

First Order of Autoregressive Air Pollution Forecasting with Symmetry Triangular Fuzzy Number based on Percentage Error

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Abstract: Autoregressive (AR) models is known best to predict multiple sets of stationary data. Previous AR model uses single point data, though uncertainties does exist in data due to various factor. When the data contain uncertainty, traditional procedure which is developed to handle the single point (crisp) data is insufficient to deal with the uncertain data. Moreover, unresolved uncertainty in data may increase error in prediction model. That is, data collected that contains uncertainty should be adequately treated before being used for analysis. Hence, this study proposes an first order of autoregressive (AR(1)) model building based on symmetry triangular fuzzy number. The triangles are established from percentage error method during data preparation of AR(1) modelling to address the uncertainty issue. In this study, AR(1) model with fuzzy data is built to forecast air pollution. The result of this study demonstrates that the proposed method of building fuzzy triangles for AR(1) model obtain smaller error in prediction. The improvement on the existing data preparation process sought from this study is expected to give benefit in achieving better forecasting accuracy and dealing with uncertainty in the analysis.

Index Terms: left-right spread, symmetry triangular fuzzy number, AR(1), percentage error, air pollutions.

I. INTRODUCTION

Air pollution phenomena are global issues for centuries. Studies are evolving to find solutions to address the impact of pollution on universal life [1]. One of air pollution data researches falls under time series analysis for building forecasting model. Air pollution analysis and reduction involves various technical disciplines [2]. As a precautionary measure, air pollution forecasts are the basis for effective pollution control measures. It makes precise predictions about air pollution is important. Broad research shows that the methods of air pollution prediction can be divided into three classical categories: statistical prediction method, artificial intelligence method, and numerical prediction method [3]. Air pollution analysis usually results in a time series data. Autoregressive [4] and Average Movement [5] are examples of widely used techniques for analyzing air pollution index behavior based on historical data. The results obtained help to predict the pattern of air pollution [6]–[10].

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Air pollution data may involve uncertainty due to measurement error which produces during pollution exposure assessment [11]. The measurement error is characterized by instrument imprecision and spatial variability [12]. Fault during measuring instrument in the technique used in the experiment give rise to uncertainty involvement in data collection[13]. In short, the collected data or measured value contain uncertainty. When the data with uncertainty is analyzed to build a forecasting model, the uncertainty is carried through to the results, and thus reduce the model's accuracy. Handling uncertainty in data is one of the main challenges in forecasting. A widely used methods to address the uncertainty lies in fuzzy theories [14]. Recent study based on fuzzy time series on air pollution case by [15], where uncertainty involving air pollution in the data collection that obtained from historical data. Another study uses fuzzy theory in air pollution forecast to deal with high uncertainty in carbon dioxide (CO) which will cause wind speed, temperature and amount of sunlight[3]. Since most of the data are obtained from secondary sources, it may contain validity, biasness, and representation issues which contributes to data error that cause less accurate forecasting models [16].

Since measurement error may have substantial implications for interpreting forecasting on air pollution, a systematic data preparation is presented in this paper for dealing uncertainty while data preparation for forecasting. Time-series records single-point data values which best suit to the existing conventional time-series analysis such as autoregressive. However, the existence of inherent uncertainties makes the conventional analysis incapable to deal with such data [17], [18]. Thus, fixing the uncertainties in data during data preparation is necessary as a data preprocessing [19]–[23]. Fuzzy approach is utilized in autoregressive to analyze time-series data which handle the uncertainties. However, the procedure to construct fuzzy number from crisp data are not discussed intensely. Most of the existing approach is focused on the forecasting model itself, while fuzzy data preparation is not thoroughly explained.

A systematic data preparation procedure which involve fuzzy data transformation from single point (crisp) to fuzzy number is important to obtain appropriate fuzzy values for forecasting. In this paper, a systematic procedure is proposed for building a triangular fuzzy number (TFN) which deals the uncertainty in measurement.



