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Ordered Neutrosophic Fuzzy Convergence Bitopological Spaces

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***Abstract:** In this paper we introduce the new concept of an ordered neutrosophic fuzzy convergence bitopo-logical spaces ($\mathcal{ONF}\mathbb{CONVG}$BTS). Besides giving some interesting propositions. Tietze extension theorem for pairwise ordered NFlim extremally disconnected space($\mathcal{PON}$Flim$\mathcal{EDS}$) is established.*

***Keywords:** $\mathcal{ONF}\mathbb{CONVG}$BTS, $\mathcal{PONFLIMEDS}$, ordered τ_{lim_i} NFlimCF, lower (resp. upper) τ_{lim_i} NFlimCF, (i =1 or 2).*

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [12]. Fuzzy sets have applications in many fields such as information [8] and control [9]. The concept of an ordered L-fuzzy bitopological spaces was introduced and studied in [2]. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy set, vague fuzzy set, interval-valued fuzzy sets, etc.

Tomasz Kubiaz[10,11] extended the Urysohn lemma and Tietze extension theorem for the L-fuzzy normal spaces. After the introduction of intuitionistic fuzzy sets and its topological spaces by Atanassov[1] and Coker[3], the concept of imprecise data called neutrosophic sets was introduced by Smarandache[4].

Later A. A. Salama [6,7] studied the neutrosophic topological space and neutrosophic filters. The concepts of separation axiom in an ordered neutrosophic bitopological space was studied by R.Narmada Devi[5].

In this paper, the concept of an ordered *NF* convergence bitopological space and pairwise ordered *NFlim* extremally disconnected spaces are introduced and studied. Some interesting propositions are discussed. Tietze extension theorem for pairwise ordered *NFlim* extremally disconnected spaces has been established.

II. PRELIMINARIES

Definition 2.1. [4] Let X be a nonempty set. A neutrosophic set A in X is defined as an object of the form $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$ such that $T_A, I_A, F_A : X \to [0,1]$. and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.2. [4] Let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be any two neutrosophic fuzzy sets in X. Then

(i) $A \cup B = \langle x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)\rangle$ where $T_{A\cup B}(x) = T_A(x) \vee T_B(x)$, $I_{A\cup B}(x) = I_A(x) \vee I_B(x)$ and $F_{A\cup B}(x) = F_A(x) \wedge F_B(x)$.

(ii) $A \cap B = \langle x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)\rangle$ where $T_{A\cap B}(x) = T_A(x) \wedge T_B(x)$, $I_{A\cap B}(x) = I_A(x) \wedge I_B(x)$ and $F_{A\cap B}(x) = F_A(x) \vee F_B(x)$.

(iii) $A \subseteq B$ if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$ and $F_A(x) \ge F_B(x)$, for all $x \in X$.

(iv) The complement of A is defined as $C(A) = \langle x, T_{C(A)}(x), I_{C(A)}(x), F_{C(A)}(x)\rangle$ where $T_{C(A)}(x) = 1 - T_A(x)$, $I_{C(A)}(x) = 1 - I_A(x)$ and $F_{C(A)}(x) = 1 - F_A(x)$.

(v) $0_N = \{\langle x,0,0,1\rangle : x \in X\}$ and $1_N = \{\langle x,1,1,0\rangle : x \in X\}$.

III. VIEW ON NF CONVERGENCE BITOPOLOGICAL SPACES

Notation 3.1.

(i) Y : nonempty set

(ii) $\mathfrak{F}, \mathfrak{G}$: neutrosophic fuzzy filters on Y

(iii) $\mathbb{U}(Y)$: set of all neutrosophic fuzzy ultrafilters on X.

(iv) $\mathbb{NF}_p(Y)$: set of all neutrosophic fuzzy prime filters on X.

(v) $\mathbb{NF}_m(\mathfrak{F})$: set of all minimal prime neutrosophic fuzzy filters finer than $\mathfrak{F}$.

(vi) ζ^Y : set of all neutrosophic fuzzy sets on Y.

(vii) $lim\mathfrak{F}$, $lim\mathfrak{G}$, $lim\mathfrak{H}$, $lim\mathfrak{M}$, $lim\mathfrak{K}$, $lim\mathfrak{L}$: neutrosophic fuzzy sets on Y.

(viii) $\zeta^{\mathbb{R}}$: set of all neutrosophic fuzzy sets on the real line $\mathbb{R}$.

(ix) $\mathbb{NF}(Y)$: set of all neutrosophic fuzzy filters on Y.

Definition 3.1. A nonempty collection $\mathfrak{F}$ of elements in the lattice ζ^Y is called a neutrosophic fuzzy filter on Y provided

(i) $0_N \notin \mathfrak{F}$,

(ii) $D, E \in \mathfrak{F}$ implies $D \cap E \in \mathfrak{F}$,

(iii) If $D \in \mathfrak{F}$ and $E \in \zeta^Y$ with $D \subseteq E$, then $D \in \mathfrak{F}$.

Definition 3.2. A neutrosophic fuzzy filter $\mathfrak{F}$ is said to be neutrosophic fuzzy prime filter (or) prime neutrosophic fuzzy filter whenever, $L \cup M \in \mathfrak{F}$ implies $L \in \mathfrak{F}$ or $M \in \mathfrak{F}$.

Definition 3.3. A base for a neutrosophic fuzzy filter is a nonempty subset $\mathfrak{B}$ of ζ^Y obeying (i) $0_N \notin \mathfrak{B}$, (ii) $L, M \in \mathfrak{B}$ implies $L \cap M \supseteq H$, for some $H \in \mathfrak{B}$.

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The neutrosophic fuzzy filter generated by $\mathscr{B}$ is denoted by $[\mathscr{B}] = \{ B \in \zeta^Y : D \supseteq E, D \in \mathscr{B} \}$.

Definition 3.4. Let *H* be any neutrosophic fuzzy set on *Y*. The characteristic $*$ set of $\mathfrak{F} \in \mathbb{NF}(Y)$ is defined by $c(\mathfrak{F}) = \cup_{H\in\mathfrak{F}} H$. Moreover, $c(\mathfrak{F}) = c(f\,\mathfrak{F})$.

Definition 3.5. Let $x_{r,p,s}$ be any *NFP*. A *NF* characteristic function of $x_{r,p,s}$ is denoted by

$$1_{x_{r,p,s}} = \langle x, T_{x_{r,p,s}}, I_{x_{r,p,s}}, F_{x_{r,p,s}} \rangle .$$

Definition 3.6. Any *NFS* $\alpha 1_{x_{r,p,s}}$ is of the form

$\alpha 1_{x_{r,p,s}} = \langle y, \alpha T_{x_{r,p,s}}(y), \alpha I_{x_{r,p,s}}(y), \alpha F_{x_{r,p,s}}(y) \rangle$, where $r + p + s \leq 3$ and $\alpha \in (0, 1)$.

