Numerical Experiment on Two-Sided Tolerance Limit for Safety Analysis of Energy Generation Systems

Seola Han, Taew n Kim

Abstract: ***It is necessary to quantify the uncertainty in the safety assessment when the protection characteristics of energy generation systems are tested through first-class-estimate methodologies. Especially, the tolerance restriction for licensing must be carefully predicted taking into account the uncertainty within the protection analysis results. The tolerance limits are usually predicted as values with a probability of 95% and a confidence level of 95% for general industrial applications. Various statistical techniques were applied a good way to estimate the tolerance restriction of the great-predicted results and a methodology based totally on non-parametric facts is one of these methods which lets in to estimate the tolerance limit with reasonable attempt. Among non-parametric methodologies, a method primarily based on order statics has been broadly used to decide the tolerance restriction for energy structures including nuclear structures. This method has benefits wherein the quantity of simulations is independent of the number of uncertainty parameters and the interpretation of the results has not anything to do with the distribution of the outcomes from uncertainty evaluation. In this technique, the quantity of simulations is determined by a formula recommended by Wilks but it is recently found that the quantity of simulation decided by Wilks' formula should be revised considering the symmetric nature of the tolerance limits in case of two-sided approach, namely centered two-sided formula. In this study, the results from methods based on Wilks' formula and centered two-sided formula are compared for three different trial output distributions. The comparison indicates that the tolerance limits estimated on the basis of Wilks' formula shows lower confidence level, 80%, whereas the centered two-sided formula results in a proper confidence level of 95%. In addition, the distribution-free and order-free characteristics of non-parametric methodology have been demonstrated on the basis of the analysis results. The results indicate that the lower order formula tends to produce more conservative results. Thus, it is recommended for regulators to use the lower order formula for safety audit analysis for energy generation systems to secure higher safety.***

Keywords: ***Wilks' formula, Tolerance limit, Best-estimate analysis, Non-parametric method, Centered two-sided tolerance limit.***

I. INTRODUCTION

It is necessary to quantify the uncertainty in the safety assessment when the safety and/or performance traits of energy systems are tested with the aid of high-quality-estimate methodologies.

Especially, the tolerance limit for regulatory activities must be carefully estimated taking into account the uncertainty in the safety evaluation results. Generally industrial applications, the analysis result should be compared with the acceptance restrict with a certain probability and confidence level (typically probability of 95% and confidence level of 95%). Many statistical approaches such as Monte-Carlo method and response surface method have been employed to evaluate the uncertainty with required probability and confidence level. However, it is well-known drawback of those methods that the number of calculations to achieve the required probability and confidence level is increased as the number of uncertainty parameters is increased. Thus, it is obvious that high calculation cost should be accompanied in order to evaluate uncertainty in the analysis where the complex physical phenomena occur and many parameters influence the figure of merits.

As a way to conquer such a limitation, an approach based totally on non-parametric statistics has been broadly used in lots of engineering applications, especially for nuclear applications [1]. A conventional instance of the application of non-parametric statistics is an order data technique suggested by Wilks [2-3]. In this approach, the range of simulation is decided by a formulation suggested by Wilks and therefore the resulted variety of simulations is independent of the number of uncertainty parameters. In addition, since the technique is based on non-parametric statistics, the interpretation of the results has nothing to try and do with the distribution of the results from uncertainty analysis. Due to such characteristics, the order statistics technique based on Wilks' formula has been applied for uncertainty analysis for complicated industrial systems to estimate the tolerance limits with reasonable calculation efforts.

In case of Wilks' technique, the tolerance limits with a probability of 95% and a confidence level of 95%, namely 95/95 values, are decided best considering the area among higher and lower tolerance limits. Thus, the method is appropriate for one-sided tests, i.e. for either higher or lower restriction, such as tests for regulatory recognition limits. However, in general uncertainty evaluation for the two-sided cases to determine the tolerance range, the lower and upper tolerance limits should be located ideally at 2.5% and 97.5%, respectively. Because Wilks' approach does not consider the locations of the tolerance limits, it is natural for the tolerance limits by Wilks' approach to be located at asymmetric locations such as 1% and 96%, for example.

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In this case, it is impossible to address proper lower and upper 95/95 values for industrial purposes and, in general, the confidence of the estimated tolerance limit should be decreased. Hong and Connolly indicated such a limitation of Wilks' approach for two-sided tests and suggested a formula where the tolerance limit is symmetrically located, namely centered two-sided formula [4-5].

