International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075, Volume-8 Issue-8, June, 2019

A New Approach for Evaluation of Volume Integrals by Haar Wavelet Method

K.T. Shivaram, N. Mahesh Kumar, Megha .V. Goudar, S. Gagandeep

Abstract**:** ***This paper presents, Numerical integration rule based on Haar wavelets method are proposed to find volume integrals of various region such as cuboids, tetrahedron, cone, cylinder, ellipsoid, sphere, etc., volume integral region are transformed to standard integrals by linear and non linear transformation method, the advantage of this method gives the efficiency and simple applicability, performances of this method is illustrated with numerical examples***

Index Terms**:** ***Numerical Integration, Haar wavelet method, Volume.***

I. INTRODUCTION

Analytical / numerical integration of functions over three dimensional regions or finding volume of various region often arises in chemical engineering, electromagnetic, field theory, fluid mechanics, biomechanics, bioinformatics. etc. mathematical modeling and computer simulation are applicable for biological system in the form of partial differential equation are to be solved by finite element method, to extract the stiffness matrix in the form of integral equations. In particular they are used for problems arriving in calculation of volume, moment of inertia, center of mass, volume of potholes and other geometric properties of solids. Numerical integration of triple integrals over various region are carryout by many authors, cuboids [Shivaram, 2014, Sarada and Nagaraja, 2015, Fengying Zhou, et.al.2017, tetrahedral region Rathod et.al.
2005, 2007, 2010,Shivaram, 2013, Mamtha and Venkatesh, 2015, Fengying Zhou, et.al.2017, cone , cylinder, ellipsoid, paraboloid [Sarada and Nagaraja, 2015, Fengying Zhou, et.al.2017, Spherical region Shivaram, 2013, Sarada and Nagaraja, 2015, Fengying Zhou, et.al.2017, numerical integration of multiple integrals by using Haar wavelet and hybrid functions are discussed in [Siraj ul Islam et.al., 2010, 2012, Imran Aziz, et.al. 2011, In This paper, we apply the wavelet based integration technique of Haar wavelet method over various region, this method is more accurate and easy to implement for variety of problems arising in science and engineering, the necessary computer program has been developed in MAPLE

The paper is organized as follows. In Section 2. mathematical preliminaries required for understanding the derivation, In Section 3. by using transformation method to convert the volume integral into standard integrals, In Section 4. We compare the numerical results with exact value.

Revised Manuscript Received on May 28, 2019.

K.T.Shivaram, Department of Mathematics, Dayananda Sagar College of Engineering, Bangalore, India

N. Mahesh Kumar, Department of Digital Electronics & Communication, Dayananda Sagar College of Engineering, Bangalore, India

Megha .V. Goudar and S. Gagandeep, Department of Digital Electronics & Communication, Dayananda Sagar College of Engineering, Bangalore, India

.

II. MATHEMATICAL PRELIMINARIES

A. Haar Wavelets method

The explicit form of the function $H_{jk}(x)$ is defined as

$$H_{jk}(x) = \begin{cases} 1, & if \ x \in [a_{jk}, \frac{a_{jk}+b_{jk}}{2}) \\ -1, & if \ x \in [\frac{a_{jk}+b_{jk}}{2}, b_{jk}) \\ 0, & otherwise \end{cases} \quad (1)$$

where $j \geq 0$, $k = 0,1,2,---,2^j - 1$

$a_{jk} = \frac{k}{2^j}$ and $b_{jk} = \frac{k+1}{2^j}$

Using the orthogonal basis of $L^2([0,1))$ the Haar wavelet function $H_{jk}(x)$ can be expressed by Haar series function $f(x)$ of infinite terms as

$$\int_a^b f(x)\,dx = \frac{(b-a)}{2^{n+1}} \sum_{i=1}^{2^{n+1}} f\left(a + \frac{(b-a)(2i-1)}{2^{n+2}}\right)$$

$$= \frac{(b-a)}{2M} \sum_{i=1}^{2M} f\left(a + \frac{(b-a)(i-0.5)}{2M}\right)$$

Where $M = 2^n$

For triple integral

$$\int_{a_1}^{a_2}\int_{a_3}^{a_4}\int_{a_5}^{a_6} f(x_1, x_2, x_3)\,dx_1 dx_2 dx_3 = ,$$

$$\frac{(a_2-a_1)(a_4-a_3)(a_6-a_5)}{8M^3} \sum_{i_1=1}^{2M} \sum_{i_2=1}^{2M} \sum_{i_3=1}^{2M} f(A, B, C)$$

