Development of Scheffe's Model to Predict the Compressive Strength of Concrete using SDA as Partial Replacement for Fine Aggregate

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Abstract*: *Saw Dust Ash (SDA) despite being an industrial waste, has played a vital role in concrete mix research. It has served as an alternative or complementary material to some of the traditional materials of concrete. In this study, it served as a fifth ingredient of concrete blend as it replaced 5% of the fine aggregate (sand). The other four ingredients were cement, sand, granite, and water. Scheffe's simplex theory was used for five mix ratios in a {5,2} experimental design which resulted in additional ten mix ratios. For purposes of verification and testing, additional fifteen mix ratios were generated. The thirty concrete mix ratios were subjected to laboratory experiments to determine the 28 days compressive strengths. The results of the first fifteen compressive strengths were used for the calibration of the model constant coefficients, while those from the second fifteen were used for the model verification using Scheffe's simplex lattice design. A mathematical regression model was derived from the experimental results, with which the compressive strengths were predicted. The derived model was subjected to a two-tailed t-test with 5% significance, which ascertained the model to be adequate with an R^2 value of 0.8336. The study revealed that SDA can replace 5% of fine aggregate and promote sustainability, without compromising the 28 days compressive strength.

Index Terms*: *Compressive strength of Concrete, Saw Dust Ash, Scheffe's simplex lattice.

I. INTRODUCTION

The use of industrial waste materials in concrete research has come to stay. Some of such materials range from fly ash, saw dust ash (SDA), rice husk ash, quarry dust, and palm kernel shell ash. They have been used in one form or another, to replace fractions of either cement or fine aggregates, while others have been used to stabilise sub-base materials for pavement construction.

Saw dust is an industrial waste or bye-product of saw mills produced after the wood has been sawn to shape in the saw mill, and comes out in powder form. It has been used in concrete construction for over 30 years [1]. When saw dust is subjected to fire, it burns to ashes. That ash is called Saw Dust Ash (SDA).

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In this study, SDA was used to partially replace 5% of the fine aggregate. A mathematical model was derived using Scheffe's simplex theory, with which the compressive strengths were predicted. There were five components in the mix (water-cement ratio, cement, sand, SDA, and granite).

II. LITERATURE REVIEW

According to [2], concrete plays the biggest material role in the construction industry. Several authors have studied and determined various means of actualizing sustainability in the construction industry with respect to concrete. [3] demonstrated in his experimental research that SDA can drastically improve the properties of lateritic soils when used as a stabiliser. His experiment shows that the optimum moisture content, maximum dry density, unconfined compressive strength, and shear strength of the lateritic soil were improved when stabilised with SDA. A similar study carried out by [4] also shows that using saw dust to stabilise lateritic soils could improve the CBR and other properties of the soil, as well as reduce the construction cost. [5] have described the production of cement as a major source of environmental degradation as about 400kg of CO_2 is being emitted for every 600kg of cement produced. They therefore replaced 10% of cement with SDA which did not negatively affect the chloride permeability and thaw resistance of the concrete, but decreased the drying shrinkage, and increased the water absorption. This also established the pozzolanic ability of SDA. Similarly, [6] found in their research that replacing 5 to 15% cement content with saw dust increased the compressive, flexural, and split tensile strengths of the concrete for 28days curing period and beyond. It also decreased the weight and cost. However, fewer researchers such as, [7] have carried out research work on replacement of fine aggregates with SDA. Their research findings revealed that 10% replacement of fine aggregate with SDA will result in an acceptable tensile, flexural, and compressive strengths as well as reduce the amount of wastes in the environment.

SDA has different particles that are mostly angular in shape. According to [7] SDA has a specific gravity of 2.5, fineness modulus of 1.78, water absorption of 0.56%, and bulk dry density of 1300kg/m^3 as against sand with specific gravity of 2.65, fineness modulus of 2.21, water absorption of 0.45%, and bulk dry density of 1512 kg/m^3. When 10% of SDA is added to the sand, these properties became 2.67, 2.2, 0.5%, and 1436kg/m^3 for specific gravity, fineness modulus, water absorption, and bulk dry density respectively.