In this study, the validity of methods based on Wilks' formula and centered two-sided formula has been assessed by means of a series of numerical experiments for three different trial distributions. The distribution-free nature of the methods has been investigated as well as the soundness of the methods. In addition, the characteristics of the results from the numerical experiments have been discussed.

II. DETERMINING TOLERANCE LIMIT

Wilks' formula

Wilks' formula for one-sided tests is given as follows:

$$1-\sum_{k=n-p+1}^{n} {}_nC_k \alpha^k (1-\alpha)^{n-k} \geq \beta \tag{1}$$

where, n and p denote the quantity of simulations and the order of Wilks' formula, respectively. α and β suggest the cumulative probability and the confidence level, respectively. The formula indicates that at least p values should be positioned in top probability range beyond α and thus, the p th largest value out of the results will becomes the primary value beyond a probability of α. Typically, the number of simulations is calculated by using Eq. (1) with other parameters which are pre-defined. For example, if the 1st order is considered, in total 59 simulations are necessary to obtain the 95/95 value and the largest value becomes the 95/95 value. In case of the 2nd order, 93 simulations are required to achieve the 95/95 value and the 95/95 value is the 2nd largest value. Eq. (1) indicates that the quantity of simulations can be determined regardless of then wide variety of uncertainty parameters. Considering that the range of simulations for the Monte-Carlo technique is increased exponentially with respect to the quantity of uncertainty parameters, this characteristic is one of the most important features of the order statistics

In case of two-sided tests, Wilks' formula is given as:

$$1-\sum_{k=n-2p+1}^{n} {}_nC_k \alpha^k (1-\alpha)^{n-k} \geq \beta \tag{2}$$

For the 1st order, Eq. (2) is reduced as:

$$1-\alpha^n - n(1-\alpha)\alpha^{n-1} \geq \beta \tag{3}$$

From Eq. (3), it is found that the number of simulations to achieve the 95/95 for two-sided tests is 93 considering n is an integer. In this case, the 95/95 values for higher and lower tolerance limits are given as the largest and smallest values out of the results, respectively.

Table 1 summarizes the wide variety of simulations required to attain 95/95 value with respect to the order of the formula. From the table and Eq. (2), it is found that the number of simulations by the p[th] order Wilks' formula for two-sided tests is the same as one by the 2p[th] order Wilks' formula for one-sided tests.

Table. 1 Number of simulations to obtain the 95/95 values

Order		1	2	3	4	5
One-sided	Wilks	59	93	124	153	181
Two-sided	Wilks	93	153	208	260	311
	Centered	146	221	286	348	406

In this method, only cumulative probability is considered to develop the formula. This feature is suitable to estimate the tolerance limit for one-sided test since the location of the tolerance limit is decided totally by the cumulative probability. However, in case of two-sided tests, the locations of upper and lower tolerance limits could be varied when the cumulative probability is fixed. For example, if the upper and lower tolerance limits are 96% and 1%, respectively, the cumulative probability with the range of upper and lower tolerance limits is 95%. However, since the cumulative probability beyond the upper tolerance limit is large, the confidence level for the upper 95/95 value must be decreased. Thus, it is impossible to address that the tolerance restrictions are estimated with a confidence level of 95%.

Wilks' formula

In order to improve the confidence level for two-sided tests, the centered two-sided formula has been proposed [4]. In this formula, the location of the lower and upper tolerance limits fixed at $(1-\alpha)/2$ and $(1+\alpha)/2$, respectively. The general form of the centered two-sided formula is given as:

$$1-\sum_{A=0}^{p-1}[2\times {}_nC_A\left(\frac{1-\alpha}{2}\right)^A \alpha^{n-A}\sum_{k=A}^{n-A}\{{}_{n-A}C_k\left(\frac{1-\alpha}{2\alpha}\right)^k\} - {}_nC_A\left(\frac{1-\alpha}{2}\right)^A {}_{n-A}C_A\left(\frac{1-\alpha}{2}\right)^A \alpha^{n-2A}] \geq \beta \tag{4}$$

In case of the 1st order, Eq. (3) is reduced as:

$$1+\alpha^n - 2\alpha^n \sum_{k=0}^{n} {}_nC_k\left(\frac{1-\alpha}{2\alpha}\right)^k \geq \beta \tag{5}$$

Using Eq. (4), the number of simulations required to achieve the 95/95 value is calculated as 146 which is much greater than one calculated by Wilks' formula. As listed in Table 1, the number of simulations calculated by the centered two-sided formula is generally larger than that calculated by Wilks' formula. This result is qualitatively reasonable considering that the number of simulations calculated by Wilks' formula for two-sided tests is expected to be small to achieve a confidence level of 95% actually.

