notice and long waiting time may cause loss of one's life in some of the emergency cases.

## III. COLLECTION OF DATA

In the present problem mainly two types of data are required.

(i) Arrival patterns of patients

(ii) The time spent in the treatment.

The data relates to the hospital of public sector with their initial distribution of beds among male, female and emergency (including maternity) wards as follows:

| Wards | No of Beds | No of patients | Average length of stay per patient |
|---|---|---|---|
| Male Ward | 16 | 2.92 | 4.32 |
| Female Ward | 12 | 2.17 | 3.87 |
| Emergency Ward | 10 | 2.75 | 2.62 |

## IV. MATHEMATICAL FORMULATION OF THE PROBLEM:

We assume first come first served queue discipline,



*Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication*

Poisson arrival of the patients and exponential service times.

The differential Equation is:

$$P_n'(t) = -\lambda P_n(t) + \mu p_1(t)$$
$$P_n'(t) = -(\lambda + n\mu)P_n(t) + \lambda p_{n-1}(t) + (n+1)\mu p_{n+1}(t), n < s$$
$$P_n'(t) = -(\lambda + s\mu)P_n(t) + \lambda p_{n-1}(t) + s\mu p_{n+1}(t), n \geq s$$

Where λ is the arrival rate, μ is the service rate and s the number of beds

By solving the above differential equations the following results of queuing theory are easily obtained .

(1) The probability that there is no patient waiting being served on bed is

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{s\mu}{s\mu - \lambda}\left(\frac{\lambda}{\mu}\right)^s \frac{1}{s!} \right]^{-1}$$

(2) The Number of patients waiting for admission is:

$$E_w = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^2}{(s-1)(s\mu - \lambda)^2} P_0$$

(3) The number of patients being served on beds in addition to those waiting for admission

$$E_n = E_w + \frac{\lambda}{\mu}$$

(4) The average time the patient must wait before being admitted is

$$E_t = \frac{E_w}{\lambda}$$

(5) The expected time in system is

$$E_n = E_w + \frac{1}{\mu}$$

For each ward , relevant data pertaining to the arrival of the patient and the length of stay has been calculated from the appreciate records and characteristics of queuing system are obtained as given in table 1,2 and 3 for male, female and emergency ward respectively.

## V. VALUES OF VARIOUS QUEUING CHARACTERISTICS

**TABLE-1**

| S | $P_0$ | $E_w$ | $E_t$ | $E_n = E_w + \frac{\lambda}{\mu}$ |
|---|---|---|---|---|
| 14 | $[46.1980 \times 10^4]^{-1}$ | 5.478 | 1.876 | 18.0642 |
| 15 | $[35.6139 \times 10^4]^{-1}$ | 2.193 | 0.7509 | 14.7792 |
| 16 | $[31.9294 \times 10^4]^{-1}$ | 1.025 | 0.3511 | 13.6112 |
| 17 | $[30.3484 \times 10^4]^{-1}$ | 0.508 | 0.1740 | 13.0942 |
| 18 | $[29.6082 \times 10^4]^{-1}$ | 0.256 | 0.0877 | 12.8422 |
| 19 | $[29.2533 \times 10^4]^{-1}$ | 0.1292 | 0.0443 | 12.7154 |
| 20 | $[29.0809 \times 10^4]^{-1}$ | 0.0644 | 0.0221 | 12.6506 |

**TABLE-2**

| S | $P_0$ | $E_w$ | $E_t$ | $E_n = E_w + \frac{\lambda}{\mu}$ |
|---|---|---|---|---|
| 9 | $[1.1269 \times 10^4]^{-1}$ | 11.2200 | 5.1705 | 19.6308 |
| 10 | $[0.6060 \times 10^4]^{-1}$ | 2.6837 | 1.2368 | 11.0945 |
| 11 | $[0.5062 \times 10^4]^{-1}$ | 1.0177 | 0.4690 | 9.4285 |
| 12 | $[0.4724 \times 10^4]^{-1}$ | 0.4340 | 0.1999 | 8.8448 |
| 13 | $[0.4590 \times 10^4]^{-1}$ | 0.1924 | 0.0886 | 8.6032 |
| 14 | $[0.4534 \times 10^4]^{-1}$ | 0.0846 | 0.0390 | 8.4954 |
| 15 | $[0.4511 \times 10^4]^{-1}$ | 0.0367 | 0.0169 | 8.4475 |

