

# Pedestrian Evacuation Route Plan using Existing Optimal Exit Selection Algorithms

Fadratul Hafinaz Hassan, Nur Syifa Abdul Razak, Najihah Ibrahim, Wong Li Pei, Mohamed Najib Salleh

**Abstract:** *The optimal exit selection by the pedestrians is important especially for the evacuation process during panic situation to reduce casualties. Nowadays, the efficiency of the evacuation process had become one of the issues during emergency incidents due to the difficulties faced by the pedestrians to access the exit points and the formation of clogging region near to the exit points and narrow pathways. Hence, the simulation of the pedestrian evacuation had been introduced as the potential solution exploration by imitating the emergency situations for predicting the survival rate and the casualties of the pedestrians involved. The pedestrian movement in the simulation will construct the movement patterns that are able to assist the development of the emergency route plan for guiding the pedestrians to find the nearest exit to escape. However, the route plan designed based on the ordinary pedestrian movement simulation will highlight the nearest exit instead of the optimal safest exit for the pedestrian to evacuate safely. Hence, to design a low-risk exit selection, this research had made a survey on three existing algorithms to optimize the exit selection process; multi-agent based algorithm, least effort algorithm and game theory algorithm. The experiments were conducted with three different structural layouts (number of doors) with variety number of dataset (number of pedestrians). The result of this research had shown that the game theory algorithm was able to assist the pedestrian movement towards the optimal nearest and safest exit for evacuation process and capable to enhance the simulation for designing a feasible emergency route plan to reduce casualties.*

**Index Terms:** *Pedestrian Movement, Evacuation Route Plan, Optimization, Exit Selection.*

## I. INTRODUCTION

The emergency cases in Malaysia has increased proportionately with the growth or the urban constructions and had destroyed approximately 6000 premises annually [1]. The numbers of emergency cases are soaring with the high complexity of the building structures, the layout arrangements that are majoring in functional utilities, and the continuous large crowd access of a space such as; shopping complexes, airports, sports venues, schools, hotels and many more.

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There are some emergency incidents had happened in these recent years such as; 1) the Mumbai Elphinstone Road railway station stampede incident in 2017 that killed 22 people and injured 39 people due to the dumbbell effect of a narrow pathways for the heavy crowd to cross the bridge that connect the two railway stations (Parel railway station and Elphinstone road railway station), 2) the traumatic fire incidents at a religious school in Malaysia that caused the death of 25 person in 2017 due to the limited movement space and alleged structural design with the limited number of egress point, 3) the stampede of students during recess break in the toilet at Number Three Experimental Primary School in Puyang county, China in 2017 that killed a student and injured 22 students due to the overcrowding, non-standardize structural design, and low quality of buildings' design and architecture, and many more incidents that caused high number of casualties and injuries [2-4].

Building's structure is one of the main contributors towards the safety of the pedestrians since the structural had become the intermediate medium that separates the spatial layout inside and the low-risk ground outside [5]. There a lot of researches on the building's structural design for reducing the layout complexity, to create a pedestrian friendly space and to reduce the physical pressure and congestion of the pedestrians especially during evacuation process [6-8]. The main structural element that separates the effected space and the outer low-risk's space are the doorways (exit points /egress points). Hence, there are also many researches on the ingress/ egress (exit points) placement, size, fire proof and many more to determine the influence of the access points towards the evacuation process of the pedestrians and also for determining the aftermath highlight of panic situation; the casualties and injuries of the pedestrians.

Based on some research findings, besides the structural of the layout, the arrangement of the materials inside the space is also becomes one of the contributing factor that is able to disrupt the pedestrian evacuation process and leads to a huge casualties incident due to the high possibility of physical collision [7, 9, 10]. During evacuation process, the pedestrians will keep on moving in fast speed, which will increase the probability of collision with the static obstacles such as furniture materials and even the dynamic obstacles such as other pedestrians. The standard reflex of the pedestrians will create some repulsive actions by avoiding the obstacles. Due to the indication of pedestrians' collision avoidance, there are several researches findings on the re-direction of the pedestrian movement that highlighted the solution condition that is when the pedestrians' direction



paths were shaped by the arrangement of the materials (obstacles, specifically the static obstacles) inside the spatial layout [7, 9].