Notation 3.2. The collection of all neutrosophic fuzzy set of the form $\alpha 1_{x_{r,p,s}}$ is denoted by $\mathfrak{C}$.

Definition 3.7. Any *NF* filter generated by $\mathfrak{C}$ is denoted and defined by $[\mathfrak{C}] = \{ B \in \zeta^X : B \supseteq A, A \in \mathfrak{C} \}$, however $[\mathfrak{C}]$ is written as $\alpha 1_{x_{r,p,s}}$.

Definition 3.8. Given a nonempty set *X*, the pair (*X*, *lim*) is said to be a neutrosophic fuzzy convergence space (*NF𝒞𝒪𝒩𝒱𝒢𝒮*) where $lim : \mathbb{NF}(X) \to \zeta^X$ provided:

(i) For every $\mathfrak{F} \in \mathbb{NF}(X)$, $lim\mathfrak{F} = \cap_{\mathfrak{G}\in\mathbb{NF}_m(\mathfrak{F})} lim\mathfrak{G}$,

(ii) For every $\mathfrak{F} \in \mathbb{NF}_p(X)$, $lim\mathfrak{F} \subseteq c(\mathfrak{F})$,

(iii) For every $\mathfrak{F},\mathfrak{G} \in \mathbb{NF}_p(X)$, $\mathfrak{F} \subseteq \mathfrak{G} \Rightarrow lim\mathfrak{G} \subseteq lim\mathfrak{F}$.

(iv) For every $\alpha \in (0,1)$ and $\alpha 1_{x_{r,p,s}} \in \mathfrak{C}$,

$$lim(1_{x_{r,p,s}}\alpha) \supseteq \alpha 1_{x_{r,p,s}} .$$

Definition 3.9. Let (*X*, *lim*) be a *NF𝒞𝒪𝒩𝒱𝒢𝒮*. An operator $NFint : \zeta^X \to \zeta^X$ is defined by

$NFint(lim\mathfrak{F}) = \cup\{lim\mathfrak{G} : lim\mathfrak{G} \subseteq lim\mathfrak{F}$, for every $\mathfrak{G} \in \mathbb{U}(X)$ and $\mathfrak{F} \in \mathbb{NF}(X)\}$.

Definition 3.10. Let (*X*, *lim*) be any *NF𝒞𝒪𝒩𝒱𝒢𝒮*. Then *NFlim* topology is defined by

$\tau_{\lim} = \{lim\mathfrak{F} \,/\, NFint(lim\mathfrak{F}) = lim(\mathfrak{F})$, for every $\mathfrak{F} \in \mathbb{NF}(X)\}$.

The pair $(X,\tau_{\lim})$ is said be a neutrosophic fuzzy convergence topological space(*NF𝒞𝒪𝒩𝒱𝒢𝒯𝒮*). Every member of *NFlim* topology is a neutrosophic fuzzy *lim* open set(*NFlimOS*).

The complement of a neutrosophic fuzzy *lim* open set is a neutrosophic fuzzy *lim* closed set(*NFlimCS*).

Definition 3.11. Let (X,τ_{lim}) be any *NF𝒞𝒪𝒩𝒱𝒢𝒯𝒮* and $lim\mathfrak{F} \in \zeta^X$. Then the *NFlim* closure and *NFlim* interior of $lim\mathfrak{F}$ are denoted and defined by

(i) $NFlimcl(lim\mathfrak{F}) = \cap\{lim\mathfrak{G}: lim\mathfrak{G}$ is a *NFlimCS* in *X* and $lim\mathfrak{F} \subseteq lim\mathfrak{G}\}$,

(ii) $NFlimint(lim\mathfrak{F}) = \cup\{lim\mathfrak{G} : lim\mathfrak{G}$ is a *NFlimOS* in *X* and $lim\mathfrak{G} \subseteq lim\mathfrak{F}\}$, $\forall\, \mathfrak{G}, \mathfrak{F} \in \mathbb{NF}(X)$.

Definition 3.12. An ordered neutrosophic fuzzy convergence topological space (*𝒪NFℂ𝕆ℕ𝕍𝔾TS*) is a triple $(X, \tau_{\lim}, \leq)$ where τ_{lim} is a *NFlim* topology on *X* equipped with a partial order $\leq$.

Definition 3.13. Let $(X, \tau_{lim}, \leq)$ be an *𝒪NFℂ𝕆ℕ𝕍𝔾TS*. Then a neutrosophic fuzzy set $lim\mathfrak{F}$ in $(X, \tau_{lim}, \leq)$ is said to be an

(i) increasing neutrosophic fuzzy set (*increasing NFS*) if $x \leq y$ implies $lim\mathfrak{F}(x) \subseteq lim\mathfrak{F}(y)$.
   That is, $T_{lim\mathfrak{F}}(x) \leq T_{lim\mathfrak{F}}(y)$, $I_{lim\mathfrak{F}}(x) \leq I_{lim\mathfrak{F}}(y)$ and $F_{lim\mathfrak{F}}(x) \geq F_{lim\mathfrak{F}}(y)$.

(ii) decreasing neutrosophic fuzzy set (decreasing NFS) if $x \leq y$ implies $lim\mathfrak{F}(x) \supseteq lim\mathfrak{F}(y)$.
   That is, $T_{lim\mathfrak{F}}(x) \geq T_{lim\mathfrak{F}}(y)$, $I_{lim\mathfrak{F}}(x) \geq I_{lim\mathfrak{F}}(y)$ and $F_{lim\mathfrak{F}}(x) \leq F_{lim\mathfrak{F}}(y)$.

Definition 3.14. Let $(X, \tau_{lim}, \leq)$ be an *𝒪NFℂ𝕆ℕ𝕍𝔾TS* and $lim\mathfrak{F}$ be any *NFS* in $(X, \tau_{lim}, \leq)$.