**TABLE-3**

| S | $P_0$ | $E_w$ | $E_t$ | $E_n = E_w + \frac{\lambda}{\mu}$ |
|---|---|---|---|---|
| 14 | $[0.2548 \times 10^4]^{-1}$ | 6.301 | 2.291 | 13.500 |
| 15 | $[0.1656 \times 10^4]^{-1}$ | 1.727 | 0.628 | 8.926 |
| 16 | $[0.1451 \times 10^4]^{-1}$ | 0.651 | 0.237 | 7.850 |
| 17 | $[0.1387 \times 10^4]^{-1}$ | 0.266 | 0.097 | 7.465 |
| 18 | $[0.1355 \times 10^4]^{-1}$ | 0.112 | 0.041 | 7.311 |
| 19 | $[0.1345 \times 10^4]^{-1}$ | 0.046 | 0.017 | 7.245 |
| 20 | $[0.1341 \times 10^4]^{-1}$ | 0.019 | 0.006 | 7.218 |

## VI. ANALYSIS AND DISCUSSION OF RESULTS

In the male ward for available strength of 16 beds as per table 1,expected waiting time is 0.3511 day ( 3 hours and 25 minutes ).such a high value of waiting time may cause inconvenience to the patient which is met by unscheduled discharge of the patients. therefore , only alternative is to increase the number of beds so as to reduce the waiting time for admission. Since the hospital administration is of the view that no patient be allowed to wait for more than 2 hours in general ward. So19 beds are recommended in male ward. Similarly in general female ward, table-2 reveals that for available strength of 12 beds, the expected waiting time is 0.20 day per patient ( 4 hours and 48 minutes). This waiting time for



Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

nearly 5 hours is alarming. Hence 14 beds are recommended in female ward to avoid a patient to wait for more than 2 hours. In emergency ward for available 10 beds, it has been found that the expected waiting time is 0.24 day ( 5 hours 45 minute )per patient. It is a glaring instance when such a high value of waiting time may cause loss of one's life in some of the emergency cases. It may not be possible to discharge the patient before the schedule. Thus it is recommended that number of beds should be increased to 13 so that the waiting time of a patient is less than half an hour.

## VII. CONCLUSION

On the basis of the above conclusion it is suggested that the number of beds allowed in different ward be as follows:

| | | |
|---|---|---|
| Male ward | : | 19 beds |
| Female ward | : | 14 beds |
| Emergency ward | : | 13 beds |
| Total | : | 46 beds |

So by increase of only 8 beds the waiting time for general patient may be reduced to less than 2 hours and for emergency cases less than half an hour. In this paper, applications of queuing theory in modeling hospital management have been reviewed and categorized. Since bed allocations are directly dealing with hospital of public sector, improving system performance of allocation of beds in emergency is a very important goal. Increasing servers' utilization and decreasing patients' waiting time can enhance system productivity. Queuing theory provides an effective and powerful modeling technique that can help managers achieve the aforementioned goals. This approach can be easily implemented and has several advantages such as providing good and rapid estimations of the system performance. According to this survey, there are several future research opportunities. Studying the effect of bed and nurse flexibility. Beds can be used with different categories of patients and nursing staff can be cross-trained to gain the flexibility to serve different patient categories. Developing efficient dynamic priority rules for expensive shared healthcare facilities.Developing dynamic resource allocation models in case of multiple patient categories and several service resources. Regional capacity allocation based on the characteristics and criticality of the candidate regions. Staff and patient planning models are designed for a specific environment and there is no general framework that works well in any environment. Developing a scheme for selecting the best architecture for the appointment system for a given healthcare facility could be a potential future research opportunity in this area.

## REFERENCE


1. Rao, D,G, (1972) : Management break through in Hospital Management opsearch 9(384), 143-167
2. Bishram Sharma, Hospital Science, Applied Science Periodical, Vol-III,Number-1 February-2001
3. Rao,Rajiv,(2000):Applications of queuing theory, International Journal of Applied Statistics,Vol-56,150-160
4. Rajesh Kanna, Theory of Queue ,III-Edition(2005), PHI publications ,Delhi
5. Foad Mahdavi Pajouh , Manjunath Kamath , Applications of Queueing Models in Hospitals, Queueing Applications in Hospitals, Proceedings of the Fifth Midwest Association for Information Systems Conference, Moorhead, MN May 21-22, 2010.
6. K.B. SUBBA RAO CHAPTER 2: PLANNING A MODERN HOSPITAL


## AUTHORS PROFILE

R.Prabakaran, M.Sc.,M.Phil.,P.hd.,B.Ed., working as an Assistant Professor in the Department of Mathematics, Coimbatore Institute of Technology, Coimbatore . He has 9 years of teaching for both under graduate and post graduate students in Engineering college.

K.Kumar, M.Sc.,M.Phil.,B.Ed., working as an Assistant Professor in the Department of science and Humanities, Sri Krishna College of Engineering and technology, oimbatore. He has 9 years of teaching for both under graduate and post graduate students in Engineering college.



*Published By:*
*Blue Eyes Intelligence Engineering*
*& Sciences Publication*