

# A Graph Theory Based Power System Restoration Plan

S. Hemalatha, R. Srinivasan, A. Ruban Raja

**Abstract:** Power system restoration planning is necessary in order to make the restoration more reliable and effective, thereby preventing further blackouts during the restoration process. In this paper an efficient graph theory based restoration plan is used in place of heuristics based algorithms. In the proposed plan, an extension of Dijkstra's algorithm is being used to obtain a reliable power system restoration. The restoration plan is executed in WSCC 9 Bus System and New England 39 Bus System.

**Index Terms:** Power System Restoration, Load Pickup, Dijkstra's Algorithm, Graph Theory Technique

## I. INTRODUCTION

Countries which are growing in terms of economy, especially when there is an economic boost, have an ever increasing power demand, which further increases the complexity of power systems upon development. This makes understanding of power networks even more difficult, which implies an even more complex understanding of power system restoration process. Even though modern control and protection devices and technologies are being employed, the chances of a blackout occurring is still inevitable in worst case conditions.

For example, the blackout in India in 30 and 31 July 2012 led to severe social impacts and economic losses [5]. The blackout happened in 2003 (USA and Canada) [6] led to severe economic losses. The power blackout of 1995 in Israel [9] is also a good example for this. Also, the power outage caused by a malicious attack in 2016 (Ukraine) explains why study of power system restoration is the need of the hour. These things show that power system blackouts are low probability high impact events; hence there should be a proper way to restore the system back to normal state. Therefore, how fast power system restoration takes place becomes important to prevent losses and havoc.

Such a power system, when has to be restored has to follow many constraints such as black-start sequencing of generators as in [3] and [11] considering the ramping rates [8], generator pickup characteristics ([3] and [7]), load characteristics, power balance between generation and load as in [12] and [13], capacity of the line, voltage control, frequency limits [7] and load priority.

There are multiple ways to solve the power restoration problem. Previously, power system restoration planning was used to be knowledge-based [10], in which the subject matter experts used to find the right solution for this. In these mechanisms in earlier case they used to sectionalize the power system into islands, restore each island and synchronize the island. Then the process such as black-start, network reconfiguration and load restoration were considered [4], and still this strategy is being adopted till date, but with the constraint as the power system should be radial in nature [7].

However, the advent of mathematical programming enabled power system engineers and researchers have a good approach to solve restoration problem in an easier manner. Algorithms involving heuristics search were involved in solving the restoration problem along with Genetic Algorithm as in [1] and [2]. For the network reconfiguration part algorithms like Prim's search algorithm was used along with Dijkstra's algorithm for finding optimal path in [2].

However the usage of graph theory technique in mathematical programming concept is to be noted here. In case of power system optimization genetic algorithm is being used, but using graph theory techniques applied for finding the optimal path in the network (to reduce power losses) can become a suitable replacement for genetic algorithm.

In this work an extended Dijkstra's algorithm is being used along with the constraints as mentioned in Section IV. Previous works insisted that the network should be radial in nature as a constraint. The proposed algorithm is applied to WSCC 9 Bus System and then applied to New England 39 Bus System. The networks are simulated using PowerWorld Simulator version 20.

Further sections are listed as follows. Section II gives an insight about the details of Dijkstra's algorithm and how it is implemented in bulk power system restoration. However Dijkstra's algorithm alone is not enough to solve power system restoration problem, hence additional constraints related to power systems should have to be added. These constraints are discussed in Section III. Section IV gives the proposed algorithm. The test results are being explained in Section V. Concluding remarks and future scope are given in Section VI.

## II. PRELIMINARIES

In graph theory, shortest path problem is the problem of finding a path between two nodes in a graph such that the sum of weights of edges of it is minimized.

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Generally shortest path problems are applied in the transportation sector, wherein the shortest path between the source node and destination node are found out. The above problem mentioned is also called as the single-pair shortest path problem, and it consists of three types – the single-source shortest path problem, single-destination shortest path problem and all-pairs shortest path problem.

Dijkstra’s algorithm solves the single-source shortest path problem with non-negative edge weight, Bellman-Ford algorithm does the same if the edge weights are negative too, A\* search algorithm solves for single pair shortest path using heuristics, Floyd-Warshall algorithm and Johnson’s algorithm solve all pairs shortest paths. When it comes to time complexity and constraints Dijkstra’s algorithm serves the need of the idea proposed.

**Dijkstra’s Algorithm**

Dijkstra’s algorithm is a single source shortest path finding algorithm for identifying the shortest path in a weighted graph with non-negative weights. It is an example for a greedy algorithm.