This is a significant indication that the mixture of sand and 10% SDA replacement gave similar physical properties with the 0% SDA replacement, making the mixture adequate for a fine aggregate. However, [8] SDA had a specific gravity of 2.19, bulk dry density of 1040kg/m^3, and moisture content of 0.3%. This gives a bigger difference in the specific gravity of SDA as compared to that of sand. Furthermore, [9] shows that 50% of the SDA grain size is passing the AASHTO sieve no. 200 (75µm) while 31% is retained by sieve no. 325 (45 µm). This according to [9] justifies the fineness of SDA.

SDA, like many other concrete construction materials, contains several chemical compounds. According to [7] SDA has the following chemical composition by mass: 65.3% SiO_2, 4% Al_2O_3, 2.23% Fe_2O_3, 9.6% CaO, 5.8% MgO, 0.01% MnO, 0.07% Na_2O, 0.11% K_2O, 0.43% P_2O_5, and 0.45% SO_2. Summing up SiO_2, Al_2O_3, and Fe_2O_3 gives 71.53%. Similar works carried out by [10] reveals 67.95% SiO_2, 4.29% Al_2O_3, 2.15% Fe_2O_3, 9.47% CaO, 5.84% MgO, 0.01% MnO, 0.06% Na_2O, 0.11% K_2O, and 0.56% SO_3. Summing up SiO_2, Al_2O_3, and Fe_2O_3 gives 74.39. These, in accordance with [11] indicate that SDA is a good pozzolanic material. The chemical compositions as found by [7], [10], [12] all show that SDA has a high percentage of SiO_2 and small percentages of Al_2O_3 and Fe_2O_3, which are similar to those of sand with high percentage of about 95% SiO_2. Hence SDA can be used with sand as fine aggregate.

A. Scheffe's Simplex Theory

Several authors such as [13]–[19] have carried out Portland cement concrete mixture research while [20] has carried out Asphalt concrete research with the development of mathematical models. Most of such works were based on Scheffe's Simplex theory.

Scheffe's model is based on the simplex lattice and simplex theory or approach [21]. The simplex approach considers a number of components, *q*, and a degree of polynomial, *m*. The sum of all the i^{th} components is not greater than 1. Hence,

$$\sum_{i=1}^{q} x_i = 1 \qquad (1)$$

$$x_1 + x_2 + \cdots + x_q = 1 \qquad (2)$$

with $0 \le x \le 1$. The factor space becomes S_{q-1}. According to [21] the {*q,m*} simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_m^{(q+m-1)}$ has (m+1) equally spaced values of x_i = 0, $^1/_m$, $^2/_m$, …. $^m/_m$. For a 3-component mixture with degree of polynomial 2, the corresponding number of points will be $C_2^{(3+2-1)}$ which gives 6 (eq. 3 or eq. 4 below) with number of spaced values, 2+1 = 3, that is x_i = 0, $^1/_2$, and 1 as design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,01/2), and (0,1/2,1/2). Similarly, for a {*5,2*} simplex, there will be 15 points with x_i = 0, $^1/_2$, and 1 as spaced values. The 15 design points are (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1), (1/2,1/2,0,0,0), (1/2,0,1/2,0,0), (1/2,0,0,1/2,0), (1/2,0,0,0,1/2), (0,1/2,1/2,0,0), (0,0,1/2,1/2,0), (0,0,0,1/2,1/2), (0,1/2,0,1/2,0), (0,0,1/2,0,1/2), (0,1/2,0,0,1/2).

$$N = C_n^{(q+n-1)} \qquad (3)$$

or

$$N = \frac{(q+n-1)!}{(q-1)!\,(n)!} \qquad (4)$$

For a polynomial of degree *m* with *q* component variables where eq. (2) holds, the general form is:

$$Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \cdots + \sum b_{i1,i2\ldots in} x_{i1} x_{i2} x_{in} \qquad (5)$$

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le k \le q$, and b_0 is the constant coefficient.

x is the pseudo component for constituents *i*, *j*, and *k*.