Then we define the increasing-*NF* closure, decreasing-*NFS* closure, increasing-*NFS* interior and decreasing-*NFS* interior of $lim\mathfrak{F}$ respectively as follows:

(i) $IncrNFcl(lim\mathfrak{F}) = \cap\, \{lim\mathfrak{G} : lim\mathfrak{G}$ is an *increasingNFlimCS* in *X* and $lim\mathfrak{F} \subseteq lim\mathfrak{G}\}$,

(ii) $DecrNFcl(lim\mathfrak{F}) = \cap\, \{lim\mathfrak{G} : lim\mathfrak{G}$ is a *decreasingNFlimCS* in *X* and $lim\mathfrak{F} \subseteq lim\mathfrak{G}\}$,

(iii) $IncrNFint(lim\mathfrak{F}) = \cup\, \{ lim\mathfrak{G} : lim\mathfrak{G}$ is an *increasingNFlimOS* in *X* and $lim\mathfrak{F} \subseteq lim\mathfrak{G}\}$,

(iv) $DecrNFint(lim\mathfrak{F}) = \cup\, \{ lim\mathfrak{G} : lim\mathfrak{G}$ is an *decreasingNFlimOS* in *X* and $lim\mathfrak{G} \subseteq lim\mathfrak{F}\}$,

for every $\mathfrak{F}, \mathfrak{G} \in \mathbb{NF}(X)$.

Proposition 3.1. For any neutrosophic fuzzy set $lim\mathfrak{F}$ in an *𝒪NFℂ𝕆ℕ𝕍𝔾TS*, the following statements holds

(i) $C(IncrNFcl(lim\mathfrak{F})) = DecrNFint(C(lim\mathfrak{F}))$,

(ii) $C(DecrNFcl(lim\mathfrak{F})) = IncrNFint(C(lim\mathfrak{F}))$,

(iii) $C(IncrNFint(lim\mathfrak{F})) = DecrNFcl(C(lim\mathfrak{F}))$,

(iv) $C(DecrNFint(lim\mathfrak{F})) = IncrNFcl(C(lim\mathfrak{F}))$.

Proof:

We shall prove (i) only, (ii),(iii) and (iv) can be proved in a similar manner.

(i) Let $lim\mathfrak{F}$ be any neutrosophic fuzzy set in *X*. Then $IncrNFcl(lim\mathfrak{F}) = \cap\, \{ lim\mathfrak{G} : lim\mathfrak{G}$ is an *increasingNFlimCS* in *X* and $lim\mathfrak{F} \subseteq lim\mathfrak{G}\}$, for every $\mathfrak{G}, \mathfrak{F} \in \mathbb{NF}(X)$.

Taking complement on both sides, we have $C(IncrNFcl(lim\mathfrak{F})) = \cup\, \{C(lim\mathfrak{G}) : C(lim\mathfrak{G})$ is a *decreasingNFlimOS* in *X* and $C(lim\mathfrak{G}) \subseteq C(lim\mathfrak{F})\}$

$= DecrNFint(C(lim\mathfrak{F}))$, for every $\mathfrak{G}, \mathfrak{F} \in \mathbb{NF}(X)$.

Definition 3.15. An ordered neutrosophic fuzzy convergence bitopological space (*𝒪NFℂ𝕆ℕ𝕍𝔾BTS*)

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is an *OrderedNF*$\mathbb{CONVG}$*TS* $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ where τ_{lim_1} and τ_{lim_2} are the *NFlim* topologies on *X* equipped with a partial order $\leq$.

IV. CHARACTERIZATION OF $\mathcal{PONF}lim\mathcal{EDS}$

Definition 4.1. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ be an $\mathcal{O}NF\mathbb{CONVG}BTS$. Let $lim\mathfrak{F}$ be any τ_{lim_1} increasing resp. decreasing) *NFlimOS* in *X*. If $IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})$ (resp. $DecrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})$) is τ_{lim_2} increasing (resp. decreasing) *NFlimOS*.

Then $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is said to be τ_{lim_1} upper (resp. lower) *NFlim* extremally disconnected space. Similarly we can define τ_{lim_2} upper (resp. lower) *NFlim* extremally disconnected space.

Definition 4.2. An $\mathcal{O}NF\mathbb{CONVG}BTS$ $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is said to be pairwise upper *NFlim* extremally disconnected ($\mathcal{PU}pperNFlim\mathcal{EDS}$) if it is both τ_{lim_1} upper *NFlim* extremally disconnected and τ_{lim_2} upper *NFlim* extremally disconnected.

Similarly we can define the pairwise lower *NFlim* extremally disconnected space($\mathcal{PL}owerNFlim\mathcal{EDS}$).

Definition 4.3. An $\mathcal{O}NF\mathbb{CONVG}BTS$ $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is said to be pairwise ordered *NFlim* extremally disconnected ($\mathcal{PONF}lim\mathcal{EDS}$)if it is both $\mathcal{PU}pperNFlim\mathcal{EDS}$ and $\mathcal{PL}owerNFlim\mathcal{EDS}$.

Proposition 4.1. For $\mathcal{O}NF\mathbb{CONVG}BTS$ $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$, the following statements are equivalent:

(i) $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is $\mathcal{PU}pperNFlim\mathcal{EDS}$.

(ii) $DecrNFint_{\tau_{lim_2}}(lim\mathfrak{F})$ is a τ_{lim_2} decreasing *NFlimCS*, for each τ_{lim_1} decreasing *NFlimCS* $lim\mathfrak{F}$. Similarly, $DecrNFint_{\tau_{lim_1}}(lim\mathfrak{F})$ is a τ_{lim_1} decreasing *NFlimCS*, for each τ_{lim_2} decreasing *NFlimCS* $lim\mathfrak{F}$.

(iii) $DecrNFcl_{\tau_{lim_2}}(C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})))$

$= DecrNFint_{\tau_{lim_2}}(C(lim\mathfrak{F}))$,

for each τ_{lim_1} increasing *NFlimOS* $lim\mathfrak{F}$.

Similarly, $DecrNFcl_{\tau_{lim_1}}(C(IncrNFcl_{\tau_{lim_1}}(lim\mathfrak{F})))$

$= DecrNFint_{\tau_{lim_1}}(C(lim\mathfrak{F}))$,

for each τ_{lim_2} increasing *NFlimOS* $lim\mathfrak{F}$.

(v) For each pair of τ_{lim_1} increasing *NFlimOS* $lim\mathfrak{F}$ and τ_{lim_2} decreasing *NFlimOS* $lim\mathfrak{G}$ with $IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}) = C(lim\mathfrak{G})$, $DecrNFcl_{\tau_{lim_2}}(lim\mathfrak{G}) = C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}))$.

Similarly, for each pair of τ_{lim_2} increasing *NFlimOS* $lim\mathfrak{F}$ and τ_{lim_1} decreasing *NFlimOS* $lim\mathfrak{G}$ with $IncrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}) = C(lim\mathfrak{G})$, $DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{G}) = C(IncrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}))$.

Proof:

(i)⇒(ii) Let $lim\mathfrak{F}$ be any τ_{lim_1} decreasing *NFlimCS*. Then $C(lim\mathfrak{F})$ is τ_{lim_1} increasing *NFlimOS*.