During evacuation, the movement process of avoiding the obstacles and also avoiding the other pedestrians will result in more collisions, tensions and pressures especially near to the ingress/egress points that forming the clogging region. This situation had been described by Helbing as the “freezing by heating” state [9, 11]. The poor layout arrangement and inadequate structural design are able to cooperatively causing the clogging region near to the ingress/egress due to; 1) the abrupt movement of the pedestrians while finding the shortest path towards the nearest exit point, 2) the heavy physical collision and 3) the opening magnitude of the ingresses/egresses [8, 12]. The clogging region formed will entrap the pedestrians in the affected space and increase the probability of casualties and injuries.

The commercial or residential buildings are one of the attraction centers among citizens that able to lead towards the serious congestion issue and forming the clogging region especially near the egress and also the narrow pathways. The continuous of clogging region formation will increase the pressure among the pedestrians towards the center of attraction (exit point/ egress) that will promote the emergency situations such as crush and stampede incidents and will cause great injuries and casualties. The entrapment of the pedestrians in the large crowd and the heavy collision was happened the most due to the wrong selection of exit and ended up caught up in the clogging region [13]. Hence, the familiarity of the pedestrians towards the surrounding arrangement and structure are the most important factors that will be able to assist the pedestrian to move towards the exit point during evacuation process. However, the spatial layout familiarity is the quality of knowledge that exclusively valuable for the pedestrians to select the right exit but will not be able to promise the safest selection of exits by the pedestrians.

In addition, the wrong selection of the exit also was made by the other two indications; 1) the nearest exit distance between the exit with the pedestrian’s standing ground and 2) the pedestrian movement was redirected by collision avoidance stimulation. Hence, the emergency route plan is important to direct the pedestrian movement towards the safest exit that provides the optimal solution on the exit selection. This emergency management will control the movement of the pedestrians by designing the suggestion directions for reducing the number of injuries and fatalities.

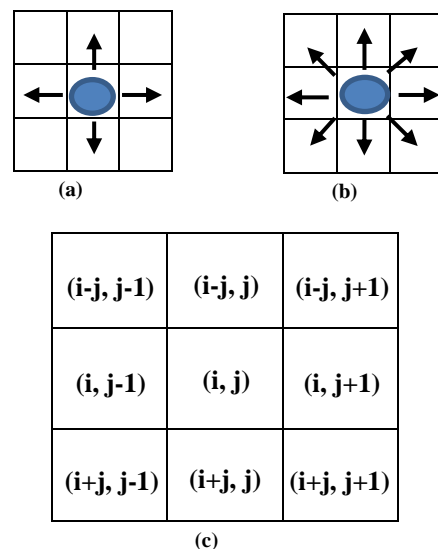
The emergency route plan can be established by simulating the spatial layout with the best number of pedestrians involved in the worst case scenarios. The simulation will take place with the modeling of the pedestrians’ movement towards the low-risk place outside of the spatial layout. The emergency route design should be providing the safest and fastest route plan for the pedestrians to optimize the exit selection. To explore and analyse this safest route plan occurrences, nowadays, there are a lot of researches on the pedestrian route selection behavior during emergency situations. There are researches on the route selection of the pedestrians are based on the impact of the social force towards the individual agent decision making.

The decision on the route selection will be made based on the selection algorithms by the researchers that had display some of the strong human decision making imitation for the realistic judgment and movement while increasing the accuracy of the optimal exit selection. Hence, this research is aiming to survey the available and reliable algorithms for optimizing the exit selection for future pedestrian evacuation route plan development to reduce the casualties and injuries.

## II. EXISTING ALGORITHMS FOR OPTIMAL EXIT SELECTION

Various researches were proposed for finding the suitable algorithms to design the emergency evacuation route plan. There are some researches on the effect of realistic movement by using the imitation model such as the social force model and agent-based model [14-18].

The realistic movement model for the pedestrian movement simulation is important for the accuracy of the result. Therefore, there are a lot of algorithms were discovered to represent the model of the pedestrian movement. In this research, the main movement and reflex of the pedestrian will be represented by using the agent-based model in referring to discrete structure of human individualistic. For each time step, the movement will be determined by the navigation concept of the enhanced Von Neumann movement model; the Moore Neighborhood movement model that had shown in Figure 1.



