On a New Notation of Fuzzy Subgroup

Sudipta Gayen, Sripati Jha, Manoranjan Singh, Ranjan Kumar

***Abstract*: The concept of fuzzy subgroup has been investigated and analysed under a different condition. It will be observed that the result discussed in this paper is more appealing than the previously defined ones. To establish that a graphical comparison and some examples are given. Under the proposed definition some important results of the fuzzy subgroup are verified and some new interesting results are observed. Further, a modified notion of the level subgroup is given.**

***Index Terms*: Fuzzy subgroup, Level set, Level subgroup.**

I. INTRODUCTION

The idea of a fuzzy subset (FS) was first mathematically formulated by Zadeh [*1*]. Nowadays it has become an important area of research in many applied as well as and pure fields. It was based on the belief that, in interpreting most of the physical problems crisp set theory was not reasonable or effective, because each object confronted in our surroundings fetches a certain amount of fuzziness. Many researchers have implemented this concept in various fields. For instance in modelling and control theory [*2*], optimization techniques [*3*], shortest path problems [*4-10*] neural network [*11*], database [*12*], transportation problem [*13*] etc.

Rosenfeld [*14*] was the first to introduce the notion of the fuzzy subgroup (FSG). He defined fuzzified versions of most of the algebraic structures like subgroupoid, subgroup, ideal, and homomorphism, lattice, etc. Later on, Das [*15*] gave us the concept of level subgroup (LSG), which play an important role in the interpretation of FSG. Mukherjee [*16*] gave us the perception of the fuzzy normal subgroup as well as fuzzy coset. He proved some interesting results on fuzzy normal subgroups related to conjugate classes and LSGs. He also established the fuzzified version of Lagrange's theorem.

Later on, Singh [*17*] redefined the notion of FSG and showed that his proposed definition was more appealing. Based on that he proved some interesting results, theorems, and propositions related to FSG. In his book, he not only redefined FSG but also redefined the notions of level set (LS) level subgroup (LSG), fuzzy normal subgroup, ring, ideal, field, linear space, module, topological space, etc.

In this paper, we have generalized the notion of FSG such way that the domain of definition becomes vaster and based on that we have redefined LS and LSG. While doing so we have introduced the 3D graphical image of a FSG. Further following the same path as mentioned in [*17*] we can generalize other algebraic structures like fuzzy normal subgroup, ring [*18*], ideal [*14*], field, linear space, module, topological space [*19*], Intuitionistic FSG [*20*], anti-FSG [*21*] etc.

The major contributions of this article are:

- We have introduced a new generalized version of FSG.
- For the first time, we have given 3D graphical image of a FSG.
- We have also given some well-known examples to prove that our proposed version of FSG is more general.
- Based on newly defined notion we have given some new propositions.
- We have also introduced a modified version of LS and LSG.
- Lastly, we have mentioned some areas in which this newly defined notion can be implemented and concluded that this article may generate more scope of future researches.

This paper has been arranged as the following: In Section II we have included some preliminary definitions and theorems. In Section III we have introduced a generalized version of FSG. We have also given 3D graphical comparisons as well as some examples to prove our claim. In Section IV we have introduced modified versions of LS, LSG and have proved some theories regarding them. Finally, in Section V we have concluded that our proposed notions are more appealing and may generate more scope of future researches.

II. PRELIMINARIES

Definition II.1 [*1*] A FS σ of a crisp set U is a function from U to [0,1] i.e. $\sigma : U \to [0,1]$.

Definition II.2 [*1*] Let α be a FS of U. Then $\forall t \in [0,1]$ the set $\alpha_t = \{x \in U : \alpha(x) \geq t\}$ is called a LS ($t-$LS) of α.

Definition II.3 [*14*] A FS α of a group H is called a FSG of the group H iff $\forall\, p, q \in H$, the subsequent conditions are fulfilled:

(i) $\alpha(mu) \geq \min\{\alpha(m), \alpha(u)\}$

(ii) $\alpha(m^{-1}) \geq \alpha(m)$

$\forall\, m \in H$.

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Here $\alpha(m^{-1}) = \alpha(m)$ and $\alpha(m) \le \alpha(e)$ (e is the neutral element of H). Again, it can be proved that α is a FSG of H iff $\alpha(mu^{-1}) \ge \min\{\alpha(m), \alpha(u)\}, \forall m,u \in H$.