When {*q,m*} = {*5,2*}, eq. (5) becomes:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 \qquad (6)$$

and eq. (2) becomes

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \qquad (7)$$

Multiplying eq. (7) by b_0 gives

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = b_0 \qquad (8)$$

Multiplying eq. (7) successively by x_1, x_2, x_3, x_4, and x_5 and making x_1, x_2, x_3, x_4, and x_5 the subjects of the respective formulas:

$$\left.\begin{aligned} x_1^2 &= x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5 \\ x_2^2 &= x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5 \\ x_3^2 &= x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5 \\ x_4^2 &= x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5 \\ x_5^2 &= x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5 \end{aligned}\right\} \qquad (9)$$

Substituting eq. (8) and eq. (9) into eq. (6) we have:

$$Y = b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5) + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5) + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5) + b_{44}(x_4 - x_1x_4\; x_2x_4 - x_3x_4 - x_4x_5) + b_{55}(x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5)$$

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$$Y = (b_0 + b_1 + b_{11})x_1 + (b_0 + b_2 + b_{22})x_2 + (b_0 + b_3 + b_{33})x_3 + (b_0 + b_4 + b_{44})x_4 + (b_0 + b_5 + b_{55})x_5 + (b_{12} - b_{11} - b_{22})x_1x_2 + (b_{13} - b_{11} - b_{33})x_1x_3 + (b_{14} - b_{11} - b_{44})x_1x_4 + (b_{15} - b_{11} - b_{55})x_1x_5 + (b_{23} - b_{22} - b_{33})x_2x_3 + (b_{24} - b_{22} - b_{44})x_2x_4 + (b_{25} - b_{22} - b_{55})x_2x_5 + (b_{34} - b_{33} - b_{44})x_3x_4 + (b_{35} - b_{33} - b_{55})x_3x_5 + (b_{45} - b_{44} - b_{55})x_4x_5 \quad (10)$$

Let

$$\left.\begin{aligned}
\beta_1 &= b_0 + b_1 + b_{11}\\
\beta_2 &= b_0 + b_2 + b_{22}\\
\beta_3 &= b_0 + b_3 + b_{33}\\
\beta_4 &= b_0 + b_4 + b_{44}\\
\beta_5 &= b_0 + b_5 + b_{55}\\
\beta_{12} &= b_{12} - b_{11} - b_{22}\\
\beta_{13} &= b_{13} - b_{11} - b_{33}\\
\beta_{14} &= b_{14} - b_{11} - b_{44}\\
\beta_{15} &= b_{15} - b_{11} - b_{55}\\
\beta_{23} &= b_{23} - b_{22} - b_{33}\\
\beta_{24} &= b_{24} - b_{22} - b_{44}\\
\beta_{25} &= b_{25} - b_{22} - b_{55}\\
\beta_{34} &= b_{34} - b_{33} - b_{44}\\
\beta_{35} &= b_{35} - b_{33} - b_{55}\\
\beta_{45} &= b_{45} - b_{44} - b_{55}
\end{aligned}\right\} \quad (11)$$

Substituting eq. (11) into eq. (10) gives

$$Y = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{15}x_1x_5 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{25}x_2x_5 + \beta_{34}x_3x_4 + \beta_{35}x_3x_5 + \beta_{45}x_4x_5 \quad (12)$$

$$Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \le i \le j \le 5} \beta_{ij} x_i x_j \quad (13)$$

Where the response, Y is a dependent variable (compressive strength of concrete). Eq. (12) is the general equation for a {5,2} polynomial, and it has 15 terms, which conforms to Scheffe's theory in eq. (3).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in i and j. If $x_i = 1$ and $x_j = 0$, sice $j \neq i$, then

$$Y_i = \beta_i \quad (14)$$

Which means

$$\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i \quad (15)$$

Hence, from eq. (14)

$$\left.\begin{aligned}
Y_1 &= \beta_1\\
Y_2 &= \beta_2\\
Y_3 &= \beta_3\\
Y_4 &= \beta_4\\
Y_5 &= \beta_5
\end{aligned}\right\} \quad (16)$$

According to [21], $\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j$ (17)

Substituting eq. (14) $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$ (18)

III. MATERIALS AND METHODS

Water, cement, sand, SDA, and granite were the materials used to produce the concrete.

The first five concrete mix ratios derived from different mix design methods given as

BRE 12 = [0.54 1 1.9475 0.1025 2.95];
BRE 22 = [0.58 1 2.1185 0.1115 3.21];
USBR 22 = [0.58 1 2.2515 0.1185 3.29];
BIS 12 = [0.43 1 1.2065 0.0635 2.88];
ACI 12 = [0.55 1 1.8335 0.0965 3.09]

These can be put in matrix form as follows:

$$S = \begin{bmatrix} 0.54 & 0.58 & 0.58 & 0.43 & 0.55\\ 1 & 1 & 1 & 1 & 1\\ 1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335\\ 0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965\\ 2.95 & 3.21 & 3.29 & 2.88 & 3.09 \end{bmatrix} \quad (19)$$