The procedure is based on percentage error method to identify the spread to construct symmetry triangular fuzzy number (STFN) within a set of air pollution data. The concentration is made in fuzzy data preparation and model forecasting which are very important to be investigated in improving the forecasting values and achieving the forecasting accuracy. The rest of the paper consists of the following. Section 2 gives a brief review of the related studies. The main method is described in Section 3. Section 4 illustrates empirical studies with developed triangular fuzzy number and its application to air pollution data analysis. Finally, concluding remarks are presented in Section 5.

II. RELATED WORKS

This section explains the fundamental theories including auto-regression model and fuzzy number.

A. Autoregressive (AR) Model

AR model predicts future behavior based on past behavior [24] when there are some correlations between values in time series. In the AR model, the value of result (Y axis) at some point t in time directly related to variable predictor (X axis). The function should include constant to improve the value of the autoregression as written in the following:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + e_t \quad (2)$$

where c is constant, e_t is white noise (error), and $y_{t-1}, y_{t-2}, \dots, y_{t-p}$ are past series. The outcome variable in AR(1) is the process at some point in time, t which is related only to line periods that are one period apart.

From Eq. (2), the expected value of y_t for AR(1) is written as follows:

$$y_t = c + \phi_1 y_{t-1} + e_t \quad (3)$$

B. Fuzzy Number (FN)

Zadeh [14] proposed a fuzzy set theory to deal with vagueness in data through membership grade of a fuzzy set between [0,1] values. The fuzzy set [14] is defined as follows:

Definition 1: Let U denote the universal set of the discourse. Then a fuzzy set A on U is defined in terms of the membership function m_A that assigns to each element of U a real value from the interval [0, 1]. A fuzzy set A in U can be written as a set of ordered pairs in the form $A = \{(x, m_A(x)) : x \in U\}^{(1)}$, where $m_A: U \rightarrow [0, 1]$.

The value $m_A(x)$, expresses the degree to which x verifies the characteristic property of A . Thus, the nearer is the value $m_A(x)$ to 1 indicates the higher membership degree of x in A . The fuzzy number is a special form of fuzzy sets of real number sets on R .

Definition 2: A fuzzy set A on U with membership function $y = m(x)$ is said to be normal, if there exists x in U , such that $m(x) = 1$.

Definition 3: Let A be as in Definition 2 and x be a real number of the interval [0, 1]. Then the x -cut of A , denoted by A^x , is defined as follows:

$$A^x = \{y \in U : m(y) \geq x\} \quad (4)$$

Definition 6: Triangular Fuzzy Number (TFN) [25]: Let \tilde{y} is a triangular fuzzy number with membership function.

$$\tilde{y} = m(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a, b] \\ \frac{c-x}{c-b}, & x \in [b, c] \\ 0, & x < a \text{ and } x > c \end{cases} \quad (5)$$

From Eq. (5), triangular fuzzy number is defined as

$$\tilde{y} = [\alpha_l, c, \alpha_r] \quad (6)$$

where c is the center of the TFN, α_l is the left spread of the TFN and α_r is the right spread of the TFN.

From Eq. (6), a symmetry triangular fuzzy number has same spread, $\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2$. Then, symmetry triangular fuzzy number \tilde{y} is denoted as follows:

$$\tilde{y} = [c, \alpha], \quad (7)$$

where c is center value and α is spread of triangular fuzzy number. \tilde{y} is non-fuzzy number if $\alpha = 0$.

The above information provides prior knowledge for dealing with autoregressive modelling based on symmetry triangular fuzzy number.

III. PROPOSED METHOD: BUILDING SYMMETRIC FUZZY NUMBER WITH PERCENTAGE ERROR (PE) METHOD FOR AR(1) MODELLING

This section discusses the procedure to build the symmetric fuzzy number with percentage error method for developing AR (1) forecasting model. The proposed procedure identifies the time series compliant model earlier before establishing fuzzy triangles from single point data. This step is important to assure only compliant AR (1) dataset model is selected. A systematic step to deal with uncertain data is presented in the step of data preparation for building symmetry triangular fuzzy number. A method called percentage error (Δ_{PE}) is used to determine the spread value for building TFN.