Note- The blue color cells are the pedestrian as the discrete microstructure element and the arrows are the possible movement direction for every timestep based on the navigation approach.

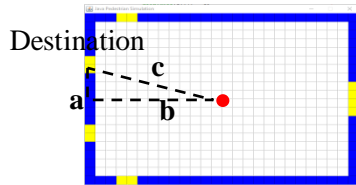
Fig. 1(a) The von Neumann approach in movement simulation model

Fig. 1(b) The Moore approach in movement simulation model

Fig. 1(c) The transition probabilities of the movement over a time step

Through this agent based model, the previous findings had highlighted the enhancement of the pedestrian movement especially for finding the exit point by using the Euclidean geometry; the Pythagorean Theorem. Equation 1 had shown the Pythagorean Theorem and Figure 2 had shown the implementation of the formulation to find the nearest exit point in the layout by using agent based model.

$$a^2 + b^2 = c^2 \quad (1)$$



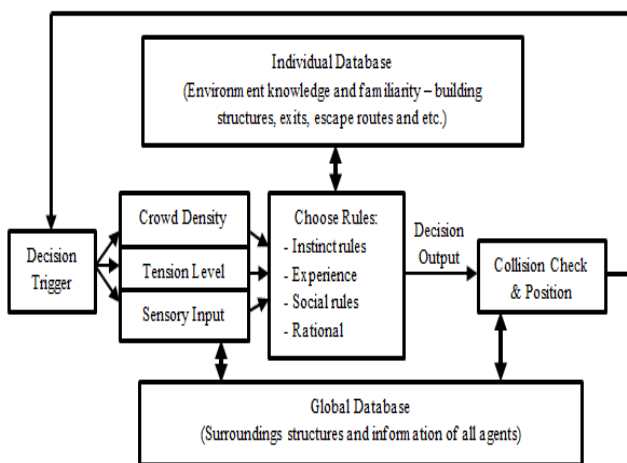
Note- The blue color cells are the walls, the yellow color cells are the exit points (pedestrian destination) and the white color cells are the floor of the spatial layout. The red color cell is the pedestrians.

**Fig. 2 The example of the implementation of Pythagorean Theorem in designing the pedestrian movement to find the nearest exit**

Based on Figure 2, the Pythagorean Theorem will cause the pedestrian to movement towards the nearest exit point instead of the safest exit point. Hence, this research had studied the existing algorithms that are able to optimize the exit selection by the pedestrian during movement process; 1) multi-agent based algorithm, 2) least effort algorithm, and 3) game theory algorithm.

### A. Multi-Agent based Algorithm

Agent-based model is the representation of the discrete microstructure of a spatial layout. However, there are some enhancements on the agent-based model for the pedestrian movement approach by being the building block for designing the multi-agent based model that display the interaction between the pedestrian based on the macroscopic or mesoscopic movement approaches. Figure 3 shown the basic individual behavior model based on the researched by [19].



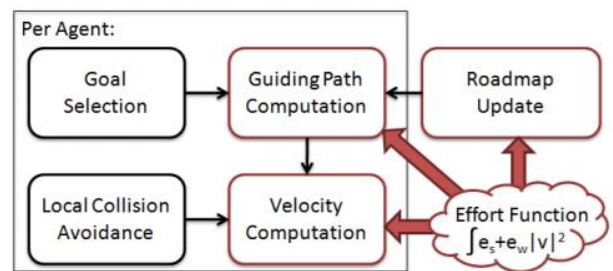
**Fig. 3 Pedestrian individual behavior movement model [19].**