By assumption(i) $IncrNFcl_{\tau_{lim_2}}(C(lim\mathfrak{F}))$ is τ_{lim_2} increasing *NFlimOS*.

Since, $IncrNFcl_{\tau_{lim_2}}(C(lim\mathfrak{F})) = C(DecrNFint_{\tau_{lim_2}}(lim\mathfrak{F}))$.

Thus $DecrNFint_{\tau_{lim_2}}(lim\mathfrak{F})$ is τ_{lim_2} decreasing *NFlimCS*.

(ii)⇒(iii) Let $lim\mathfrak{F}$ be any τ_{lim_1} increasing *NFlimOS*. Then $C(lim\mathfrak{F})$ is τ_{lim_1} decreasing *NFlimCS*.

By assumption(ii) $DecrNFint_{\tau_{lim_2}}(lim\mathfrak{F})$ is τ_{lim_2} decreasing *NFlimCS*. Consider

$DecrNFcl_{\tau_{lim_2}}(C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})))$

$=DecrNFcl_{\tau_{lim_2}}(DecrNFint_{\tau_{lim_2}}(C(lim\mathfrak{F})))$

$= DecrNFint_{\tau_{lim_2}}(C(lim\mathfrak{F}))$.

(iii)⇒ (iv) For each pair of τ_{lim_1} increasing *NFlimOS* $lim\mathfrak{F}$ and τ_{lim_2} decreasing *NFlimOS* $lim\mathfrak{G}$ with $IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}) = C(lim\mathfrak{G})$.

By assumption (iii),

$DecrNFcl_{\tau_{lim_2}}(C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})))$

$= DecrNFint_{\tau_{lim_2}}(C(lim\mathfrak{F}))$.

By using Proposition 3.1 and by the hypothesis, we have

$DecrNFcl_{\tau_{lim_2}}(lim\mathfrak{G})$

$= DecrNFcl_{\tau_{lim_2}}(C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})))$

$= DecrNFint_{\tau_{lim_2}}(C(lim\mathfrak{F}))$

$= C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}))$.

(iv)⇒(i) Let $lim\mathfrak{F}$ be any τ_{lim_1} increasing *NFlimOS*. Put $lim\mathfrak{G} = C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}))$. Clearly, $lim\mathfrak{G}$ is τ_{lim_2} decreasing *NFlimOS*.

By assumption (iv), it follow that

$DecrNFcl_{\tau_{lim_2}}(lim\mathfrak{G}) = C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}))$.

That is, $C(IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F}))$ is τ_{lim_2} decreasing *NFlimCS*. This implies that $IncrNFcl_{\tau_{lim_2}}(lim\mathfrak{F})$ is τ_{lim_2} increasing *NFlimOS*. Thus, $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is τ_{lim_1} upper *NFlim* extremally disconnected space.

Similarly, we can show that $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is τ_{lim_2} upper *NFlim* extremally disconnected space.

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Hence $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is 𝒫UpperNFlimℰ𝒟𝒮.

Proposition 4.2. An 𝒪𝒩𝔽ℂ𝕆ℕ𝕍𝔾BTS $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is 𝒫𝒪𝒩FlimℰDS iff for each τ_{lim_1} decreasing *NFlimOS* $lim\mathfrak{F}$ and τ_{lim_2} decreasing *NFlimCS* $lim\mathfrak{G}$ such that $lim\mathfrak{F} \subseteq lim\mathfrak{G}$, we have $DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}) \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$.

Proof:

Suppose $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is 𝒫UpperNFlimℰ𝒟𝒮. Let $lim\mathfrak{F}$ be any τ_{lim_1} decreasing *NFlimOS* and $lim\mathfrak{G}$ be any τ_{lim_2} decreasing *NFlimCS* such that $lim\mathfrak{F} \subseteq lim\mathfrak{G}$. Then by (ii) of Proposition 4.1, $DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$ is τ_{lim_1} decreasing *NFlimCS*.

Also, since $lim\mathfrak{F}$ is τ_{lim_1} decreasing *NFlimOS* and $lim\mathfrak{F} \subseteq lim\mathfrak{G} \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$. It follows that $DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}) \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$.

Conversely, let $lim\mathfrak{G}$ be any τ_{lim_2} decreasing *NFlimCS*. By Definition 3.14, $DecrNFint\tau_{lim_1}(lim\mathfrak{G})$ is τ_{lim_1} decreasing *NFlimOS* and $DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}) \subseteq lim\mathfrak{G}$.

By assumption,

$$DecrNFcl_{\tau_{lim_1}}(DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})) \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}).$$

Also we know that

$$DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}) \subseteq DecrNFcl_{\tau_{lim_1}}(DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})).$$

Thus $DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$

$$= DecrNFcl_{\tau_{lim_1}}(DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})).$$

Therefore, $DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$ is τ_{lim_1} decreasing *NFlimCS*. Hence by (ii) of Proposition 4.1, it follows that $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is τ_{lim_1} upper *NFlim* extremally disconnected space. Similarly we can prove the other cases. Therefore, $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is 𝒫𝒪𝒩FlimℰDS.

Notation 4.1. A *NFS* which is both increasing (resp. decreasing)*NFlimOS* and increasing (resp. decreasing) *NFlimCS* is increasing (resp. decreasing) *NFlim* clopen set.

Remark 4.1. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ be a 𝒫UpperNFlimℰ𝒟𝒮. Let $\{ lim\mathfrak{F}_i, C(lim\mathfrak{G}_i): i \in \mathbb{N} \}$ be a collection such that $lim\mathfrak{F}_i$'s are τ_{lim_1} decreasing *NFlimOSs* and $lim\mathfrak{G}_i$'s are τ_{lim_2} decreasing *NFlimCSs*.

Let $lim\mathfrak{F}$ and $C(lim\mathfrak{G})$ be τ_{lim_1} decreasing *NFlimOS* and τ_{lim_2} increasing *NFlimOS* respectively.

If $lim\mathfrak{F}_i \subseteq lim\mathfrak{F} \subseteq lim\mathfrak{G}_j$ and $lim\mathfrak{F}_i \subseteq lim\mathfrak{G} \subseteq lim\mathfrak{G}_j$ for all $i,j \in \mathbb{N}$, then there exists a τ_{lim_1} and τ_{lim_2} decreasing *NFlim* clopen sets $lim\mathfrak{H}$ such that

$$DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_i) \subseteq lim\mathfrak{H} \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_j),$$

for all $i, j \in \mathbb{N}$.

Proof: By Proposition 4.2,

$DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_i)$

$$\subseteq DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}) \cap DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$$

$$\subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_j),$$

for all $i,j \in \mathbb{N}$.