The procedure for building AR(1) with Δ_{PE} is explained as follows:

Step 1. Select n times series datasets in a form of input data as shown in Table 1.

Table. 1 Input data format

Data	1	2	...	n
Input	y_1	y_2	...	y_n

Step 2. AR(1) times series model identification.

- i. Prepare datasets as Table 1.
- ii. Run Final Estimate Parameter test using statistical software to obtain the coefficient, standard error of the coefficient and p -value.
- iii. Examine the result from (ii) to determine either the dataset fulfills AR(1) based on the p -value. The p -value represents the probability of an error when considering the real value of estimated parameter differs from the computed. The dataset comply with AR(1) model if the p -value is 0. Only AR(1) compliant dataset will be used.

Step 3. Build symmetry triangular fuzzy number based on Percentage Error (PE) method, Δ .



i. Generate spread, p .

The method is stimulated from confident interval concept. Based on the confidence interval, the spread of symmetry fuzzy number is adjusted to 5%, 3% and 1%. A fuzzy time series data \tilde{y}_t^p at time, t with triangular fuzzy number data is written in Eq. (8).

$$\tilde{y}_t^p = [y_t - \Delta, y_t, y_t + \Delta] \quad (8)$$

where y_t is time series data at time, t ($t = 1, 2, \dots, n$) and Δ is the spread of triangle which is characterized by $(y_t \cdot p)$ using percentage error method.

ii. Forecast generated spread, s using Ordinary Least Square (OLS).

iii. Find AR(1) model.

$$y_t = \beta_1 - \beta_2 y_{t-1} \quad (9)$$

where y_t is times series, β_1 is constant, β_2 is coefficient and y_{t-1} is past times series.

The time series model based on symmetry triangular fuzzy number y_t^p is written as follows:

$$y_t^p = (\beta_1^L, \beta_1^R) - (\beta_2^L, \beta_2^R) y_{t-1}. \quad (10)$$

(β_1^L, β_1^R) is the left and right constant, and (β_2^L, β_2^R) contain left and right coefficient.

Eq. (9) shows the conventional AR(1) model while Eq. (10) presents AR(1) model based on PE approach.

iv. Find center point for symmetry triangular fuzzy number, \bar{y}_t^p .

The average predicted value is written as follows:

$$\bar{y}_t^p = \frac{(\tilde{y}_t^p + p) + (\tilde{y}_t^p - p)}{2}. \quad (11)$$

\bar{y}_t^p is average predicted value (\tilde{y}_t^p), \tilde{y}_t^p is a left predicted value, \tilde{y}_t^p is a right predicted value and p is the triangular spread based on percentage error of y_t .

Table 2 shows the data format for center point, (\bar{y}_t^p).

Table. 2 Data format for center point, (\bar{y}_t^p)

y_t	y_1	y_2	...	y_n
\bar{y}_t^p	\bar{y}_1^p	\bar{y}_2^p	...	\bar{y}_n^p

Step 4. Calculate the Mean Square of Error (MSE) of AR (1).

After all datasets has been tested, the result of training and testing is analyzed. Eq (12) is used to obtain the total MSE for each y_t^p .

$$MSE = \frac{\sum_{i=1}^n (y_t - \bar{y}_t^p)^2}{n}. \quad (12)$$

where y_t is a time series data and \bar{y}_t^p is a predicted times series data at time, t ($t = 1, 2, \dots, n$) and n is sample size.

Step 5. Validate AR (1) accuracy based on MSE value. The model with lower MSE shows the better accuracy of prediction.

The systematic steps presented here explains AR(1) model building incorporated with fuzzy number construction from single point value to address uncertainties co-exist. This step is important during data preparation ([26], [27]) since the crisp value which is observed as single points are insufficient to interpret uncertainties inherent in data.

IV. EMPIRICAL STUDIES

Four air pollutant index (API) dataset from four locations has been selected, which are SMK Pasir Gudang, SMK Bukit Rambai, Kompleks Sukan Langkawi, and SK Cenderawasih. The timeframe for the data used is from January 1, 2015 to March 31, 2015 with a total of 90 data for each location.