III. NUMERICAL EXPERIMENTS

In order to examine the confidence level resulted by Wilks' formula and centered two-sided formula, a sequence of numerical experiments has been conducted. Actually, the uncertainty quantification method is employed to estimate the uncertainty propagated through the simulations. However, in this numerical experiment, random samples are generated from trial distributions and the cumulative probability of each sample is used as an output from arbitrary simulations, rather than conducting actual simulations. The normal, log-normal, and uniform distributions are employed as trial distributions.

The process of the mathematical experiments are as follows:

- Step 1: An unknown code output parameter is assumed to follow a given trial allocation.
- Step 2: Random values and also the cumulative probability of each value are generated from a given trial distribution. The quantity of samples is decided by Wilks' formula and centered two-sided formula. Here, the order examined is ranged from 1 to 5.
- Step 3: The 95/95 values for every order are determined.
- Step 4: Steps 2 and 3 are repeated for a given range of sets that is one million in this study.
- Step 5: The confidence of every order is estimated for every range of sets.
- Step 6: Steps 2 to 5 are repeated for three various trial distributions

The designed numerical experiments have been carried out by using MATLAB® R2017b [6].

Table. 2 Confidence level for the number of simulations determined by Wilks' formula

Order		1	2	3	4	5
Number of simulations		93	153	208	260	311
Wilks	Analytical confidence	0.9500	0.9506	0.9508	0.9502	0.9504
Centered		0.8186	0.8052	0.7989	0.7935	0.7909
Normal	Numerical confidence	0.8194	0.8053	0.8001	0.7938	0.7923
Log-normal		0.8190	0.8066	0.7991	0.7947	0.7915
Uniform		0.8192	0.8058	0.7999	0.7937	0.7918

IV. RESULTS

Simulations determined by Wilks' formula

Table 2 summarizes the confidence levels calculated by the formulas and numerical experiments on the basis of the number of simulations determined by Wilks' formula. From the analytical result, it is found that the centered two-sided formula estimates lower confidence level than Wilks' formula for the given number of simulations by Wilks' formula. Considering that the required number of simulation by the centered two-sided formula is larger than that by Wilks' formula, it is obvious to obtain lower confidence level by the centered two-sided formula. The results from the numerical experiments for three different output distribution clearly shows that the confidence level by the numerical experiment is very close to the confidence level predicted by the centered two-sided formula. Thus, it is concluded that a method based on Wilk's formula will result in the confidence lower than 95% and thus, the tolerance limit predicted by Wilks' formula cannot be used for the safety analysis of energy generation systems.

In general, the numerical confidence by each output distribution is slightly higher than that predicted by the centered two-sided formula. However, the maximum difference is 0.14 percentage points which is negligibly small. Since the non-parametric approach is a distribution-free approach, if the result is produced by the non-parametric approach appropriately, the result should be totally independent of the shape of output distributions. The result listed in Table 2 strongly supports such a characteristic of the method based on non-parametric statistics.

Table. 3 Confidence level for the number of simulations determined by the centered two-sided formula

Order		1	2	3	4	5
Number of simulations		146	221	286	348	406
Centered	Analytical confidence	0.9509	0.9510	0.9502	0.9507	0.9501
Normal	Numerical confidence	0.9511	0.9509	0.9499	0.9514	0.9499
Log-normal		0.9510	0.9514	0.9499	0.9505	0.9502
Uniform		0.9508	0.9510	0.9504	0.9510	0.9504

Simulations determined by the centered two-sided formula

Table 3 summarizes the confidence levels calculated by the formulas and numerical experiments on the basis of the number of simulations determined by the centered two-sided formula. The analytical confidence by the centered two-sided formula is slightly higher than 95% since the quantity of simulations should be an integer.

It is indicated by Table 3 that the numerical confidence for each output distribution is very close to the analytical confidence by the centered two-sided formula, on the contrary to the simulations determined by Wilks' formula. The maximum difference between analytical and experimental confidence is 0.07 percentage points which is negligible. In addition, the distribution-free characteristics of the method are clearly supported by the results listed in Table 3. Thus, it is concluded that the number of simulations determined by the centered two-sided formula should be used in order to obtain the 95/95 tolerance limit for safety analysis of energy generation systems.