Letting $lim\mathfrak{H} = DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}) \cap DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G})$ in the above, we have $lim\mathfrak{H}$ is τ_{lim_1} and τ_{lim_2} decreasing *NFlim* clopen set satisfying the required condition.

Proposition 4.3. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ be a 𝒫𝒪𝒩FlimℰDS. Let $\{ lim\mathfrak{F}_q \}_{q\in\mathbb{Q}}$ and $\{ lim\mathfrak{G}_q \}_{q\in\mathbb{Q}}$ be monotone increasing collections of τ_{lim_1} decreasing *NFlimOSs* and τ_{lim_2} decreasing *NFlimCSs* of $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ respectively.

Suppose that $lim\mathfrak{F}_{q_1} \subseteq lim\mathfrak{G}_{q_2}$ whenever $q_1 < q_2$ ($\mathbb{Q}$ is the set of all rational numbers). Then there exists a monotone increasing collection $\{ lim\mathfrak{H}_q \} q\in\mathbb{Q}$ of τ_{lim_1} and τ_{lim_2} decreasing *NFlim* clopen sets of $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ such that

$$DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_{q_1}) \subseteq lim\mathfrak{H}_{q_2} \text{ and}$$

$$lim\mathfrak{H}_{q_1} \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_{q_2})$$

whenever $q_1 < q_2$.

Proof:

Let us arrange all rational numbers into a sequence $\{q_n\}$ (without repetitions). For every $n \geq 2$, we shall define inductively a collection $\{lim\mathfrak{H}_{q_i} \ / 1 \leq i \leq n\} \subset \zeta^X$ such that

$DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_q) \subseteq lim\mathfrak{H}_{q_i}$ if $q < q_i$ and

$lim\mathfrak{H}_{q_i} \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_q)$ if $q_i < q$, for all $i < n$ **(S_n)**

By Proposition 4.2, the countable collections $\{DecrNFcl\tau_{lim_1}(lim\mathfrak{F}_q)\}$ and $\{DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_q)\}$ satisfy $DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_{q_1}) \subseteq DecrNFint\tau_{lim_1}(lim\mathfrak{G}_{q_2})$ if $q_1 < q_2$.

By Remark 4.1, there exists a τ_{lim_1} and τ_{lim_2} decreasing *NFlim* clopen set $lim\mathfrak{K}_1$ such that

$DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_{q_1}) \subseteq lim\mathfrak{K}_1 \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_{q_2})$. Letting $lim\mathfrak{H}_{q_i} = lim\mathfrak{K}_1$, we get **($S_2$)**.

Assume that τ_{lim_1} *NFSs* $lim\mathfrak{H}_{q_i}$ are already defined for $i < n$ and satisfy **(S_n)**.

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Define $lim\mathfrak{L} = \cup\{ lim\mathfrak{H}_{q_i} /i < n, q_i < q_n \} \cup lim\mathfrak{F}_{q_n}$ and

$$lim\mathfrak{M} = \cap\{lim\mathfrak{H}_{q_j} /j < n, q_j > q_n \} \cap lim\mathfrak{G}_{q_n} .$$

Then we have that

$DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{H}_{q_i})$

$\subseteq DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{L}) \subseteq DecrNFint_{\tau_{lim_1}} (lim\mathfrak{H}_{q_j})$

and

$DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{H}_{q_i})$

$\subseteq DecrNFint_{\tau_{lim_1}} (lim\mathfrak{M}) \subseteq DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{H}_{q_j})$

whenever $q_i < q_n < q_j$ $(i,j < n)$.

As well as $lim\mathfrak{F}_q \subseteq DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{L}) \subseteq lim\mathfrak{G}_{q'}$ and $lim\mathfrak{F}_q \subseteq DecrNFint_{\tau_{lim_1}} (lim\mathfrak{M}) \subseteq lim\mathfrak{G}_{q'}$ whenever $q < q_n < q'$.

This shows that the countable collections

$\{ lim\mathfrak{H}_{q_i} | i < n, q_i < q_n \} \cup \{ lim\mathfrak{F}_q \mid q < q_n \}$ and

$\{lim\mathfrak{H}_{q_j} \mid j < n, q_j > q_n \} \cup \{lim\mathfrak{G}_q \mid q > q_n \}$ together with $lim\mathfrak{L}$ and $lim\mathfrak{M}$ fulfil the conditions of Remark 4.1.

Hence, there exists a τ_{lim_1} and τ_{lim_2} decreasing *NFlim* clopen set $lim\mathfrak{K}_n$ such that

$DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{K}_n) \subseteq lim\mathfrak{G}_q$ if $q_n < q$,

$lim\mathfrak{F}_q \subseteq DecrNFint_{\tau_{lim_1}} (lim\mathfrak{K}_n)$ if $q < q_n$, $DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{H}_{q_i}) \subseteq DecrNFint_{\tau_{lim_1}} (lim\mathfrak{K}_n)$ if $q_i < q_n$, $DecrNFcl_{\tau_{lim_1}} (lim\mathfrak{K}_n) \subseteq DecrNFint_{\tau_{lim_1}} (lim\mathfrak{H}q_j)$ if $q_n < q_j$ where $1 \le i, j \le n-1$.

Letting $lim\mathfrak{H}_{q_n} = lim\mathfrak{K}_n$ we obtain a τ_{lim_1} *NFSs* $lim\mathfrak{H}_{q_1}, lim\mathfrak{H}_{q_2}, lim\mathfrak{H}_{q_3}, \ldots, lim\mathfrak{H}_{q_n}$ that satisfy **(S_n+1).**

Therefore, the collection $\{ lim\mathfrak{H}_{q_i} /i = 1,2,\ldots\}$ has the required property. This completes the proof.

Definition 4.4. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ and $(Y, \sigma_{lim_1}, \sigma_{lim_2}, \le)$ be any two *𝒪NF𝒞𝒪𝒩𝒱𝒢BTSs*.

Let $f : (X, \tau_{lim_1}, \tau_{lim_2}, \le) \to (Y, \sigma_{lim_1}, \sigma_{lim_2}, \le)$ be any *NF* function. Then *f* is said to be a

(i) τ_{lim_1} increasing *NFlim* continuous function (τ_{lim_1} increasing *NFlimCF*) if for every σ_{lim_1} (or) σ_{lim_2} *NFlimOS* $lim\mathfrak{F}$ in $(Y, \sigma_{lim_1}, \sigma_{lim_2}, \le)$, $f^{-1}(lim\mathfrak{F})$ is a τ_{lim_1} increasing *NFlimOS* in $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$.