Step 1. Select four times series datasets as shown in Table 3.

Table. 3 Datasets from four location

Data	1	2	...	89	90
SMK Pasir Gudang	50	52	...	40	37
SMK Bukit Rambai	57	59	...	52	50
Komplek Sukan	53	58	...	46	43
Langkawi					
SK Cenderawasih	51	46	...	65	57

Step 2. Identify AR(1) compliant data series.

The dataset from SMK Pasir Gudang is used to demonstrate the time series model identification.

i. Prepare datasets as in Table 4.

Table. 4 Dataset of air pollution index from SMK Pasir Gudang

n	1	2	...	89	90
y_t	50	52	...	40	37

ii. Run Final Estimate Parameter test using Statistical Software and identify Coefficient, SE Coefficient, T-value and P-value as in Table 5.

Table. 5 Final Estimation of Parameters for air pollution index from SMK Pasir Gudang

IPU	Final Estimates of Parameters			
	Type	Coefficient	SE Coefficient	P
SMK Pasir Gudang	AR1	0.9287		0.0659
	Constant	130.861		1.4330
	0.00			
	Mean	1835.88	20.10	

iii. From the result shown in Table 5, dataset of air pollution index from SMK Pasir Gudang fulfills AR(1) requirement. This is based on the p -value which is 0.00. The lowest p -value mean it is more meaningful to the model because changes in the predictor's value are related to the changes in the response variable.

Step 3. Build Triangular Fuzzy Number.

i. Determine spread, p .

The spread of TFN are determined by using percentage error approach (see Section 3 step 3) and is calculated based on Eq. (8). The TFN is constructed as $(\tilde{y}_t^p, y_t, \tilde{y}_t^p)$. Table 6 shows the spread, p of symmetry TFN for 1%, 3% and 5% percentage error.



Table. 6 The possibilities spread, s of TFN

n	y_1	y_2	...	y_{89}	y_{90}
y_t	50	52	...	40	37
$\tilde{y}_t^{0.05}$	47.5	49.4	...	38	35.15
$\tilde{y}_t^{0.05}$	52.5	54.6	...	42	38.85
$\tilde{y}_t^{0.03}$	48.5	50.44	...	38.8	35.89
$\tilde{y}_t^{0.03}$	51.5	53.56	...	41.2	38.11
$\tilde{y}_t^{0.01}$	49.5	51.48	...	39.6	36.63
$\tilde{y}_t^{0.01}$	50.5	52.52	...	40.4	37.37

ii. OLS is used to obtain the forecast model. The predicted result for spread, p is as shown in Table 7.

Table. 7 Predicted result for spread, p of TFN

n	y_1	y_2	y_3	...	y_{90}
y_t	-	49.1372	50.5226	...	42.2102
$\tilde{y}_t^{0.05}$	-	46.6803	47.9964	...	40.0997
$\tilde{y}_t^{0.05}$	-	51.5941	53.0488	...	44.3208
$\tilde{y}_t^{0.03}$	-	47.6631	49.0069	...	40.9439
$\tilde{y}_t^{0.03}$	-	50.6113	52.0383	...	43.4765
$\tilde{y}_t^{0.01}$	-	48.6458	50.0173	...	41.7881
$\tilde{y}_t^{0.01}$	-	49.6286	51.0279	...	42.6323

iii. Find AR(1) models

Four equations are obtained from the information in Table 7 as follows:

$$TFN = 14.5022 - 0.6927y_{t-1} \quad (13)$$

$$TFN_{0.05} = (13.7771, 15.2274) - 0.6927y_{t-1} \quad (14)$$

$$TFN_{0.03} = (14.0672, 14.9373) - 0.6927y_{t-1} \quad (15)$$

$$TFN_{0.01} = (14.3572, 14.6473) - 0.6927y_{t-1} \quad (16)$$

Eq. (13) represents conventional approach while Eq. (14) to Eq. (16) presents the percentage error based for 5%, 3% and 1% respectively.

iv. Find center point of STF, \tilde{y}_t^p based on Eq. (11). This is to transform the predicted values from TFN into a single point (defuzzify) to measure the error. Center point value for spread, p of STF is depicted in Table 8.