**Pseudocode for Dijkstra’s algorithm**

The pseudocode for Dijkstra’s algorithm is given as follows:  
 Define array NW of size [V], a variable infinite and assign a big number to it, define a variable MIN.  
 BEGIN  
 Step 1: for all node n do  
     NW[n] = infinite  
 for end

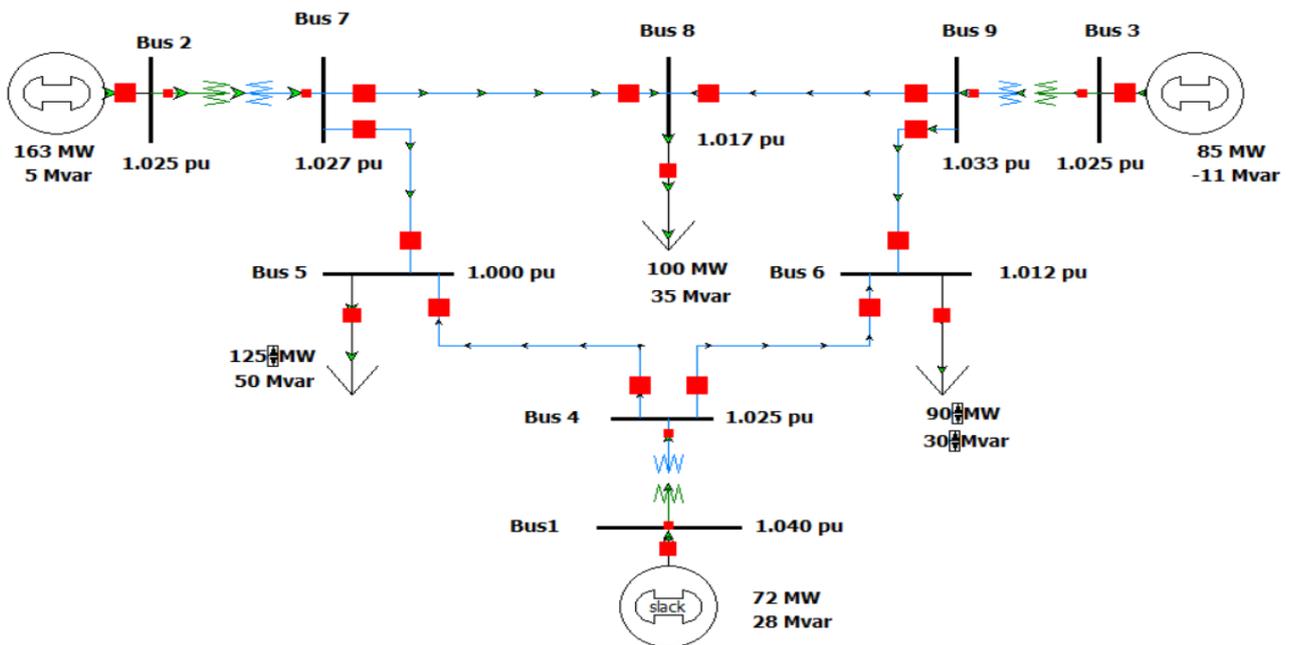
Step 2: Add the node S in queue  
 Step 3: while (queue is not empty) do  
     MIN = infinite  
     For all queued node m do  
         MIN = minimum(MIN, NW[m])  
     for end  
     for all node m in queue do  
         if (NW[m]=MIN)then  
             remove m from queue and add to settled group.  
             for all edge [m,p] (member of) E do  
                 NW[p] = minimum (NW[p], NW[m] + W[m,p])  
                 Add node p in queue if it is not present in the queue.  
             for end  
         if end  
     for end  
 while end  
 END

Dijkstra’s algorithm in the list form has a time complexity of  $O(V^2)$ .

**Implementation of Dijkstra’s Algorithm in 9 Bus System**

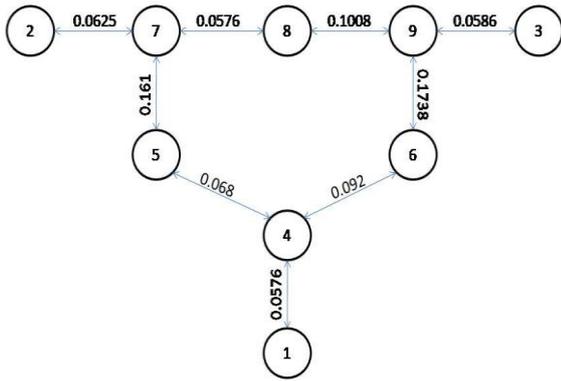
The transmission line impedances are taken in the form of an adjacency matrix, wherein the line impedance shows the impedance between the source node and nearby node (i.e., bus). The adjacency matrix values are the weights across the vertices.