(ii) τ_{lim_1} decreasing *NFlim* continuous function (τ_{lim_1} decreasing *NFlimCF*) if for every σ_{lim_1} (or) σ_{lim_2} *NFlimOS* $lim\mathfrak{F}$ in $(Y, \sigma_{lim_1}, \sigma_{lim_2}, \le)$, $f^{-1}(lim\mathfrak{F})$ is a τ_{lim_1} decreasing *NFlimOS* in $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$.

(iii) If *f* is both τ_{lim_1} increasing *NFlimCF* and τ_{lim_1} decreasing *NFlimCF* then it is said to be an ordered τ_{lim_1} *NFlim* continuous function($\mathcal{O}\tau_{lim_1}$ *NFlimCF*). Similarly we can define $\mathcal{O}\sigma_{lim_2}$ *NFlimCF*.

V. TIETZE EXTENSION THEOREM FOR *𝒫𝒪NF𝒞𝒪𝒩𝒱𝒢BTS*

Definition 5.1. A *NF* real line $\mathbb{R}_{\mathbb{I}}(I)$ is the set of all monotone decreasing *NFS* $A \in \zeta^{\mathbb{R}}$ satisfying

$$\cup\{A(t) : t \in \mathbb{R}\} = 1^N \text{ and } \cap\{A(t) : t \in \mathbb{R}\} = 0^N$$

after the identification of an *NFSs* $A,B \in \mathbb{R}_{\mathbb{I}}(I)$ if and only if $A(t-) = B(t-)$ and $A(t+) = B(t+)$ for all $t \in \mathbb{R}$ where $A(t-) = \cap\{A(s) : s < t\}$ and $A(t+) = \cup\{A(s) : s > t\}$.

The *NF* unit interval $\mathbb{I}_{\mathbb{I}}(I)$ is a subset of $\mathbb{R}_{\mathbb{I}}(I)$ such that $[A] \in \mathbb{I}_{\mathbb{I}}(I)$ if the degrees of membership, indeterminate-membership and nonmembership of *A* are defined by

$$T_A(t) = \begin{cases} 1, t < 0 \\ 0, t > 1 \end{cases}, \text{ and } I_A(t) = \begin{cases} 1, t < 0 \\ 0, t > 1 \end{cases} \quad F_A(t) = \begin{cases} 0, t < 0 \\ 1, t > 1 \end{cases}$$

respectively.

The natural *NF* topology on $\mathbb{R}_{\mathbb{I}}(I)$ is generated from the subbasis $\{L^{\mathbb{I}}_t, R^{\mathbb{I}}_t : t \in \mathbb{R}\}$ where $L^{\mathbb{I}}_t, R^{\mathbb{I}}_t : \mathbb{R}_{\mathbb{I}}(I) \to \mathbb{I}_{\mathbb{I}}(I)$ are given by $L^{\mathbb{I}}_t[A] = C(A(t-))$ and $R^{\mathbb{I}}_t [A] = A(t+)$ respectively.

Definition 5.2. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ be any *𝒪NF𝒞𝒪𝒩𝒱𝒢BTS*. A function $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ is said be

(i) lower τ_{lim_1} *NFlimCF* if $f^{-1}(R^{\mathbb{I}}_t)$ is τ_{lim_1} increasing or decreasing *NFlimOS*,

(ii) upper τ_{lim_1} *NFlimCF* if $f^{-1}(L^{\mathbb{I}}_t)$ is τ_{lim_1} increasing or decreasing *NFlimOS*, for each $t \in \mathbb{R}$.

Similarly we can define lower τ_{lim_1} *NFlimCF* and upper τ_{lim_2} *NFlimCF* respectively.

Notation 5.1. Let *X* be any nonempty set and $lim\mathfrak{F} \in \zeta^X$. Then for $x \in X$, $\langle T_{lim\mathfrak{F}}(x), I_{lim\mathfrak{F}}(x), F_{lim\mathfrak{F}}(x) \rangle$ is denoted by $lim\mathfrak{F}^N$.

Proposition 5.1. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ be any *𝒪NF𝒞𝒪𝒩𝒱𝒢BTS* and let $lim\mathfrak{F}$ be *NFS* in *X*.

Let $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ be such that $f(x)(t) = \begin{cases} 1^N \text{ if } t < 0 \\ lim\mathfrak{F}^N \text{ if } 0 \le t \le 1 \\ 0^N \text{ if } t > 1 \end{cases}$

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for all $x \in X$ and $t \in \mathbb{R}$. Then f is lower (resp. upper) τ_{lim_1} *NFlimCF* iff $lim\mathfrak{F}$ is τ_{lim_1} increasing or decreasing *NFlimOS* (*NFlimCS*).

Proof:

$$f^{-1}(R^{\mathbb{I}}{}_t)(t) = \begin{cases} 1_N \text{ if } t < 0 \\ lim\mathfrak{F} \text{ if } 0 \le t \le 1 \\ 0_N \text{ if } t > 1 \end{cases}$$

implies that f is lower τ_{lim_1} *NFlimCF* continuous function iff $lim\mathfrak{F}$ is τ_{lim_1} increasing or decreasing *limOS*.

$$f^{-1}(C(L^{\mathbb{I}}{}_t))(t) = \begin{cases} 1_N \text{ if } t < 0 \\ lim\mathfrak{F} \text{ if } 0 \le t \le 1 \\ 0_N \text{ if } t > 1 \end{cases}$$

implies that f is upper τ_{lim_1} *NFlimCF* iff $lim\mathfrak{F}$ is τ_{lim_1} increasing or decreasing *NFlimCS*. Hence the proof is complete.

Definition 5.3. A *NF* characteristic function of a *NFS* $lim\mathfrak{F}$ in X is a map $\psi_{lim\mathfrak{F}} : X \to \mathbb{I}_{\mathbb{I}}(I)$ is defined by $\psi_{lim\mathfrak{F}}(x) = lim\mathfrak{F}^N$, for each $x \in X$.

Remark 5.1. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ be any $\mathcal{ONFCONVGBTS}$. Let $\psi_{lim\mathfrak{F}}$ be *NF* characteristic function of a *NFS* $lim\mathfrak{F}$ in X. Then $\psi_{lim\mathfrak{F}}$ is lower (resp. upper) τ_{lim_1} *NFlimCF* iff $lim\mathfrak{F}$ is τ_{lim_1} increasing or decreasing *NFlimOS* (*NFlimCS*).

Proof: The proof follows Proposition 5.1.

Proposition 5.2. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ be any $\mathcal{ONFCONVGBTS}$. Then the following conditions are equivalent

(i) $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ is $\mathcal{PONFlimEDS}$.