From Table 3, it is revealed that there is no significant difference in confidence level with regard to the order of the formula. This indicates that the formula with any order can be used to estimate the 95/95 tolerance limit. It is true that the confidence level is independent of the order of the formula since the confidence level and also the order are independent variables of the centered two-sided formula, as given in Eq. (4). However, the numerical experiment provides additional important insight for the order of the formula. Fig. 1 depicts the probability density of the result from the numerical experiment for normal distribution. As shown in Fig. 1, although there is no significant discrepancy by the order, the probability density distributions of the 95/95 tolerance limit present different behaviors with respect to the order.

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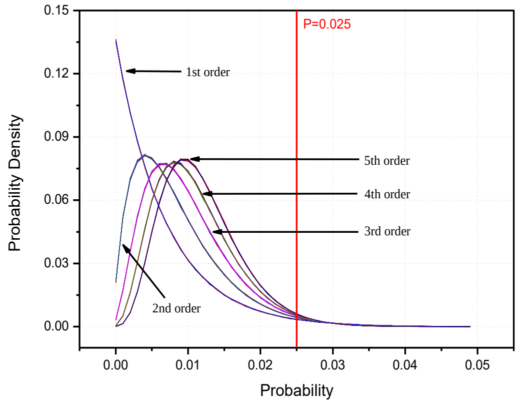
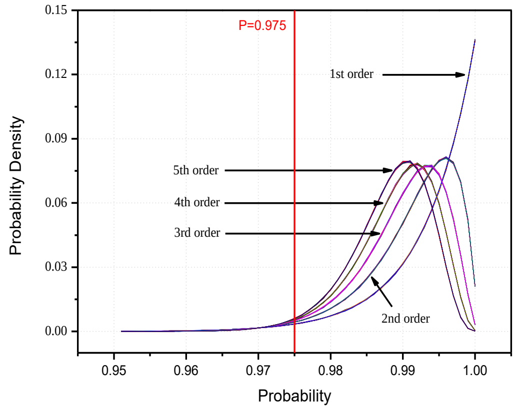
In case of the 1st order, the probability density increases as the value of the 95/95 tolerance limit goes to the end, i.e. zero or unity. As the order increases, the peak of the distribution moves toward 95 percentile. This indicates that there is a larger chance to have the 95/95 tolerance limit far from the 95 percentile when the lower order formula is utilized. Considering that the 95/95 tolerance limit estimated by the non-parametric statistics is the first value beyond 95 percentile, it is concluded that the application of lower order formula will produce more conservative results, whereas higher order formula will result in more realistic result. This result could give guidance to both industry and regulator for the use of non-parametric statistics to the safety analysis of a certain energy system. For example, since a regulator wants to be conservative to secure the safety with a higher confidence, the 1st order formula should be used for the safety audit analysis for licensing to have more margins for safety improvement. In case of industry, it is necessary to use upper order formula to achieve the 95/95 tolerance restriction as realistic as possible.



(a)



(b)

Fig. 1 Probability density of the 95/95 tolerance limits: (a) lower and (b) upper

V. CONCLUSION

In this study, the validity of non-parametric methods to obtain the 95/95 tolerance restriction for the best-estimate safety analysis of the energy generation systems have been assessed by means of a series of numerical experiments for three different trial distributions. The methods based on Wilks' formula and the centered two-sided formula are examined. The numerical experiments were performed for different order of the formula ranged from 1st to 5th orders and trial output distributions examined are normal, lognormal, and uniform distributions.

The result indicates that the number of simulations determined by Wilks' formula is insufficient to obtain a confidence level of 95%. However, the centered two-sided formula estimates the quantity of simulations required to attain the 95/95 tolerance limit appropriately. The difference between analytical and numerical confidences is negligibly small and the results confirm the soundness of the method based on the centered two-sided formula. It is also found that the result from non-parametric method is independent from the shape of the output distributions and the order of the formula. From the probability density distribution of the 95/95 tolerance limits, it is found that the lower order formula will produce more conservative tolerance restriction in comparison to the higher order formula. Thus, it is suggested for regulators to utilizing the 1st order formula for safety audit analysis to secure the safety of the energy generation systems.

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