Their corresponding pseudo components are given as:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

With centre points

X_{12} = [0.5 0.5 0 0 0]; X_{13} = [0.5 0 0.5 0 0];
X_{14} = [0.5 0 0 0.5 0]; X_{15} = [0.5 0 0 0 0.5];
X_{23} = [0 0.5 0.5 0 0]; X_{24} = [0 0.5 0 0.5 0];
X_{25} = [0 0.5 0 0 0.5]; X_{34} = [0 0 0.5 0.5 0];
X_{35} = [0 0 0.5 0 0.5]; X_{45} = [0 0 0 0.5 0.5]

According to [21],

$$S_{ij} = XS_i \quad (21)$$

Substituting,

$$\begin{bmatrix} S_{12}\\ S_{13}\\ S_{14}\\ S_{15}\\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0\\ 0.5 & 0 & 0.5 & 0 & 0\\ 0.5 & 0 & 0 & 0.5 & 0\\ 0.5 & 0 & 0 & 0 & 0.5\\ 0 & 0.5 & 0.5 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0.54\\ 0.58\\ 0.58\\ 0.43\\ 0.55 \end{bmatrix} \quad (22)$$

This process is repeated for S_{24}, S_{25}, S_{34}, S_{35}, and S_{45}. Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular pentagons for the actual components with their corresponding pseudo components are given in figures (1) and (2) respectively.

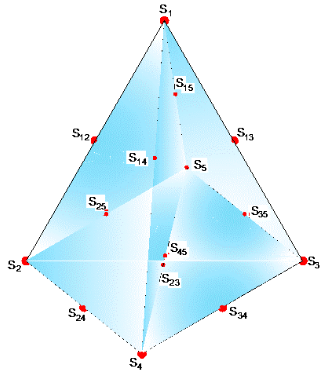

Figure 1 – Simplex plot for actual components

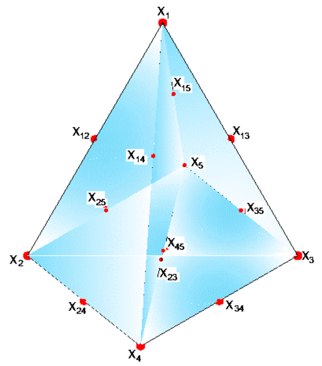

Figure 2 – Simplex plot for pseudo components

Table I - Model Mix Ratios

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
BRE12	0.54	1	1.9475	0.1025	2.95	Y_1	1	0	0	0	0
BRE22	0.58	1	2.1185	0.1115	3.21	Y_2	0	1	0	0	0
USBR22	0.58	1	2.2515	0.1185	3.29	Y_3	0	0	1	0	0
BIS12	0.43	1	1.2065	0.0635	2.88	Y_4	0	0	0	1	0
ACI12	0.55	1	1.8335	0.0965	3.09	Y_5	0	0	0	0	1
N1	0.56	1	2.033	0.107	3.08	Y_{12}	0.5	0.5	0	0	0
N2	0.56	1	2.0995	0.1105	3.12	Y_{13}	0.5	0	0.5	0	0
N3	0.485	1	1.577	0.083	2.915	Y_{14}	0.5	0	0	0.5	0
N4	0.545	1	1.8905	0.0995	3.02	Y_{15}	0.5	0	0	0	0.5
N5	0.58	1	2.185	0.115	3.25	Y_{23}	0	0.5	0.5	0	0
N6	0.505	1	1.6625	0.0875	3.045	Y_{24}	0	0.5	0	0.5	0
N7	0.565	1	1.976	0.104	3.15	Y_{25}	0	0.5	0	0	0.5
N8	0.505	1	1.729	0.091	3.085	Y_{34}	0	0	0.5	0.5	0
N9	0.565	1	2.0425	0.1075	3.19	Y_{35}	0	0	0.5	0	0.5
N10	0.49	1	1.52	0.08	2.985	Y_{45}	0	0	0	0.5	0.5