(ii) If $g, h : X \to \mathbb{R}_{\mathbb{I}}(I)$, g is lower τ_{lim_1} *NFlimCF*, h is upper τ_{lim_2} *NFlimCF* and $g \subseteq h$, then there exists increasing τ_{lim_1} and τ_{lim_2} *NFlimCF* $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ such that $g \subseteq f \subseteq h$.

(iii) If $C(lim\mathfrak{F})$ is increasing τ_{lim_2} *NFlimOS* and $lim\mathfrak{G}$ is decreasing τ_{lim_1} *NFlimOS* such that $lim\mathfrak{G} \subseteq lim\mathfrak{F}$, then there exists an increasing τ_{lim_1} and τ_{lim_2} *NFlimCF* $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ such that $lim\mathfrak{G} \subseteq f^{-1}(C(L^{\mathbb{I}}{}_1)) \subseteq f^{-1}(R^{\mathbb{I}}{}_0) \subseteq lim\mathfrak{F}$.

Proof:

(i) ⇒(ii) Define $lim\mathfrak{F}_r = h^{-1}(L^{\mathbb{I}}{}_r)$ and $lim\mathfrak{G}_r = g^{-1}(C(R^{\mathbb{I}}{}_r))$, for all $r \in \mathbb{Q}$ ($\mathbb{Q}$ is the set of all rationals). Clearly, $\{lim\mathfrak{F}_r\}_{r\in\mathbb{Q}}$ and $\{lim\mathfrak{G}_r\}_{r\in\mathbb{Q}}$ are monotone increasing families of decreasing τ_{lim_1} *NFlimOSs* and decreasing τ_{lim_2} *NFlimCSs* respectively. Moreover $lim\mathfrak{F}_r \subseteq lim\mathfrak{G}_s$ if $r < s$.

By Proposition 4.3, there exists a monotone increasing family $\{lim\mathfrak{H}_r\}_{r\in\mathbb{Q}}$ of τ_{lim_1} and τ_{lim_2} decreasing *NFlim* clopen sets of $(X, \tau_{lim_1}, \tau_{lim_2}, \le)$ such that $DecrNFcl_{\tau_{lim_1}}(lim\mathfrak{F}_r) \subseteq lim\mathfrak{H}_s$ and

$lim\mathfrak{H}_r \subseteq DecrNFint_{\tau_{lim_1}}(lim\mathfrak{G}_s)$ whenever $r < s$ $(r,s \in \mathbb{Q})$.

Letting $lim\mathfrak{V}_t = \bigcap_{r<t}\{C(lim\mathfrak{H}_r)\}$ for all $t \in \mathbb{R}$, we define a monotone decreasing family $\{lim\mathfrak{V}_t \mid t \in R\} \subseteq \zeta^X$.

Moreover we have

$IncrNFcl_{\tau_{lim_1}}(lim\mathfrak{V}_t) \subseteq IncrNFint_{\tau_{lim_1}}(lim\mathfrak{V}_s)$ whenever $s < t$.

We have,

$$\begin{aligned}\bigcup_{t\in\mathbb{R}} lim\mathfrak{V}_t &= \bigcup_{t\in\mathbb{R}} \bigcap_{r<t} C(lim\mathfrak{H}_r) \\ &\supseteq \bigcup_{t\in\mathbb{R}} \bigcap_{r<t} C(lim\mathfrak{H}_r) \\ &= \bigcup_{t\in\mathbb{R}} \bigcap_{r<t} g^{-1}(R^{\mathbb{I}}{}_r) \\ &= \bigcup_{t\in\mathbb{R}} g^{-1}(C(L^{\mathbb{I}}{}_t)) = g^{-1}\Big(\bigcup_{t\in\mathbb{R}} C(L^{\mathbb{I}}{}_t)\Big) = 1_N\end{aligned}$$

Similarly, $\bigcap_{t\in\mathbb{R}} lim\mathfrak{V}_t = 0_N$. Now define a function $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ possessing required conditions.

Let $f(x)(t) = lim\mathfrak{V}_t(x)$, for all $x \in X$ and $t \in \mathbb{R}$. By the above discussion, it follows that f is well defined.

To prove f is increasing τ_{lim_1} and τ_{lim_2} *NFlimCF*. Observe that $\bigcup_{s>t} lim\mathfrak{V}_s = \bigcup_{s>t} IncrNFint_{\tau_{lim_1}}(lim\mathfrak{V}_s)$ and $\bigcap_{s<t} lim\mathfrak{V}_s = \bigcap_{s<t} IncrNFcl_{\tau_{lim_1}}(lim\mathfrak{V}_s)$. Then $f^{-1}(R^{\mathbb{I}}{}_t) = \bigcup_{s>t} lim\mathfrak{V}_s = \bigcup_{s>t} IncrNFint_{\tau_{lim_1}}(lim\mathfrak{V}_s)$ is τ_{lim_1} increasing or decreasing *NFlimOS* and $f^{-1}(L^{\mathbb{I}}{}_t) = \bigcap_{s<t} lim\mathfrak{V}_s = \bigcap_{s<t} IncrNFcl_{\tau_{lim_1}}(lim\mathfrak{V}_s)$ is τ_{lim_1} increasing or decreasing *NFlimCS*. Hence f is increasing τ_{lim_1} *NFlimCF*.

Similarly, we can prove f is increasing τ_{lim_2} *NFlimCF* in same manner. Therefore, f is increasing τ_{lim_1} and τ_{lim_2} *NFlimCF*.

To conclude the proof it remains to show that $g \subseteq f \subseteq h$. That is $g^{-1}(C(L^{\mathbb{I}}{}_t)) \subseteq f^{-1}(C(L^{\mathbb{I}}{}_t)) \subseteq h^{-1}(C(L^{\mathbb{I}}{}_t))$ and $g^{-1}(R^{\mathbb{I}}{}_t) \subseteq f^{-1}(R^{\mathbb{I}}{}_t) \subseteq h^{-1}(R^{\mathbb{I}}{}_t)$ for each $t \in \mathbb{R}$. We have,

$$\begin{aligned} g^{-1}(C(L^{\mathbb{I}}{}_t)) &= \bigcap_{s<t} g^{-1}(C(L^{\mathbb{I}}{}_s)) \\ &= \bigcap_{s<t}\bigcap_{r<s} g^{-1}(R^{\mathbb{I}}{}_r) \\ &= \bigcap_{s<t}\bigcap_{r<s} C(lim\mathfrak{G}_r) \\ &\subseteq \bigcap_{s<t}\bigcap_{r<s} C(lim\mathfrak{H}_r)\end{aligned}$$