(2)

where $A = a_1 + \frac{(a_2-a_1)(i_1-0.5)}{2M}$, $B = a_3 + \frac{(a_4-a_3)(i_2-0.5)}{2M}$,

$C = a_5 + \frac{(a_6-a_5)(i_3-0.5)}{2M}$

we shall be using these formula to evaluate the volume integral by Haar wavelet method

B. Volume Integral over xyz – plane

In this section is devoted to the numerical integration of arbitrary function over sphere, cylinder, cuboids, cone, ellipsoid,tetrahedral region is of the form

Region

$R = \{(x, y, z)|\ a \leq x \leq b, f_1(x) \leq y \leq f_2(x), g_1(x, y) \leq z \leq g_2(x, y)\}$

` having linear or non linear faces are plotted in figure.1

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

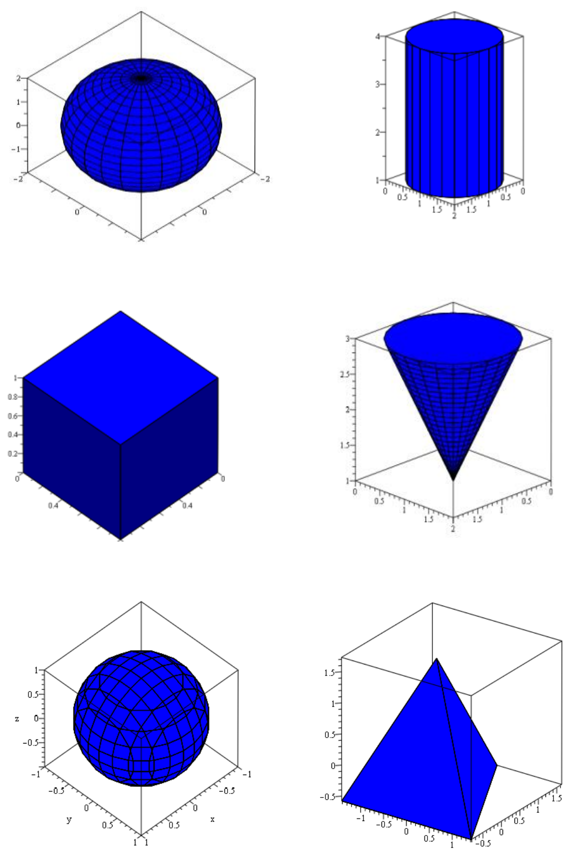

Fig.1: Three dimensional boundary regions

III. NUMERICAL INTEGRATION OVER THREE DIMENSIONAL REGION

A. Numerical Integration over sphere

Consider the volume integral of arbitrary function $f(x,y,z)$ over sphere $x^2+y^2+z^2 \leq 1$

$$I=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} f(x,y,z)\ dzdydx \qquad (3)$$

The volume integral of Eq.(3) can be transformed to standard integration form by assuming

$x=\alpha\cos\beta,\ y=\alpha\sin\beta\cos\gamma,\ z=\alpha\sin\beta\sin\gamma$

The Jacobian of the transformation is $J=\frac{\partial(x,y,z)}{\partial(\alpha,\beta,\gamma)}=\alpha^2\sin\beta$

Hence, eq. (3) becomes

$$I=\int_0^a\int_0^{\pi}\int_0^{2\pi}\alpha^2\sin\beta\ \ d\alpha d\beta d\gamma \qquad (4)$$

We approximate the integral Eq.(4) by applying Haar wavelet method is described in the table.1, and compare the obtained results with exact value

Table : 1

Exact value	Order N	Computed value	Error
$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dzdydx$ = 4.18879020478638	N=10	4.19047904594451	0.001688841
	N=100	4.18880708916149	1.68844 E-05
	N=200	4.18879442587249	4.22109 E-06
$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}dxdydz$ = 3.14159265358980	N=10	3.14089376895842	0.000698885
	N=100	3.14158568171523	6.97187 E-06
	N=200	3.14159091065343	1.74294 E-06
$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}}(x^2+y^2)$ = 1.24688523775579	N=10	1.24771857423142	0.000833336
	N=100	1.24688366805113	1.5697E -06
	N=200	1.24688520553791	3.22179 E-08

B. Numerical Integration over cylinder

Consider the volume integral of arbitrary function $f(x,y,z)$ over cylinder with base $y^2+z^2=a^2$ and height h along the X - direction

$$I=\int_0^h\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} f(x,y,z)\ dzdydx \qquad (5)$$