The next definition is the redefined notion of FSG by [*17*].

Definition II.4 [*17*] A FS α of a group H is called a FSG of H if $\forall m,u \in H$, the subsequent conditions are fulfilled:

(i) $\alpha(mu) \ge \alpha(m)\alpha(u)$

(ii) $\alpha(m^{-1}) \ge \alpha(m)$

Now, if α is a FSG over H according to Definition II.3, then $\forall m,u \in H$ we have

$$\alpha(mu) \ge \min\{\alpha(m), \alpha(u)\} \ge \alpha(m)\alpha(u) \quad (1)$$

Clearly, from Equation 1 when α is a FSG according to Definition II.3 then it is a FSG according to Definition II.4 but the converse is not always true.

Here $\forall m \in H$, $\alpha(m^{-1}) = \alpha(m)$, $\alpha^2(m) \le \alpha(e)$, where e is the neutral element of H. Also, observe that $\alpha(e) = 1$, if $\alpha(m) = 1$ for at-least one $m \in H$.

Definition II.5 [*15*] Let α be a FSG of a group H according to **Definition II.3**. Then the subgroups α_t for all $t \in [0, 1]$ are called LSG of α.

A. List of Abbreviations

FS stands for "Fuzzy subset"

FSG stands for "Fuzzy subgroup".

LS stands for "Level subset".

LSG stands for "Level subgroup".

$K \lesssim H$ stands for "K is a subgroup of H".

In the next section, the notion of FSG has been general -ized and compared with the previously defined ones.

III. SOME GENERALIZED NOTATION OF FS

Definition III.1 A FS α of a group H is called a FSG of H iff $\forall m,u \in H$, the subsequent conditions are fulfilled

(i) $\alpha(mu) \ge \begin{cases} \alpha(m)+\alpha(u)-1 & \text{if } \alpha(m)+\alpha(u) \ge 1 \\ \min\{\alpha(m), \alpha(u)\} & \text{if } \alpha(m)+\alpha(u) < 1 \end{cases}$

(ii) $\alpha(m^{-1}) \ge \alpha(m)$

The above mentioned definition is more general than Definition II.3 but some part of it (when $\alpha(m)+\alpha(u) < 1$) follows Definition II.4 and some part of it (when $\alpha(m)+\alpha(u) \ge 1$) is more general than the Definition II.3 as well as Definition II.4.

Definition III.2 A FS α of a group H is called a FSG of H iff $\forall m,u \in H$, subsequent conditions are fulfilled:

(i) $\alpha(mu) \ge \begin{cases} \alpha(m)+\alpha(u)-1 & \text{if } \alpha(m)+\alpha(u) \ge 1 \\ \alpha(m)\alpha(u) & \text{if } \alpha(m)+\alpha(u) < 1 \end{cases}$

(ii) $\alpha(m^{-1}) \ge \alpha(m)$

Here observe that $\forall m,u \in H$

$$\begin{aligned} \alpha(mu) &\ge \min\{\alpha(m), \alpha(u)\} \\ &\ge \begin{cases} \alpha(m)+\alpha(u)-1 & \text{if } \alpha(m)+\alpha(u) \ge 1 \\ \min\{\alpha(m), \alpha(u)\} & \text{if } \alpha(m)+\alpha(u) < 1 \end{cases} \end{aligned} \quad (2)$$

Again,

$$\begin{aligned} \alpha(mu) &\ge \min\{\alpha(m), \alpha(u)\} \\ &\ge \alpha(m)\alpha(u) \\ &\ge \begin{cases} \alpha(m)+\alpha(u)-1 & \text{if } \alpha(m)+\alpha(u) \ge 1 \\ \alpha(m)\alpha(u) & \text{if } \alpha(m)+\alpha(u) < 1 \end{cases} \end{aligned} \quad (3)$$

By Equation 2 and Equation 3 it is clear that the proposed Definition III.2 of FSG is more general than the Definition II.3and Definition II.4

A. A Graphical Comparison of the Definitions

Let α be a FSG of a group H such that $\alpha(m)$, $\alpha(u) \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. Here, we have plotted $\alpha(mu)$ on the basis of the proposed definitions of a FSG. In those proposed definitions we have replaced '≥' operator with '=' and plotted $\alpha(mu)$