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$$= \bigcap_{s<t} lim\mathfrak{V}_s = f^{-1}(C(L^{\mathbb{I}}_t))$$

and

$$f^{-1}(C(L^{\mathbb{I}}_t)) = \bigcap_{s<t} lim\mathfrak{V}_s$$
$$= \bigcap_{s<t}\ \bigcap_{r<s} C(lim\mathfrak{H}_r)$$
$$\subseteq \bigcap_{s<t}\ \bigcap_{r<s} C(lim\mathfrak{F}_r)$$
$$= \bigcap_{s<t}\ \bigcap_{r<s} h^{-1}(C(L^{\mathbb{I}}_r))$$
$$= \bigcap_{s<t} h^{-1}(C(L^{\mathbb{I}}_s)) = h^{-1}(C(L^{\mathbb{I}}_t))$$

Similarly,

$$g^{-1}(R^{\mathbb{I}}_t) = \bigcup_{s>t} g^{-1}(R^{\mathbb{I}}_s)$$
$$= \bigcup_{s>t}\ \bigcup_{r>s} g^{-1}(R^{\mathbb{I}}_r)$$
$$= \bigcup_{s>t}\ \bigcup_{r>s} C(lim\mathfrak{G}_r)$$
$$\subseteq \bigcup_{s>t}\ \bigcap_{r<s} C(lim\mathfrak{H}_r)$$
$$= \bigcup_{s>t} lim\mathfrak{V}_s = f^{-1}(R^{\mathbb{I}}_t)$$

and

$$f^{-1}(R^{\mathbb{I}}_t) = \bigcup_{s>t} lim\mathfrak{V}_s$$
$$= \bigcup_{s>t}\ \bigcap_{r<s} C(lim\mathfrak{H}_r)$$
$$\subseteq \bigcup_{s>t}\ \bigcup_{r>s} C(lim\mathfrak{F}_r)$$
$$= \bigcup_{s>t}\ \bigcup_{r>s} h^{-1}(C(L^{\mathbb{I}}_r))$$
$$= \bigcup_{s>t} h^{-1}(R^{\mathbb{I}}_s) = h^{-1}(R^{\mathbb{I}}_t)$$

Hence, the condition (ii) is proved.

(ii)⇒(iii) Let $C(lim\mathfrak{F})$ be an increasing τ_{lim_2} *NFlimOS* and $lim\mathfrak{G}$ be a decreasing τ_{lim_1} *NFlimOS* such that $lim\mathfrak{G} \subseteq lim\mathfrak{F}$. Then $\psi lim\mathfrak{G} \subseteq \psi lim\mathfrak{F}$ where $\psi lim\mathfrak{G}$ and $\psi lim\mathfrak{F}$ are lower τ_{lim_1} *NFlimCF* and upper τ_{lim_2} *NFlimCF* respectively.

By (ii), there exists increasing τ_{lim_1} and τ_{lim_2} *NFlimCF* $f : X \to \mathbb{R}_{\mathbb{I}}(I)$ such that $\psi_{lim\mathfrak{G}} \subseteq f \subseteq \psi_{lim\mathfrak{F}}$. Clearly, $f(x) \in \mathbb{I}_{\mathbb{I}}(I)$ for all $x \in X$ and $lim\mathfrak{G} = \psi^{-1}_{lim\mathfrak{G}}(C(L^{\mathbb{I}}_1)) \subseteq f^{-1}(C(L^{\mathbb{I}}_1)) \subseteq f^{-1}(R^{\mathbb{I}}_0) \subseteq \psi^{-1}_{lim\mathfrak{F}}(R^{\mathbb{I}}_0) = lim\mathfrak{F}$.

Therefore, $lim\,\mathfrak{G} \subseteq f^{-1}(C(L^{\mathbb{I}}_1)) \subseteq f^{-1}(R^{\mathbb{I}}_0) \subseteq lim\mathfrak{F}$.

(iii)⇒(i) Since $f^{-1}(C(L^{\mathbb{I}}_1))$ and $f^{-1}(R^{\mathbb{I}}_0)$ are decreasing τ_{lim_1} *NFlimCS* and decreasing τ_{lim_2} *NFlimOS* respectively. By Proposition 4.2, $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is $\mathcal{PONFlimEDS}$.

Notation 5.2. Let (X,τ_{lim}) be any $NF\mathcal{CONVGTS}$. Let $A \subset X$. Then $NFlim\chi_A$ is the form $\langle x, \chi_A(x), \chi_A(x), 1 - \chi_A(x) \rangle$.

Proposition 5.3. Let $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$ is $\mathcal{PONFlimEDS}$. Let $A \subset X$ such that $lim\chi_A$ is τ_{lim_1} and τ_{lim_2} *NFlimOS* in X respectively.

Let $f : (A, \tau_{lim_1}/A, \tau_{lim_2}/A) \to \mathbb{I}_{\mathbb{I}}(I)$ be an increasing τ_{lim_1} and τ_{lim_2} *NFlimCF*. Then f has an increasing τ_{lim_1} and τ_{lim_2} *NFlim* continuous extension over $(X, \tau_{lim_1}, \tau_{lim_2}, \leq)$.

Proof:

Let $g,h : X \to \mathbb{I}_{\mathbb{I}}(I)$ be such that $g = f = h$ on A and $g(x) = 0^N$, $h(x) = 1^N$ if $x \notin A$. For every $t \in \mathbb{R}$, We have,

$$g^{-1}(R^{\mathbb{I}}_t) = \begin{cases} lim\mathfrak{F}_t \cap lim\chi_A, & t \geq 0 \\ 1_N, & t < 0 \end{cases}$$

where $lim\mathfrak{F}_t$ is τ_{lim_1} increasing or decreasing *NFlimOS* such that $lim\mathfrak{F}_t/A = f^{-1}(R^{\mathbb{I}}_t)$ and

$$h^{-1}(L^{\mathbb{I}}_t) = \begin{cases} lim\mathfrak{G}_t \cap lim\chi_A, & t \leq 1 \\ 1_N, & t > 1 \end{cases}$$

where $lim\mathfrak{G}_t$ is τ_{lim_1} increasing or decreasing *NFlimOS* such that $lim\mathfrak{G}_t/A = f^{-1}(L^{\mathbb{I}}_t)$. Thus g is lower τ_{lim_1} *NFlimCF* and h is upper τ_{lim_2} *NFlimCF* such that $g \subseteq h$.

Hence by Proposition 5.2, there exists an increasing τ_{lim_1} and τ_{lim_2} *NFlimCF* $F : X \to \mathbb{I}_{\mathbb{I}}(I)$ such that $g(x) \subseteq F(x) \subseteq h(x)$ for all $x \in X$. Hence for all $x \in A$, $f(x) \subseteq F(x) \subseteq f(x)$. So that F is the required extension of f over X.

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