Based on Figure 3, the pedestrian decision will be

triggered by the crowd density, tension level and the sensory input for the physical collision or any harmful gestures. The pedestrian will interpret the trigger points and act based on the reflex actions based on the instinct rules, former experiences, social rules and rational judgment. The pedestrian will make the collision check for the neighboring surroundings and will update to the next position. Multi-agent based algorithm had cooperate the agent-based model of the human individual behavior characteristics with the human social behavior such as herding, shoving, queuing, competing and many more that impactful towards the pedestrian movement direction especially during panic situation. The pedestrian behavior and the movement characteristics on the spatial layout were shaped based on the reflex actions towards the surroundings' element [19, 20]. This fact was discussed by Sime on the impact of the surrounding's elements towards human psychology for decision-making or stimulating any actions [10, 21]. The multi-agent algorithm is one of a computational process that enables the construction of a simulation with environment of autonomous agents that are capable to interact with each other. This algorithm approach is suitable for simulating individual cognitive processes and behavior and for exploring emergent phenomena, i.e. social or collective behaviors. The research by [22] had stated that a multi-agent based evacuation simulation is used to study the pedestrian dynamics and learning process.

### B. Least Effort Algorithm

Least effort algorithm is the one of the method on decision making for exit selection based on the impatient of evacuate and the crowd density [23, 24]. This algorithm process was adapted from the biomechanical principles for the agent's guidance in the crowd movement simulation, specifically to find the exit point. The pedestrian movement will be designed for finding the shortest path towards the nearest exit while avoiding the collision and congestion with the minimum of energy usage [25-27]. Figure 4 show the basic principle of least effort movement process for exit selection by the pedestrian to evacuate.



**Fig. 4 Principle of Least Effort Algorithm [28]**

Based on Figure 4, [28] had specified that the least effort algorithm for the pedestrian movement decision on the exit selection will be adjusted based on the four combination of trigger points for the pedestrian to move and update for the next position.



The pedestrian will set the goal selection for evacuating that is to find the exit point while validating the nearest exit for the path guidance. However, the pedestrian will consider the least effort action that will cause minimum energy consumption that decrease the probability of physical collision and will move at preferred speed [24, 28].

These characteristics of the pedestrian movement had shown the impact of the multi-agent and the social force towards the pedestrian movement direction and movement speed [24].

**C. Game Theory Algorithm**

Game theory offers insights of fundamental importance for scholars in all branches of the social sciences, as well as for practical decision-makers. In this research context, the game theory algorithm is the procedure to overcome the conflicts and cooperation between the pedestrians and the other elements in the layout by making an intelligent and rational decision. In this approach, the situation will be analyzed and the decision and the interactive behaviors of the pedestrian will be influenced by every neighboring agent [18, 29]. The pedestrian will update the movement directions periodically and will change the target exit a few times before actually exiting from the space [30]. The framework was made by [29] for pedestrian evacuation process to find the optimal exit point based on the game theory algorithm is shown in Figure 5.

The pedestrians will strive to maximize the movement direction selection for optimal result of selecting the best exit point for the fastest route [31]. Based on the basic framework of the game theory approach for exiting the space in Figure 5, the pedestrian will move to the next node with a collective of strategies to optimize the response to other pedestrians' actions (Nash equilibrium). The strategies are based on the decision making judgment; 1) the familiarization of the pedestrian with the exits, 2) the visibility of the exits, and 3) the length queue of the pedestrians at the exits [30].

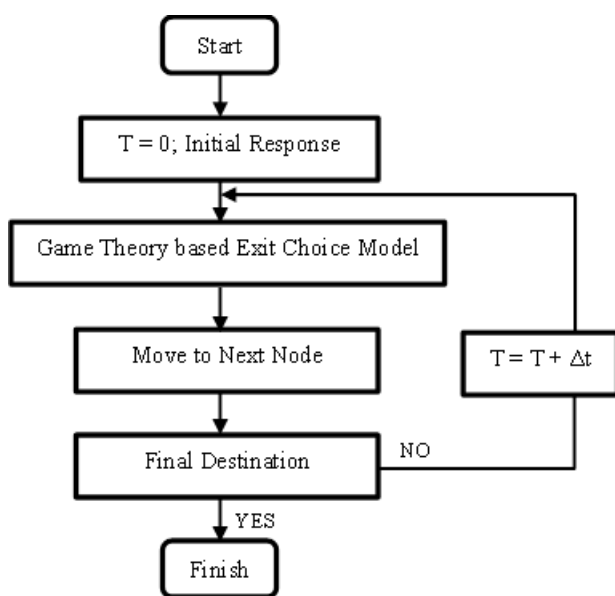


Fig. 5 The Evacuation Process for Each Person [32]