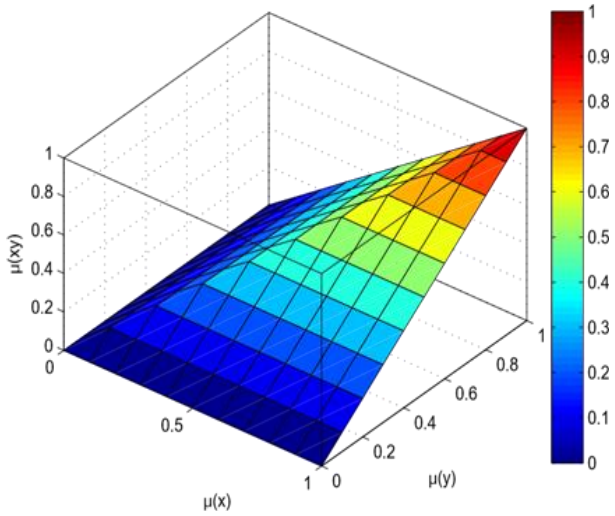

Figure III.1

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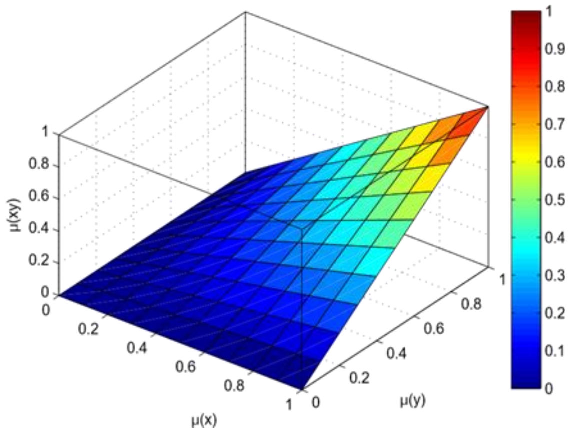

Figure III.2

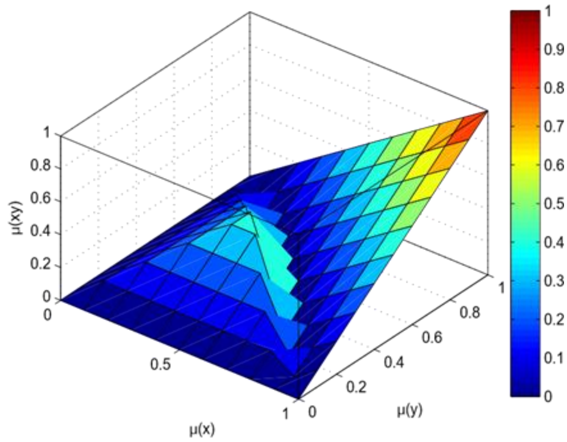

Figure III.3

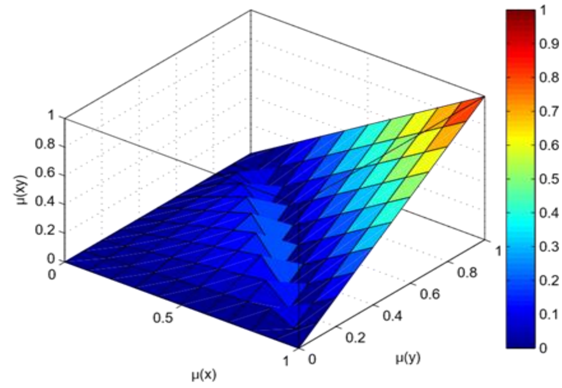

Figure III.4

Figure III.1- Figure III.4: Comparison of proposed definitions:

Figure III.1 $\alpha(mu)=\min\{\alpha(m),\alpha(u)\}$,

Figure III.2 $\alpha(mu)=\alpha(m)\alpha(u)$,

Figure III.3 $\alpha(mu)\geq\begin{cases}\alpha(m)+\alpha(u)-1 & if\ \alpha(m)+\alpha(u)\geq 1\\ \min\{\alpha(m),\alpha(u)\} & if\ \alpha(m)+\alpha(u)<1\end{cases}$,