The volume integral of Eq.(5) can be transformed to standard integration form by assuming

$x=h\,\alpha,\ y=a\,\beta\cos(2\pi\gamma),\ z=a\beta\sin(2\pi\gamma),\ \ J=2\pi a^2 h\beta$

hence, eq. (5) becomes

$$I=2\pi a^2 h\int_0^1\int_0^1\int_0^1 f(h\,\alpha, a\,\beta\cos(2\pi\gamma), a\beta\sin(2\pi\gamma))\ \beta\ d\alpha d\beta d\gamma$$

(6)

We approximate the integral Eq.(6) by applying Haar wavelet method is described in the table.2, and compare the obtained results with exact value

Table: 2

Exact value	Order N	Computed value	Error

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$\int_0^3 \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (y^2+z^2)e^{\frac{x}{z}}$ $= 8.09721235362566$	N=10	8.08624849 430927	0.010963 859
	N=100	8.09710270 398664	0.000109 65
	N=200	8.09718494 119490	2.74124E -05
$\int_0^3 \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x\sqrt{y^2+z^2}$ dzdydx $= 9.42477796076938$	N=10	9.40121601 586743	1.304003 662
	N=100	9.42471905 590711	1.327506 702
	N=200	9.42476323 455381	1.327550 881
$\int_0^4 \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \sin x\ (y^2+z$ $= 2.59753732548037$	N=10	2.60186071 852135	0.004323 393
	N=100	2.59754814 851649	1.0823E- 05
	N=200	2.59754003 124615	2.70577E -06

C. NUMERICAL INTEGRATION OVER CUBOIDS

Consider the volume integral of arbitrary function $f(x,y,z)$ over cube $a \le x \le b$,
$c \le y \le d, \quad e \le z \le f$

$$I = \int_a^b \int_c^d \int_e^f f(x,y,z)\, dzdydx \qquad (7)$$

We approximate the integral Eq.(7) by applying Haar wavelet method is described in the table.3, and compare the obtained results with exact value

Table: 3

Exact value	Order N	Computed value	Error
$\int_1^2 \int_3^4 \int_5^6 \sqrt{(x+y+z)}\, dzdydx$ $= 3.23945017707172$	N=10	3.239452483 07745	2.3E-06
	N=100	3.239450200 13191	2.3E-08
	N=200	3.239450182 83688	5.8E-09
$\int_0^\pi \int_0^\pi \int_0^{\pi/2} \cos(x+y+$ $= -4$	N=10	-4. 00926521275 107	0.009265 2127510 7
	N=100	-4. 00009252878 604	0.000092 5287860 4
	N=200	-4. 00002313196 620	0.000023 1319662 0
$\int_0^1 \int_0^1 \int_0^1 \frac{8}{1+2(x+y+z)}$ dzdydx $= 2.15214283259589$	N=10	2.151626716 74065	0.000516 116
	N=100	2.152137664 74776	5.16785E -06
	N=200	2.152141540 62114	1.29197E -06

D. NUMERICAL INTEGRATION OVER CONE

Consider the volume integral of arbitrary function $f(x,y,z)$ over cone $z=\sqrt{x^2+y^2}$ lies between $z=0$ and $z=a$ is given by

$$I = \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{\sqrt{x^2+y^2}}^{a} f(x,y,z)\, dzdydx \qquad (8)$$

The volume integral of Eq.(8) can be transformed to standard integration form by assuming
$x = a\,\alpha\cos(2\pi\beta),\ y = a\,\alpha\sin(2\pi\beta),\ z = a(1-\alpha)\gamma + a\,\alpha$,
$J = 2\pi a^3\,\alpha(1-\alpha)$
hence, Eq. (8) becomes

$$I = \int_0^1 \int_0^1 \int_0^1 f(a\,\alpha\cos(2\pi\beta)\ a\,\alpha\sin(2\pi\beta), a(1-\alpha)\gamma + a\,\alpha)\ 2\pi a^3\,\alpha(1-\alpha)\, d\alpha d\beta d\gamma \qquad (9)$$

We approximate the integral Eq.(9) by applying Haar wavelet method is described in the table.4, and compare the obtained results with exact value

Table: 4

Exact value	Order N	Computed value	Error
$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} 1\ dzdydx$ $= 1.0471975511966$	N=10	1.0485065 4813559	0.00130 8997
	N=100	1.0472106 4116599	1.309E- 05
	N=200	1.0472008 2368894	3.27249 E-06
	N=10	0.2565606 15630912	0.00030 933
	N=100	0.2562543 79246485	3.09412 E-06