**III. RESULT AND DISCUSSION**

In this research, the surveys were made on the algorithms used for the pedestrian exit selection process during pedestrian evacuation simulation; 1) multi-agent based algorithm, 2) least effort algorithm, and 3) game theory algorithm. These existing algorithms were analyzed and compared based on; 1) number of pedestrians, 2) evacuation time (time taken in seconds for all of the pedestrians in the spatial layout to exit and clear the effected space), and 3) number of exits. These experiments were conducted to find the suitable algorithm to assist the pedestrian movement for finding the optimal exit with the values and the parameters described;

Parameters	Value(s)
Number of pedestrians	100, 200, 300, 400, 500
Size of pedestrians (radius)	0.75 m
Number of exits	2, 3, 4
Size of exits (cell size)	1.0 m
Layout size	24x24 cells (Each cells = 1.0 m)
Pedestrian speed	5.0 ms <sup>-1</sup>

Based on the parameters and the values setup, the simulation experiments were compiled and implemented using MATLAB R2017b. Table 1 shows the result of average evacuation time from different algorithms.

**Table. 1 Experimental results based on the experiments for multi-agent based algorithm, least effort algorithm, and game theory algorithm using the parameters and values setup**

Algorithm	Number of Pedest.	Evacuation Time (seconds)		
		2 Exits	3 Exits	4 Exits
Multi-Agent based Algorithm	100	56.2	46.4	39.5
	200	65.7	52.3	46.3
	300	69.8	57.0	50.2
	400	74.2	61.4	53.4
	500	78.7	65.2	57.3
Least Effort Algorithm	100	57.2	45.8	39.3
	200	63.6	53.4	46.8
	300	69.9	57.9	50.6
	400	75.8	62.1	52.7
	500	78.3	65.0	56.7
Game Theory Algorithm	100	52.9	44.3	38.9
	200	61.5	51.7	44.8
	300	65.6	56.5	49.9
	400	70.5	60.4	52.8
	500	72.5	62.8	56.7



Table 1 shows the result of the evacuation time for different algorithms with variety number of pedestrians and structural effect (number of exits). Based on the results, the number of pedestrians shows a great impact on the time consumption for the evacuation process. The algorithms had shown the exponential growth in evacuation time (seconds) with the increment of the number of pedestrians. However, the number of exit also had played a crucial gap of highlighting the precision of the algorithms' process in assisting the increasing number of pedestrian to evacuate from the spatial layout safely within short amount of time. The pressure on the distribution of the clogging region on the exits can be define by manipulating the basic mathematical formulation in Equation 2 and the clogging region distribution will be listed on Table 2.

$$\rho = \frac{F}{A} \quad (2)$$

where the pressure,  $\rho$  is the amount of the force (number of pedestrians),  $F$  applied to the surface on contact (number of exits),  $A$ .

Based on Table 2, the pressure at the clogging region will be increased with the number of pedestrians. Hence, the exponential growth of the evacuation time towards the number of pedestrian can be overcome with the structural enhancement of the spatial layout by adding more ingresses/egresses to reduce the higher force at the exit points. For all of the algorithms, the process in finding the triggering points are also included the pedestrian crowdedness at the exit points that display the sensory input and tension level for the multi agent based algorithm, the usage of maximum energy due to the physical collision and the visibility of the exit and the length of queue for game theory algorithm.

**Table. 2 The clogging regions' pressure's distribution based on the number of exits.**

Number of Pedestrians	Clogging Pressure per Exit		
	2 Exits	3 Exits	4 Exits
100	50.00	33.33	25.00
200	100.00	66.67	50.00
300	150.00	100.00	75.00
400	200.00	133.33	100.00
500	250.00	166.67	125.00

Based on the results in Table 1 with the assist of the clogging pressure growth per exit in Table 2 as the benchmark, the Game theory algorithms had shown a good short amount of time (seconds) taken by the pedestrian to evacuate from the layout. The pedestrian evacuation simulation based on the multi agent based had slower movement due to the collective behavior decision making that had cause the pedestrian to be more careful for not to reach the exit point that have a high crowd density, tension level and also high sensory input. Hence, the pedestrian will involve in hesitation of moving that had caused the decision making in movement direction become slower and had impacted on the time taken by the pedestrian to evacuate from the spatial layout.