Table II – Control Points

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
C1	0.558	1	2.0463	0.1077	3.114	Y_{C1}	0.4	0	0.4	0	0.2
C2	0.52	1	1.7537	0.0923	3.078	Y_{C2}	0	0.6	0	0.4	0
C3	0.548	1	2.0083	0.1057	3.018	Y_{C3}	0.8	0	0.2	0	0
C4	0.49	1	1.5713	0.0827	3.012	Y_{C4}	0	0.4	0	0.6	0
C5	0.544	1	1.9019	0.1001	3.006	Y_{C5}	0.6	0	0	0	0.4
C6	0.55	1	2.0425	0.1075	3.208	Y_{C6}	0	0	0.8	0.2	0
C7	0.55	1	1.9589	0.1031	3.03	Y_{C7}	0.6	0.2	0	0	0.2
C8	0.514	1	1.6967	0.0893	3.054	Y_{C8}	0	0.4	0	0.4	0.2
C9	0.548	1	1.8563	0.0977	3.062	Y_{C9}	0.2	0	0	0	0.8
C10	0.46	1	1.4155	0.0745	2.962	Y_{C10}	0	0	0.2	0.8	0
C11	0.566	1	2.1071	0.1109	3.182	Y_{C11}	0.2	0	0.6	0	0.2
C12	0.544	1	1.9323	0.1017	3.152	Y_{C12}	0	0.2	0.4	0.2	0.2
C13	0.58	1	2.1451	0.1129	3.226	Y_{C13}	0	0.8	0.2	0	0
C14	0.532	1	1.7651	0.0929	3.072	Y_{C14}	0	0.2	0	0.2	0.6
C15	0.536	1	1.8715	0.0985	3.084	Y_{C15}	0.2	0.2	0.2	0.2	0.2

A. Uniaxial Compressive Strengths of Concrete

Two replicate concrete cubes were made for each of the thirty mix ratios in 150mmX150mmX150mm moulds and allowed to harden. The concrete cubes were removed from the mould after 24 hours and were soaked in water to cure for 28dyas. On the 28^{th} day, the cubes were removed from the water and subjected to crushing with the aid of a uniaxial compressive strength machine. The compressive strengths were determined with eq. (23) and recorded in table (III).

$$f_c = \frac{P}{A}$$

(23)

Where P = the applied compressive load at failure (KN)
A = the cross-sectional area of the specimen (mm^2)

Table III – Uniaxial Compressive Strengths of Concrete

Sample	Load (KN)		Area (mm^2)	Compressive Strength (N/mm^2)		
	A	B		A	B	Average
BRE12	640	640	22500	28.444	28.444	28.444
BRE22	520	550	22500	23.111	24.444	23.778
USBR22	589.3	592.05	22500	26.191	26.313	26.252
BIS12	815	805	22500	36.222	35.778	36.000
ACI12	710.2	719	22500	31.564	31.956	31.760
N1	660	641	22500	29.333	28.489	28.911
N2	494	506	22500	21.956	22.489	22.222
N3	590	600	22500	26.222	26.667	26.444
N4	600	669	22500	26.667	29.733	28.200
N5	661	624	22500	29.378	27.733	28.556
N6	652	663	22500	28.978	29.467	29.222
N7	558	654	22500	24.800	29.067	26.933
N8	698	668	22500	31.022	29.689	30.356
N9	515	554	22500	22.889	24.622	23.756
N10	635	629	22500	28.222	27.956	28.089
C1	525	548	22500	23.333	24.356	23.844
C2	645	649	22500	28.667	28.844	28.756
C3	556	555	22500	24.711	24.667	24.689
C4	700	721	22500	31.111	32.044	31.578
C5	650	639	22500	28.889	28.400	28.644
C6	623	621	22500	27.689	27.600	27.644
C7	630	645	22500	28.000	28.667	28.333
C8	675	665	22500	30.000	29.556	29.778
C9	642	612	22500	28.533	27.200	27.867
C10	715	716	22500	31.778	31.822	31.800
C11	522	515	22500	23.200	22.889	23.044
C12	625	633	22500	27.778	28.133	27.956
C13	588	569	22500	26.133	25.289	25.711
C14	669	651	22500	29.733	28.933	29.333
C15	587	591	22500	26.089	26.267	26.178

IV. RESULTS AND DISCUSSIONS

Table IV - Sieve Analysis Data for Fine Aggregate with 5% SDA replacement

Standard Sieve Opening Sizes			Mass Retained (g)	Cumulative Mass Retained (g)	% Retained	Cumulative % Retained	% Passing
Sieve Number	Sieve size (in)	Sieve size (mm)					
1/4"	0.25	6.3	0	0	0.00%	0.00%	100.00%
#4	0.187	4.75	2.2	2.2	0.73%	0.73%	99.27%