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} \frac{z\ln(1+x^2+y^2+z^2)}{1+x^2+y^2+z^2}dzdydx$ = 0.256251285130943	N=200	0.256252058661300	7.7353E-07
$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} \sqrt{x^2+y^2}dzdydx$ =0.523598775598299	N=10	0.524253274067797	0.000654498
	N=100	0.523605320582994	6.54498E-06
	N=200	0.523600411844472	1.63625E-06

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \sin(x^2+y$ = 2.28847389189686		06763277	68824
	N=100	2.28846124417661	1.26477E-05
	N=200	2.28847073004351	3.16185E-06
$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \frac{xyz}{\sqrt{x^2+y^2+z^2}}dzdydx$ = 0	N=10	0	0
	N=100	0	0
	N=200	0	0

E. NUMERICAL INTEGRATION OVER ELLIPSOID

Consider the volume integral of arbitrary function $f(x,y,z)$ over ellipsoid $\{(x,y,z)|\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$ is given by

$$I=\int_{-a}^{a}\int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}}\int_{-c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}^{c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}} f(x,y,z)\,dzdydx \qquad (10)$$

The volume integral of Eq.(10) can be transformed to standard integration form by assuming

$x=a\alpha\sin\beta\cos\gamma$, $y=b\,\alpha\sin\beta\sin\gamma, z=c\alpha\sin\beta$,
$J=abc\,\alpha^2\sin\beta$

hence, eq. (10) becomes

$$I=\int_0^1\int_0^1\int_0^1 f(a\alpha\sin\beta\cos\gamma,\ b\,\alpha\sin\beta\sin\gamma,\ c\alpha\sin\beta)\ abc\,\alpha^2\sin\beta\ d\alpha d\beta d\gamma$$

(11)

We approximate the integral Eq.(11) by applying Haar wavelet method is described in the table.5, and compare the obtained results with exact value

Table: 5

Exact value	Order N	Computed value	Error
$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} (x^2+y^2+$ =2.51327412287183	N=10	2.51062075840151	0.002653364
	N=100	2.51324760143495	2.65214E-05
	N=200	2.51326749253596	6.63034E-06
	N=10	2.287205	0.0012

F. NUMERICAL INTEGRATION OVER TETRAHEDRAL REGION

Consider the volume integral of arbitrary function $f(x,y,z)$ over tetrahedral regio $\{(x,y,z)|0\le x,y,z\le a, x+y+z\le a\}$ is given by

$$I=\int_0^a\int_0^{a-x}\int_0^{a-x-y} f(x,y,z)\,dzdydx \qquad (12)$$

Three dimensional linear tetrahedron region bounded by a tetrahedral surface S is given by

$$I=\int_0^a\int_0^{a-x}\int_0^{a-x-y} f(x,y,z)\,dzdydx=\frac{|\det J|}{\det J}\iint_K [A(u,v)-B(u,v)-C(u,v)-D(u,v)]dudv$$

(13)

Where K is the triangle bounded by $\{(0,0),(a,0),(0,a)\}$ in the $uv-$ plane, volume integral is convert to surface integral is given in the ref[6

The surface integral of Eq.(13) can be transformed to standard integration form by assuming

$x=\alpha$, $y=(1-\alpha)\beta$, $J=1-\alpha$

hence, eq. (13) becomes

$$I=\int_0^1\int_0^1 f(\alpha,\ (1-\alpha)\beta)(1-\alpha)\,d\alpha d\beta \qquad (14)$$

We approximate the integral Eq.(13) by applying Haar wavelet method is described in the table.6, and compare the obtained results with exact value

Table: 6

Exact value	Order N	Computed value	Error
	N=10	0.0207878678613931	4.54655E-05

$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{(1+x+y+z)^4} dzdydx$	N=100	0.0208315105 892720	1.82274E-06
$= \int_0^1 \int_0^{1-x} [\frac{1}{3(1+x+y)^3} - \frac{1}{24}]$ =0.0208333333333333	N=200	0.0208328776 149558	4.55718E-07
$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \sin(x+$ =	N=10	0.1319058925 87466	3.5657E-06
$\int_0^1 \int_0^{1-x} \frac{1}{4}[\cos(x+2y)$	N=100	0.1319023631 65432	3.62752E-08
= 0.131902326890182	N=200	0.1319023359 60108	9.06993E-09
$\int_0^1 \int_0^{1-x} \int_0^{1-x-y}$	N=10	0.1428063854 90012	5.07574E-05
$\sqrt{x+y+z}dzdydx$ =	N=100	0.1428566368 54196	5.06003E-07
$\int_0^1 \int_0^{1-x} \frac{2}{3}[1-(x+y)^{\frac{3}{2}}$ =0.142857142857143	N=200	0.1428570163 64584	1.26493E-07