The least effort algorithm had shown some good results that being on par with the game theory algorithm during the involvement of more exits spatial layout (4 exits). The least effort action that requires minimum energy usage had really impacted the pedestrian evacuation time (seconds). To decrease the probability of physical collision, the pedestrian will be wandering in the spatial layout to find the right exit point to cause least amount energy to be used. The fact that the pedestrian will move at their preferable speed in the spatial layout for the least effort algorithm process will be delayed the movement and increase the time taken for the evacuation process. Hence, the pedestrian might entrap in the spatial layout for a long time and will increase the chances of injuries and casualties. Based on the result, the game theory algorithm had shown the best evacuation time (seconds) compared to the other two algorithms for both the condition as for different exits and the condition on the increment of the number of pedestrians. The result in Table 1 also shows that the time taken by the pedestrian to move out from the layout is more consistent with game theory algorithm implementation, compared to the other two algorithms.

#### IV. CONCLUSION

From the studies, it can be concluded that every algorithm have different approach in choosing the optimal exit point by depending on the model, parameters and objectives. Additionally, the result obtained by different algorithms shows that:

- The number of pedestrians able to influence the movement performance during evacuation process;
- The number of exits and size of exits also able to influence the evacuation time;
- The game based theory is the suitable algorithm for optimizing the exit selection as the result had shown the minimum time taken by the pedestrians to evacuate from the spatial layout.

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#### REFERENCES

1. M. Muhamading, "About 6,000 premises destroyed by fire annually in Malaysia," in News Straits Times, ed, 2016.
2. Eloise Stevens and V. Doshi, "Stampede at Indian train station kills at least 22," in The Washington Post, ed. Washington, D.C, USA: The Washington Post, 2017.
3. B. N. Jay, "Tahfiz did not have fire exit; bodies found piled on top of each other," in New Straits Times, ed. Malaysia: New Straits Times Press (M) Berhad, 2017.