Figure III.4 $\alpha(mu)\geq\begin{cases}\alpha(m)+\alpha(u)-1 & if\ \alpha(m)+\alpha(u)\geq 1\\ \alpha(m)\alpha(u) & if\ \alpha(m)+\alpha(u)<1\end{cases}$

Let $\beta(mu)\geq\alpha(mu)$. Clearly, Figure III.4 produces more possible $\beta(mu)$ than that of Figure III.1, Figure III.2 as well as Figure III.3. Clearly, our proposed Definition III.2 of FSG is followed by the Definition II.3 and Definition II.4. As a result, the domain of definition becomes vaster and hence our version of FSG becomes more general.

B. Examples

Example III.1 [22] Let $H=\{1,-1,i,-i\}$ be a group with regular multiplication. Let α is defined as $\alpha: H\rightarrow[0,1]$ by

$$\alpha(p)=\begin{cases}1 & if\ p=1\\ 0.5 & if\ p=-1\\ 0 & if\ p=i,-i\end{cases}$$

Evidently α is a FSG and the comparison mentioned in Figure III.1- Figure III.4 is also applicable for this example.

Example III.2 [22] Let $H=S_3$ be the symmetric group of degree 3. Let e represents the neutral element of H. Define $\alpha: H\rightarrow[0,1]$ by

$$\alpha(p)=\begin{cases}1 & if\ p=e\\ 0.5 & if\ p=(123),(132)\\ 0 & otherwise\end{cases}$$

For this example, also α is a FSG and the comparison given in Figure III.1- Figure III.4 is also applicable here.

Example III.3 [22] Consider the infinite group of integers, Z with regular addition. Define $\alpha: Z\rightarrow[0,1]$ by

$$\alpha(p)=\begin{cases}0.9 & if\ p\in 2\mathbb{Z}\\ 0.8 & if\ p\in 2\mathbb{Z}+1\end{cases}$$

Similarly, for this example also α is a FSG and the comparison mentioned in Figure III.1- Figure III.4 is also app -licable.

But there are some examples which fail to become FSG according to **Definition II.3**. For instance:

Example III.4 Consider the Klien's 4-group K_4.

Let $\alpha=\{(e,0.2),(m,0.4),(u,0.3),(mu,0.8)\}$ be a FS of K_4. Notice that,

$$\begin{aligned}\alpha(m\cdot mu) &= \alpha(m^2u)\\ &= \alpha(eu)\\ &= \alpha(u)\\ &= 0.3\\ &\ngeq \min\{0.4,0.8\}\end{aligned}$$

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$$=\min\{\alpha(m),\alpha(mu)\}$$

Hence, α is not a FSG according to **Definition II.3**. But α is a FSG according to **Definition III.2** proposed in this article.

Proposition III.1 Let α be a FSG of H and e be the neutral element of H, then

(i) $\alpha(m^{-1})=\alpha(m)\ \forall m\in H$

(ii) $\alpha(e)\geq\begin{cases}2\alpha(m)-1 & if\ 2\alpha(m)\geq 1\\ \alpha^2(m) & if\ 2\alpha(m)<1\end{cases}$

(iii) $\alpha(e)=1$ if $\alpha(m)=1$ for at-least one $m\in H$.

Theorem III.1 If α is a FSG a group H then

$$\alpha(mu^{-1})\geq\begin{cases}\alpha(m)+\alpha(u)-1 & \text{if } \alpha(m)+\alpha(u)\geq 1\\ \alpha(m)\alpha(u) & \text{if } \alpha(m)+\alpha(u)<1\end{cases}.$$

Again, if $\alpha(e)=1$ with

$$\alpha(mu^{-1})\geq\begin{cases}\alpha(m)+\alpha(u)-1 & \text{if } \alpha(m)+\alpha(u)\geq 1\\ \alpha(m)\alpha(u) & \text{if } \alpha(m)+\alpha(u)<1\end{cases}$$

Then α is a FSG H.