Table. 8 Center point value for spread, p of TFN

y_t	y_1	y_2	y_3	...	y_{90}
$\tilde{y}_t^{0.05}$	-	46.8030	48.1226	...	40.2050
$\tilde{y}_t^{0.03}$	-	47.7071	49.0522	...	40.9817
$\tilde{y}_t^{0.01}$	-	48.6505	50.0222	...	41.7921

Step 4. The result of MSE for SMK Pasir Gudang is obtained based on Eq. (12) and is presented in Table 9.

Table. 9 MSE for SMK Pasir Gudang

Location	TFN	AR(1)	AR(1) _{0.05}	AR(1) _{0.03}	AR(1) _{0.01}
SMK Pasir	Traini	18.149	22.3489	19.5370	*
	ng	3			18.2008
Gudan	Testin	151.89	171.415	162.845	**
	g	17	6	7	161.330
					6

* Smallest MSE for Training

** Smallest MSE for Testing

(Training – 72 data, Testing – 18 data.)

Step 5. Validate AR(1) p accuracy based on MSE

In the early stage, four API dataset are evaluated and all four API dataset fulfill the requirement of AR(1) in Step 1. The accuracy of forecasting error can be verified by comparing MSE of each TFN. Table 10 shows the summary of MSE.

Table. 10 Summary of MSE

Location	TFN	AR(1)	AR(1) _{0.05}	AR(1) _{0.03}	AR(1) _{0.01}
SMK Pasir Gudang	Traini	18.149	22.348	19.537	*
	ng	30	9	0	18.2008
	Testin	151.89	171.41	162.84	**
	g	17	56	57	161.3306
SMK Bukit Rambai	Traini	19.066	19.003	19.038	*
	ng	4	8	9	19.0630
	Testin	62.403	62.836	62.553	**
	g	9	2	5	62.4203
Komplek Sukan Langkawi	Traini	42.424	*	42.417	42.423
	ng	0	42.416	2	0
	Testin	44.580	44.678	**	44.584
	g	9	6	43.541	1
SK Cenderawasih	Traini	89.218	*	89.134	89.207
	ng	3	88.995	1	9
	Testin	139.69	140.74	140.06	**
	g	99	06	93	139.7444

* Smallest MSE for Training

** Smallest MSE for Testing

(Training – 72 data, Testing – 18 data.)

In the validation step, although a result of MSE for training data is better, we consider a result of MSE for testing data. This is because testing MSE is made using its own data whilst training MSE uses previous data to forecast. By referring to Table 10, training AR(1)_{0.01} and AR(1)_{0.05} score an equal result. Even though the score for training is equal, but when it comes to testing, the decision suggests otherwise. Additionally, other testing for SMK Pasir Gudang, SMK Bukit Rambai and SK Cenderawasih shows the impressive result because the AR(1)_{0.01} is better than AR(1)_{0.03}, AR(1)_{0.05} and conventional approaches. This is also proven that the idea to construct STF from confidence interval is relevant because in the confidence interval, the smaller its spread, the better the result. From Table 10 also, the proposed model at least can achieve or have better accuracy result than conventional method.



V. CONCLUSION

Constructing appropriate triangular fuzzy number to represent the fuzzy data is very important in forecasting because it contribute in a model building and affect the outcomes. In this study, a procedure to build an adjusted STFV for AR(1) forecasting is presented. The proposed procedure is important in handling uncertainty contained in the data used for forecasting. This procedure also suggests the technique to identify the spread of TFN clearly as compared with other studies. From the experiment, the result shows that $AR(1)_{0.01}$ is better than other approaches. Furthermore, result concludes that the proposed procedure is able to compete efficiently the conventional approach, yet it solves the uncertainty issues in the input data. Additionally, this procedure also can serve as a guideline to build STFV systematically.

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