The single line diagram for WSCC 9 Bus System is given as follows:



**Fig. 1 Single line diagram of WSCC 9 Bus System**

The nodal diagram is obtained considering the transmission lines as vertices, line impedance as weights across the vertices and buses as nodes. Hence the nodal diagram of the above WSCC 9 Bus System is given as follows:



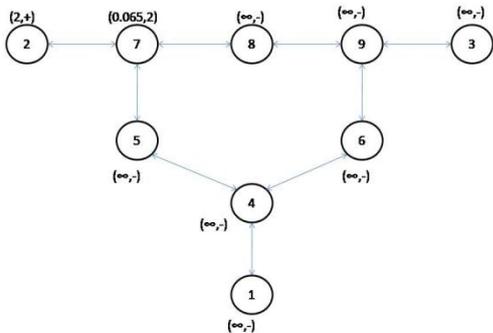
**Fig. 2 Nodal Diagram for WSCC 9 Bus System**

In the implementation described below, the source node is taken as node 2, the generator bus. In this implementation the shortest path is found between node 2 and node 6, which means that the optimal path for power flow is found out from generator connected at bus 2 to load connected at bus 6.

The implementation of Dijkstra’s algorithm is done with the help of the following steps (iterations):

**Step 1:** The process starts from node 2. Since the length of the shortest path from node 2 to node 2 is 0  $d_{22} = 0$ . The immediate predecessor node of node 2 will be denoted by the symbol + so that  $q_2 = +$ . Since the lengths of the shortest paths from node 2 to all other nodes on the shortest path are unknown,  $q_i = -$  for all. The only node which is now in a closed state is node 2. Therefore  $c = 2$ .

The nodal diagram for step 1 is as follows:



**Fig. 3 Iteration 1**

**Step 2:** In order to transform some of the temporary labels into permanent labels, examine all branches (c,i) which exit from last node which is in a closed state (node c). If node I is also in a closed state, pass the examination on to the next node. If node I is in an open state its first label  $d_{ai}$  is obtained based on the equation

$$d_{ai} = \min[d_{ai}, d_{ac} + l(c, i)] \quad (1)$$

in which the left side of the equation is the new label of node i.  $d_{2i}$  appearing on the right side of the equation is the old label for node i.

Here  $d_{27} = \min(\infty, 0+0.0625) = 0.0625$

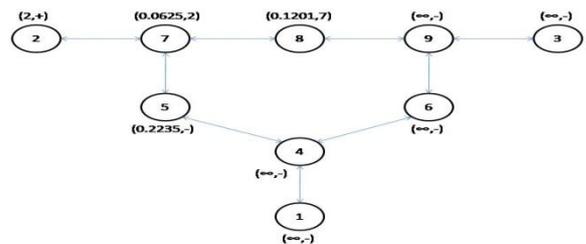
**Step 3:** In order to determine which node will be the next to go from an open to a closed state, value  $d_{2i}$  is compared for all nodes which are in an open state and choose the node with the smallest  $d_{2i}$ . Let this be the same node j. Node j passes from an open state to a closed state since there is no path from a to j shorter than  $d_{aj}$ . The path through any other node would be longer.

**Step 4:** Now that j is the next node to pass from an open state to a closed one, the immediate predecessor node of node j is determined and the shortest path which leads from node a to node j is found out. The length of all branches (I,j) which lead from closed state nodes to node j are examined until the following equation is satisfied

In the above case  $d_{27} - l(2,7) = 0.0625 - 0.0625 = 0 = d_{27}$ . Let this equation be satisfied for some node t. This means that node t is the immediate predecessor of node j on the shortest path which leads from node a to node j. Therefore,  $q_j = t$ .

**Step 5:** Node j is in a closed state. When all nodes in the network are in a closed state, we have completed the process of finding the shortest path. Should any node still be in an open state, we return to step 2.