4. K. Xie, Y. Song, J. Liu, B. Liang, and X. Liu, "Stampede prevention design of primary school buildings in china: a sustainable built environment perspective," *International journal of environmental research and public health*, vol. 15, p. 1517, 2018.
5. J. Huixian and Z. Shaoping, "Navigation system design of fire disaster evacuation path in buildings based on mobile terminals," in *2016 11th International Conference on Computer Science & Education (ICCSE)*, 2016, pp. 327-331.
6. Y. Gan and Z. P. Feng, "The Cognitive Airport Signage System Design: Comparative Case Study Between American Airport and Chinese Airport," *Cham*, 2018, pp. 69-75.
7. F. H. Hassan, N. Ibrahim, H. W. Kit, and R. Abdullah, "Incorporating Genetic Algorithm Operators in Optimization Spatial Layout Design," presented at the *Proceedings of the International Conference on Advances in Image Processing*, Bangkok, Thailand, 2017.
8. F. H. Hassan, "Using microscopic pedestrian simulation statistics to find clogging regions," in *2016 SAI Computing Conference (SAI)*, 2016, pp. 156-160.
9. D. Helbing, I. Farkas, and T. Vicsek, "Simulating dynamical features of escape panic," *Nature*, vol. 407, pp. 487-490, 2000.
10. J. F. Kihlstrom, "The person-situation interaction," *The Oxford handbook of social cognition*, pp. 786-805, 2013.
11. D. Helbing and A. Johansson, "Pedestrian, Crowd, and Evacuation Dynamics," in *Encyclopedia of Complexity and Systems Science*, R. A. Meyers, Ed., ed New York, NY: Springer New York, 2009, pp. 1-28.
12. N. Ibrahim, F. Hafnaz Hassan, N. Muzlifah Mahyuddin, and N. Abd Ra, "Cellular Automaton based Fire Spreading Simulation in Closed Area: Clogging Region Detection," 2018, vol. 7, p. 6, 2018-12-01 2018.
13. N. Ibrahim and F. H. Hassan, "Optimal Moore Neighborhood Approach of Cellular Automaton Based Pedestrian Movement: A Case Study on the Closed Area," *Cham*, 2019, pp. 57-71.
14. D. Li and B. Han, "Behavioral effect on pedestrian evacuation simulation using cellular automata," *Safety science*, vol. 80, pp. 41-55, 2015.
15. L. Wang, J.-H. Zheng, X.-S. Zhang, J.-L. Zhang, Q.-Z. Wang, and Q. Zhang, "Pedestrians' behavior in emergency evacuation: Modeling and simulation," *Chinese Physics B*, vol. 25, p. 118901, 2016.
16. L. Hou, J.-G. Liu, X. Pan, and B.-H. Wang, "A social force evacuation model with the leadership effect," *Physica A: Statistical Mechanics and its Applications*, vol. 400, pp. 93-99, 2014.
17. M. Apel and K. WALDEER, "Simulation of pedestrian flows based on the social force model using the verlet link cell algorithm," *Karl-Scharfenberg-Fakult at Salzgitter, Institut fur Simulation und Modellierung*, 2004.
18. F. Farina, D. Fontanelli, A. Garulli, A. Giannitrapani, and D. Prattichizzo, "Walking ahead: The headed social force model," *PLoS one*, vol. 12, p. e0169734, 2017.
19. X. Pan, C. S. Han, K. Dauber, and K. H. Law, "A multi-agent based framework for the simulation of human and social behaviors during emergency evacuations," *Ai & Society*, vol. 22, pp. 113-132, 2007.
20. Y. Bo, W. Cheng, H. Hua, and L. Lijun, "A multi-agent and PSO based simulation for human behavior in emergency evacuation," in *Computational Intelligence and Security, 2007 International Conference on*, 2007, pp. 296-300.
21. J. D. Sime, "Crowd psychology and engineering," *Safety Science*, vol. 21, pp. 1-14, 11// 1995.
22. M. E. Yuksel, "Agent-based evacuation modeling with multiple exits using NeuroEvolution of Augmenting Topologies," *Advanced Engineering Informatics*, vol. 35, pp. 30-55, 2018.
23. S. Mejia, J. Lugo, R. Doti, and J. Faubert, "Pedestrian modeling using the least action principle with sequences obtained from thermal cameras in a real life scenario," *International Journal of Computer Information Systems and Industrial Management Applications*, vol. 9, pp. 145-152.
24. L. Fu, W. Song, W. Lv, and S. Lo, "Simulation of exit selection behavior using least effort algorithm," *Transportation Research Procedia*, vol. 2, pp. 533-540, 2014.
25. G. K. Zipf, *Human behavior and the principle of least effort: An introduction to human ecology*: Ravenio Books, 2016.
26. S. Sarmady, F. Haron, and A. Z. Talib, "Simulating Crowd Movements Using Fine Grid Cellular Automata," in *2010 12th International Conference on Computer Modelling and Simulation*, 2010, pp. 428-433.
27. S. Sarmady, F. Haron, and A. Z. H. Talib, "Modeling groups of pedestrians in least effort crowd movements using cellular automata," in *2009 Third Asia International Conference on Modelling & Simulation*, 2009, pp. 520-525.
28. S. J. Guy, J. Chhugani, S. Curtis, P. Dubey, M. Lin, and D. Manocha, "Pedestrians: a least-effort approach to crowd simulation," in *Proceedings of the 2010 ACM SIGGRAPH/Eurographics symposium on computer animation*, 2010, pp. 119-128.
29. R.-Y. Guo, "New insights into discretization effects in cellular automata models for pedestrian evacuation," *Physica A: Statistical Mechanics and its Applications*, vol. 400, pp. 1-11, 2014.
30. H. Ehtamo, S. Heliövaara, S. Hostikka, and T. Korhonen, "Modeling evacuees' exit selection with best response dynamics," in *Pedestrian and Evacuation Dynamics 2008*, ed: Springer, 2010, pp. 309-319.
31. T. Korhonen and S. Heliövaara, "Fds+ evac: herding behavior and exit selection," *Fire Safety Science*, vol. 10, pp. 723-734, 2011.
32. S. M. Lo, H.-C. Huang, P. Wang, and K. Yuen, "A game theory based exit selection model for evacuation," *Fire Safety Journal*, vol. 41, pp. 364-369, 2006.