IV. CONCLUSIONS

In this paper, Wavelet based integration method is applied for numerical integration of arbitrary function over three dimensional region like cuboids, cone, cylinder, tetrahedron, ellipsoid, sphere , it converges to the exact value of the volume integral, sufficient value of order N for this proposed method

REFERENCES

1. O.C. Zienkiewicz, The Finite element method, McGraw Hill London, 3rd Edn. 1977
2. K.T. Shivaram, Generalized Gaussian quadrature over an arbitrary cube in Euclidean three-dimensional space, International journal of current engineering and technology, 4, 2014, 1712-1714.
3. Sarada Jayan, K.V.Nagaraja, Numerical Integration over N- dimensional cubes using Generalized Gaussian quadrature, Proceedings of the Jahjeon mathematical society, 17, 2014, pp.63-69.
4. Fengying Zhou, Xiaoyong Xu, Xijing Zhang, Numerical integration method for triple integrals using the second kind chebyshev wavelets method, Journal of Computational and Applied Mathematics, 5, 1-26doi.org/10.1007/s40314-017-0494-1
5. H.T. Rathod, B. Venkatesudu, Gauss Legendre quadrature formulae for tetrahedra. Int J Comput Methods. Eng. Sci. Mech. 6, 2005, pp. 179–186
6. H.T.Rathod , Venkatesudu .B, Nagaraja, On the application of two Gauss-Legendre quadrature rules for composite numerical integration over a tetrahedral region. Appl. Math. Comput., 190,2007, pp. 21–39
7. H.T. Rathod , Nagaraja K.V, Venkatesudu, Numerical, integration of some functions over an arbitrary linear tetrahedra in Euclidean three-dimensional space. Appl. Math. Comput. 191, 2007, pp.397– 409
8. H.T. Rathod, B. Venkatesudu, Gauss Legendre quadrature formulas over a tetrahedron. Numer. Meth. Part D E 22, 2010, pp. 197–219
9. K.T. Shivaram, Generalized Gaussian Quadrature Rules over An arbitrary Tetrahedron in Euclidean Three- Dimensional Space' International Journal of Applied Engineering Research, 8,2013, 1533-1538
10. T.M. Mamatha , Venkatesh, Gauss quadrature rules for numerical integration over a standard tetrahedral element by decomposing into hexahedral elements. Appl. Math. Comput. 271, 2015, pp.1062–1070
11. S. Jayan , Nagaraja, A general and effective numerical integration method to evaluate triple integrals using generalized Gaussian quadrature. Procedia Eng. 127, 2015, pp.1041–1047
12. K.T. Shivaram, Generalised Gaussian quadrature over a sphere. Int.J. Sci. Eng. Res. 4, 2013, pp.1530–1534
13. Sarada Jayan, K.V. Nagaraja(2015),, Numerical integration over Three –Dimensional regions Bounded by one or more circular edges, Procedia Engineering 127, 347 – 353
14. Sarada Jayan, K.V. Nagaraja, A General and Effective numerical integration method to evaluate triple integrals using generalized Gaussian quadrature, Procedia Engineering 127, 2015, pp. 1041 - 1047
15. Imran Aziz , Siraj-ul-Islam, Wajid Khan, A quadrature rule for numerical integration based on Haar wavelets and hybrid functions, Journal of Computers and Mathematics with Applications, 61, 2011, pp. 2770-2781
16. Siraj ul Islam, Imran Aziz, Fazal Haq, A comparative study of numerical integration based on haar wavelets and hybrid functions, Comput. Math. Appl., 59, 2010, pp. 2026-2036
17. Siraj ul Islam, Imran Aziz, Wajid Khan, Numerical integration of multidimensional highly oscillatory, gentle oscillatory and non-oscillatory integrands based on wavelets and radial basis functions, Journal of Engineering analysis with Boundary elements, 36,2012, pp. 1284-1295
18. K.T.Shivaram, Numerical Integration of Highly Oscillating Functions using Quadrature Method, Global Journal of Pure and applied Mathematics, 12, 2016, pp. 2683–2690