Proof: Let α is a FSG a group H then

$$\alpha(mu^{-1})\geq\begin{cases}\alpha(m)+\alpha(u^{-1})-1 & if\ \alpha(m)+\alpha(u^{-1})\geq 1\\ \alpha(m)\alpha(u^{-1}) & if\ \alpha(m)+\alpha(u^{-1})<1\end{cases}$$

$$\text{or, } \alpha(mu^{-1})\geq\begin{cases}\alpha(m)+\alpha(u)-1 & if\ \alpha(m)+\alpha(u)\geq 1\\ \alpha(m)\alpha(u) & if\ \alpha(m)+\alpha(u)<1\end{cases}$$

[As $\alpha(m^{-1})=\alpha(m)$]

Now let

$$\alpha(e)=1 \text{ with } \alpha(mu^{-1})\geq\begin{cases}\alpha(m)+\alpha(u)-1 & if\ \alpha(m)+\alpha(u)\geq 1\\ \alpha(m)\alpha(u) & if\ \alpha(m)+\alpha(u)<1\end{cases}$$

Replacing m with e we will have

$$\alpha(eu^{-1})\geq\begin{cases}\alpha(e)+\alpha(u)-1 & if\ \alpha(e)+\alpha(u)\geq 1\\ \alpha(e)\alpha(u) & if\ \alpha(e)+\alpha(u)<1\end{cases}$$

$$\Rightarrow\alpha(u^{-1})\geq\begin{cases}1+\alpha(u)-1 & if\ 1+\alpha(u)\geq 1\\ 1.\alpha(u) & if\ 1+\alpha(u)<1\end{cases}$$

$$\Rightarrow\alpha(u^{-1})\geq\alpha(u).$$

Again, $\alpha(u)=\alpha((u^{-1})^{-1})\geq\alpha(u)$ i.e $\alpha(q)=\alpha(q)\ \forall u\in H$

So, $\alpha(mu)=\alpha(m(u^{-1})^{-1})$

$$\geq\begin{cases}\alpha(m)+\alpha(u^{-1})-1 & if\ \alpha(m)+\alpha(u^{-1})\geq 1\\ \alpha(m)\alpha(u^{-1}) & if\ \alpha(m)+\alpha(u^{-1})<1\end{cases}$$

$$\Rightarrow\alpha(mu)\geq\begin{cases}\alpha(m)+\alpha(u)-1 & if\ \alpha(m)+\alpha(u)\geq 1\\ \alpha(m)\alpha(u) & if\ \alpha(m)+\alpha(u)<1\end{cases}.$$

Hence α is a FSG of H.

IV. A REDEFINED NOTATION OF LEVEL SUBGROUP

Let $\forall t\in[0,1]$ α_t be LSs of a FS α of a group H. Evidently, α_t are subsets of H. Here notice that if α is a FSG of H according to the **Definition III.2**, then $\forall m\in H$, we have

$$\alpha(e)=\alpha(mm^{-1})$$

$$\Rightarrow\alpha(e)\geq\begin{cases}\alpha(m)+\alpha(m^{-1})-1 & if\ \alpha(m)+\alpha(m^{-1})\geq 1\\ \alpha(m)\alpha(m^{-1}) & if\ \alpha(m)+\alpha(m^{-1})<1\end{cases}$$

$$\Rightarrow\alpha(e)\geq\begin{cases}\alpha(m)+\alpha(m)-1 & if\ \alpha(m)+\alpha(m)\geq 1\\ \alpha(m)\alpha(m) & if\ \alpha(m)+\alpha(m)<1\end{cases}$$

$$\Rightarrow\alpha(e)\geq\begin{cases}2\alpha(m)-1 & if\ 2\alpha(m)\geq 1\\ \alpha^2(m) & if\ 2\alpha(m)<1\end{cases}$$

Under the light of above, we may redefine the notion of a LS of α as:

Definition IV.1 Let α be any FS of H and let $t\in[0,1]$. Then the set $\alpha_{2t-1}=\begin{cases}2t-1 & if\ 2t\geq 1\\ t^2 & if\ 2t<1\end{cases}$ is termed as a LS of α.

Theorem IV.1 Let α be a FS of a group H such that $\alpha_{2t-1}\lesssim H$ for all $t\in[0,1]$ with $\alpha(e)\geq 2t-1$, if $2t\geq 1$ and $\alpha(e)\geq t^2$ if $2t<1$. Then α is a FSG of H.