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#8	0.0929	2.36	10.6	12.8	3.51%	4.24%	95.76%
#16	0.0465	1.18	29.7	42.5	9.84%	14.08%	85.92%
#30	0.0236	0.6	89.1	131.6	29.52%	43.61%	56.39%
#50	0.0118	0.3	111.3	242.9	36.88%	80.48%	19.52%
#100	0.00591	0.15	52.5	295.4	17.40%	97.88%	2.12%
#200	0.00295	0.075	6.1	301.5	2.02%	99.90%	0.10%
Pan	0	0	0.3	301.8	0.10%	100.00%	
		Total mass	301.8				

Fineness modulus, $FM = \frac{0.73+4.24+14.08+43.61+80.48+97.88}{100} = 2.41$

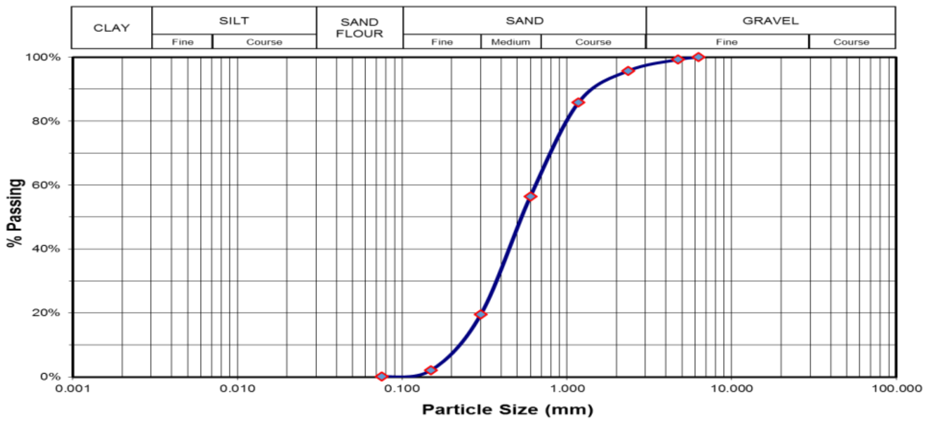

Figure 3 - Particle Size Distribution for Fine Aggregate with 5% SDA replacement

A. Scheffe's Model for 28 days Compressive Strength

The coefficients of polynomial from table (3), eq. (16), and eq. (18) are:

β_1 = 28.444, β_2 = 23.778, β_3 = 26.252, β_4 = 36, β_5 = 31.76, $\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$

$\beta_{12} = 4 * 28.911 - 2 * 28.444 - 2 * 23.778 = 11.2$

Similarly, β_{13} = -20.504, β_{14} = -23.112, β_{15} = -7.608, β_{23} = 14.164, β_{24} = -2.668, β_{25} = -3.344, β_{34} = -3.08, β_{35} = -21, β_{45} = -23.164.

Substituting the above coefficients into eq. (12) gives

$$\boldsymbol{Y = 28.444x_1 + 23.778x_2 + 26.252x_3 + 36x_4 + 31.76x_5 + 11.2x_1x_2 - 20.504x_1x_3 - 23.112x_1x_4 - 7.608x_1x_5 + 14.164x_2x_3 - 2.668x_2x_4 - 3.344x_2x_5 - 3.08x_3x_4 - 21x_3x_5 - 23.164x_4x_5} \quad \textbf{(24)}$$

Eq. (24) above is the mathematical model to predict the 28 days compressive strength of concrete using SDA to replace 5% of fine aggregate.