Proof: Let H be a group and α be a FS of H. Let α_{2t-1} be the LS of α, $\forall t\in[0,1]$. We also have $\alpha(e)\geq 2t-1$, if $2t\geq 1$ and $\alpha(e)\geq t^2$ if $2t<1$. From supposition $\alpha_{2t-1}\lesssim H$. Let us choose m,u as an arbitrary element of H. Let $t_1,t_2\in[0,1]$.

Case(i): Let $2t_1\geq 1$ and $2t_2\geq 1$ such that $\alpha(m)=2t_1-1$ and $\alpha(u)=2t_2-1$. Which in turn implies that $m\in\alpha_{2t_1-1}$ and $u\in\alpha_{2t_2-1}$. Without losing any generality let $t_1<t_2$. Then from the definition of LS we have $\alpha_{2t_2-1}\subseteq\alpha_{2t_1-1}$. Thus, $m,u\in\alpha_{2t_1-1}$. Since $\alpha_{2t_1-1}\lesssim H$ by hypothesis $mu\in\alpha_{2t_1-1}$.

As $mu\in\alpha_{2t_1-1}$ by the definition of the LS

$$\alpha(mu)\geq 2t_1-1 \tag{4}$$

Now, as $t_2\in[0,1]$, we will have

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$1 \geq 2t_2 - 1 \Rightarrow 2t_1 - 1 + 1 \geq 2t_1 - 1 + 2t_2 - 1$

$$\Rightarrow 2t_1 - 1 \geq 2t_1 - 1 + 2t_2 - 1 - 1 \quad (5)$$

Using Equation 4 we can write Equation 5 as

$$\alpha(mu) \geq 2t_1 - 1 + 2t_2 - 1 - 1 \geq \alpha(m) + \alpha(u) - 1 \quad (6)$$

Here observe that $2 \geq \alpha(m) + \alpha(u) \geq 0$ or

$1 \geq \alpha(m) + \alpha(u) - 1 \geq -1$.

Here, for $1 \geq \alpha(m) + \alpha(u) - 1 \geq 0$ or $\alpha(m) + \alpha(u) \geq 1$ Equation 6 holds.

Also, for $0 > \alpha(m) + \alpha(u) - 1 \geq -1$ or $\alpha(m) + \alpha(u) < 1$,

Here, for $1 \geq \alpha(m) + \alpha(u) - 1 \geq 0$ or $\alpha(m) + \alpha(u) \geq 1$

Equation 6 holds.

Since α is a FS $\alpha(mu^{-1})$ can never be negative.

Again, for $\alpha(m) + \alpha(u) < 1$ from Using Equation 4

$$\alpha(mu^{-1}) \geq 2t_1 - 1 \geq (2t_1 - 1)(2t_2 - 1) = \alpha(m)\alpha(u).$$

So, $\alpha(mu) \geq \begin{cases} \alpha(m) + \alpha(u) - 1 \; if \; \alpha(m) + \alpha(u) \geq 1 \\ \alpha(m)\alpha(u) \quad if \; \alpha(m) + \alpha(u) < 1 \end{cases}$.

<u>Case(ii):</u> Let $2t_1 \geq 1$ and $2t_2 < 1$ such that $\alpha(m) = 2t_1 - 1$ and $\alpha(u) = t_2^2$. Let $2t_1 - 1 \geq t_2^2$ then $\alpha_{2t_1 - 1} \subseteq \alpha_{2t_2 - 1}$.

Let $m, u \in \alpha_{2t_2 - 1}$. As $\alpha_{2t_2 - 1} \lesssim H$, $mu \in \alpha_{2t_2 - 1}$.

$$\therefore \alpha(mu) \geq t_2^2 \quad (7)$$

Now, as $t_1 \in [0,1]$, we will have

$1 \geq 2t_1 - 1 \Rightarrow t_2^2 + 1 \geq 2t_1 - 1 + t_2^2$

$$\Rightarrow t_2^2 \geq 2t_1 - 1 + t_2^2 - 1 \quad (8)$$

Using Equation 7 we can write Equation 8 as

$\alpha(mu) \geq 2t_1 - 1 + t_2^2 - 1$

$$\Rightarrow \alpha(mu) \geq \alpha(m) + \alpha(u) - 1 \quad (9)$$

Here observe that $2 \geq \alpha(m) + \alpha(u) \geq 0$ or

$1 \geq \alpha(m) + \alpha(u) - 1 \geq -1$.

For $1 \geq \alpha(m) + \alpha(u) - 1 \geq 0$ or $\alpha(m) + \alpha(u) \geq 1$ Equation 9 holds.

Again for $0 > \alpha(m) + \alpha(u) - 1 \geq -1$ or $\alpha(m) + \alpha(u) < 1$ For $1 \geq \alpha(m) + \alpha(u) - 1 \geq 0$ or $\alpha(m) + \alpha(u) \geq 1$ Equation 9 hold -s. As α is a FS $\alpha(mu)$ can never be negative.

Again, for $\alpha(m) + \alpha(u) < 1$ from Using Equation 7 we have

$$\alpha(mu) \geq t_2^2 \geq (2t_1 - 1)t_2^2 = \alpha(m)\alpha(u).$$

So,

$$\alpha(mu) \geq \begin{cases} \alpha(m) + \alpha(u) - 1 \; if \; \alpha(m) + \alpha(u) \geq 1 \\ \alpha(m)\alpha(u) \quad if \; \alpha(m) + \alpha(u) < 1 \end{cases} \quad (10)$$

Now, let $t_2^2 \geq 2t_1 - 1$. Then $\alpha_{2t_2 - 1} \subseteq \alpha_{2t_1 - 1}$ and again proceeding like above, same proposition as Equation 10 can be conclu -ded.

<u>Case (iii):</u> Let $2t_1 < 1$ and $2t_2 < 1$ such that $\alpha(m) = t_1^2$ and $\alpha(u) = t_2^2$. Without losing any generality, we may suppose that $t_1 < t_2$. So, $\alpha_{2t_2 - 1} \subseteq \alpha_{2t_1 - 1}$ and hence $m, u \in \alpha_{2t_1 - 1}$. As $\alpha_{2t_1 - 1} \lesssim H$, $mu \in \alpha_{2t_1 - 1}$.

$\therefore \alpha(mu) \geq t_1^2 \geq t_1^2 t_2^2 = \alpha(m)\alpha(u)$, where $\alpha(m) + \alpha(u) < 1$.

From Case (i), Case (ii) and Case (iii) it is evident that $\forall m, u \in H$, the following condition is satisfied

$$\alpha(mu) \geq \begin{cases} \alpha(m) + \alpha(u) - 1 \; if \; \alpha(m) + \alpha(u) \geq 1 \\ \alpha(m)\alpha(u) \quad if \; \alpha(m) + \alpha(u) < 1 \end{cases} \quad (11)$$

Again, let $2t \geq 1$ where $t \in [0, 1]$ and $m \in H$ such that $\alpha(m) = 2t - 1$ which implies that $m \in \alpha_{2t-1}$. As $\alpha_{2t-1} \lesssim H$, $m^{-1} \in \alpha_{2t-1}$.

So, $\alpha(m^{-1}) \geq 2t - 1 = \alpha(m)$.

Similarly, for $2t < 1$ we have $\alpha(m^{-1}) \geq t^2 = \alpha(m)$. From this and Equation 11 it can be concluded that α is a FSG of H.

Under the light of **Definition II.5**, we can assume that $\alpha_{2t-1} \lesssim \alpha$ for all $t \in [0, 1]$ with $\alpha(e) \geq 2t - 1$ if $2t \geq 1$ and $\alpha(e) \geq t^2$ if $2t < 1$.

Definition IV.2 Let α be a FSG of H according to **Definition III.2** then the subgroup α_{2t-1} for all $t \in [0, 1]$ with $\alpha(e) \geq 2t - 1$ if $2t \geq 1$ and $\alpha(e) \geq t^2$ if $2t < 1$ is called LSG of α.

V. CONCLUSION

This paper proposed a more general notion of FSG. To prove it a graphical comparison was given and also some examples were discussed. Based on that some important propositions and results were concluded. Further, redefined notions of LS and LSG were given. Some important theorems related to LSG and FSG were modified according to the proposed generalized notion of FSG. Based on the

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proposed notion of FSG, one can further redefine and generalized most of the other algebraic structures like normal subgroup, ring, ideal, field, linear space, module, etc. So, this newly defined notions may generate more

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