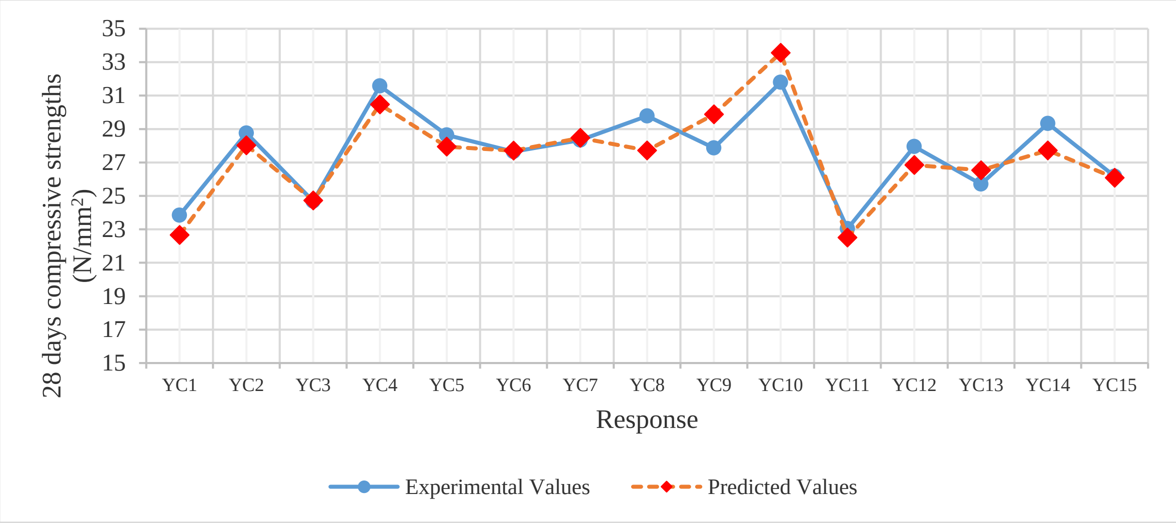

Figure 4 - Comparison between Experimental and Predicted 28days Compressive Strengths

Table V – Experimental and predicted values of 28days compressive strength of Concrete

Sample Points	Response Y	Pseudo Components					Comp. strength Y_{exp}(N/mm^2)	Comp. strength Y_{pred}(N/mm^2)
		w-c ratio	Cement	Sand	SDA	Granite		
		X_1	X_2	X_3	X_4	X_5		
BRE12	Y_1	1	0	0	0	0	28.444	28.444
BRE22	Y_2	0	1	0	0	0	23.778	23.778
USBR22	Y_3	0	0	1	0	0	26.252	26.252
BIS12	Y_4	0	0	0	1	0	36.000	36.000
ACI12	Y_5	0	0	0	0	1	31.760	31.760
N1	Y_{12}	0.5	0.5	0	0	0	28.911	28.911
N2	Y_{13}	0.5	0	0.5	0	0	22.222	22.222
N3	Y_{14}	0.5	0	0	0.5	0	26.444	26.444
N4	Y_{15}	0.5	0	0	0	0.5	28.200	28.200
N5	Y_{23}	0	0.5	0.5	0	0	28.556	28.556
N6	Y_{24}	0	0.5	0	0.5	0	29.222	29.222
N7	Y_{25}	0	0.5	0	0	0.5	26.933	26.933
N8	Y_{34}	0	0	0.5	0.5	0	30.356	30.356
N9	Y_{35}	0	0	0.5	0	0.5	23.756	23.756
N10	Y_{45}	0	0	0	0.5	0.5	28.089	28.089
C1	Y_{C1}	0.4	0	0.4	0	0.2	23.844	22.661
C2	Y_{C2}	0	0.6	0	0.4	0	28.756	28.026
C3	Y_{C3}	0.8	0	0.2	0	0	24.689	24.725
C4	Y_{C4}	0	0.4	0	0.6	0	31.578	30.471
C5	Y_{C5}	0.6	0	0	0	0.4	28.644	27.944
C6	Y_{C6}	0	0	0.8	0.2	0	27.644	27.709
C7	Y_{C7}	0.6	0.2	0	0	0.2	28.333	28.471
C8	Y_{C8}	0	0.4	0	0.4	0.2	29.778	27.716
C9	Y_{C9}	0.2	0	0	0	0.8	27.867	29.880
C10	Y_{C10}	0	0	0.2	0.8	0	31.800	33.558
C11	Y_{C11}	0.2	0	0.6	0	0.2	23.044	22.507
C12	Y_{C12}	0	0.2	0.4	0.2	0.2	27.956	26.848
C13	Y_{C13}	0	0.8	0.2	0	0	25.711	26.539
C14	Y_{C14}	0	0.2	0	0.2	0.6	29.333	27.724
C15	Y_{C15}	0.2	0.2	0.2	0.2	0.2	26.178	26.082

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B. Test of Adequacy of the Model

A two-tailed student t-test was carried out at 95% confidence level, which implies 100 – 95 = 5% significance. Since it is a two-tailed, significance = 5/2 = 2.5%

Hence significance level = 100 – 2.5 = 97.5%

Let D be difference between the experimental and predicted responses

The mean of the difference,

$$D_a = \frac{1}{n}\sum_{i=1}^{n} D_i \quad (25)$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(D - D_a)^2{}_i \quad (26)$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \quad (27)$$

Where n = number of observations with degree of freedom n – 1.

$$S^2 = \frac{18.637}{15-1}$$

$S^2 = 1.331$, $S = \sqrt{1.331} = 1.154$

$t_{calculated} = 0.961$

Table VI – Student t-test for 28days compressive strength of Concrete

Sample	Curing	Compressive Strength (N/mm^2)		t-test		
		$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D_a\text{-}D)^2$
C1	28 Days	23.844	22.661	1.183	-0.897	0.804
C2	28 Days	28.756	28.026	0.730	-0.444	0.197
C3	28 Days	24.689	24.725	-0.036	0.322	0.104
C4	28 Days	31.578	30.471	1.107	-0.821	0.674
C5	28 Days	28.644	27.944	0.700	-0.414	0.171
C6	28 Days	27.644	27.709	-0.065	0.351	0.123
C7	28 Days	28.333	28.471	-0.138	0.424	0.180
C8	28 Days	29.778	27.716	2.062	-1.776	3.153
C9	28 Days	27.867	29.880	-2.013	2.299	5.287
C10	28 Days	31.800	33.558	-1.758	2.044	4.179
C11	28 Days	23.044	22.507	0.537	-0.251	0.063
C12	28 Days	27.956	26.848	1.108	-0.822	0.675
C13	28 Days	25.711	26.539	-0.828	1.114	1.242
C14	28 Days	29.333	27.724	1.609	-1.323	1.750
C15	28 Days	26.178	26.082	0.096	0.190	0.036
TOTAL				4.294		18.637
AVERAGE D_a				0.286		

From the t-table, $t_{(\beta,v)}$ can be determined where v = 15 – 1 = 14, and β = significance level. $t_{(0.975,14)} = 2.145$

Since $t_{calculated} < t_{(0.975,14)}$, and lies between -2.145 and 2.145, therefore there is no significant difference between the experimental and predicted responses, H_0 is accepted, and H_a is rejected. The model is confirmed to be adequate.

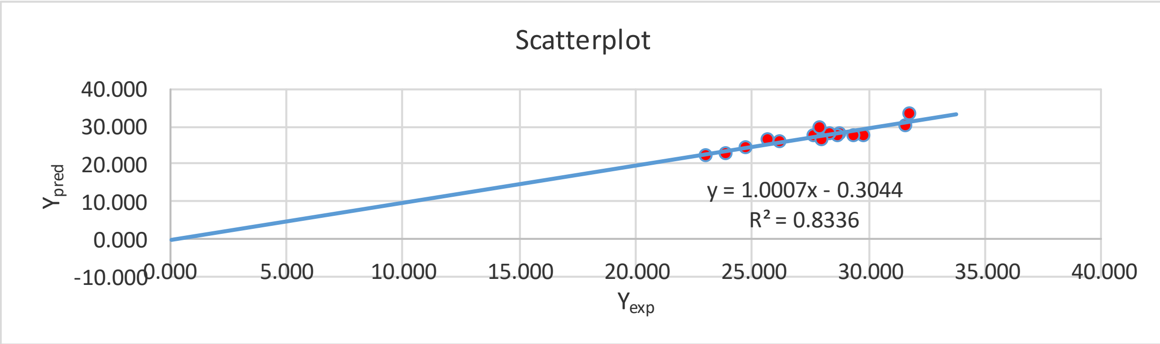

Figure 5 – Scatterplot of Predicted vs. Experimental 28days Compressive Strengths

V. CONCLUSION AND RECOMMENDATIONS

Replacement of fine aggregate with 5% SDA has resulted in acceptable 28 days compressive strengths (between 22 and 36N/mm2) with concrete mix ratios resulting from different design methods. A regression model has been generated from the resulting laboratory experiments using Scheffe's simplex theory. A two-tailed t-test was carried out, which confirmed the adequacy of the derived model with an R^2 value of 0.8336. The results also confirmed that SDA is a suitable material to replace a small fraction of fine aggregate in a bid to promote sustainability.

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