The nodal diagram after performing the above steps is given as follows:



**Fig. 4 Iteration 2**

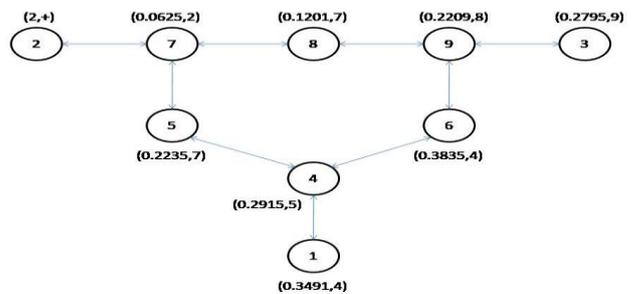
The second iteration progresses with the following flow as per Fig. 3 as follows:

$$d_{28} = \min(\infty, 0.0625 + 0.0576) = 0.1201$$

$$d_{25} = \min(\infty, 0.0625 + 0.161) = 0.2235$$

$d_{28} < d_{25}$ , so the predecessor node is assigned as ‘7’ for node 8.

Similarly the steps required are applied for the WSCC 9 Bus System and the resultant nodal diagram with bus 2 as source node is given as follows:



**Fig. 5 Resultant Nodal Diagram**



Now that the effective weights of all nodes are found out, and the shortest path can be found out as follows: While finding the shortest path from node 2 to node 6, from the above diagram we observe that while tracking the path from node 6 to node 2 as 6 -> 4 -> 5 -> 7 -> 2 and hence the shortest path is found out.

Implementation of the shortest path finding alone doesn't solve the restoration problem, so additional constraints are included in solving the bulk power restoration problem. Bulk power restoration is taken into account while solving the problem.

### III. PROBLEM FORMULATION

The primary goal of this work is to provide a reliable power system restoration strategy, and maximizing the power restored by reconfiguring the unenergized network. This restructuring is done through the optimal path in order to prevent further blackouts during the process.

#### Objective Function

The power system restoration process should be done in a systematic process subject to the following constraints: The power balance between generation and demand should be satisfied, and has to be managed properly.

The mathematical expression for the power balance can be given as

$$\sum_{k \in T_i} P_k - \sum_{k \in F_i} P_k = L_i, y_i = 0 (i \in N) \quad (2)$$

where  $T_i$  is the set of branches incident to bus I,  $F_i$  is the set of branches with originating from bus I,  $L_i$  is the load at bus I and N is the set of buses.

The power flow through a line should not exceed its capacity. This is because while rerouting the power the transmission line should have to be energized properly and power supply has to be given within limits, or else further blackout may happen during the process of restoration. This is given with the help of the following mathematical expression:

$$|P_k| - U_k \leq 0 (k \in B) \quad (3)$$

The voltage limits have to be maintained within limits throughout the restoration process. This is represented with the help of the mathematical expression

$$V_{\min} \leq V_i \leq V_{\max} \quad (4)$$

where  $V_{\min}$  is the minimum voltage limit,  $V_{\max}$  is the maximum voltage limit and  $V_i$  is the voltage at  $i^{\text{th}}$  bus. The system frequency at each generating station should have to be kept under limits. The 'system load regulation coefficient' is an important factor for consideration of frequency regulation because depending on the load characteristics the total system load experiences changes which affects system frequency. The load regulation coefficient ( $\alpha$ ) is defined as

$$\alpha = \frac{\Delta P}{\Delta f} \quad (5)$$

The mathematical expression of frequency is given as

$$f(t) = \left( 1 + \frac{\left( \sum_{i=1}^n P_{gi}(t) - \sum_{i=1}^m P_i(t) \right)}{\sum_{i=1}^m P_i(t) * \alpha} \right) * f_o \quad (6)$$

Where  $P_{gi}(t)$  is the output of  $i$ th generator calculated according to the generator ramp rates as per their characteristics at every minute, and  $n$  is the number of generators operating at that time. The value of alpha has been taken as 1.5 .

$f_o$  is assigned as 50 plus or minus 0.5 Hz.

### IV. PROPOSED ALGORITHM

- Step 1: Start
- Step 2: Identify the generator, load and slack buses from the given power system network.
- Step 3: Find the generator ramp rate and find load pickup characteristics.
- Step 4: Find the adjacency matrix for all transmission lines in the network.
- Step 5: Set  $\min = 0$
- Step 6: Assign initial values of all loads and generators for the unenergized network to zero.
- Step 7: Check if all nodes are connected. If so, go to step 9.
- Step 8: Execute Dijkstra's algorithm for the shortest path and connect the respective node.
- Step 9: Perform load flow analysis for the power system network.
- Step 10: If power balance condition is met go to step 12 else go to step 8.
- Step 11: Change the load/generator values based on their characteristics.
- Step 12: If voltage limits are matched go to step 13 else go to step 11.
- Step 13: If power flow across all branches doesn't exceed its respective capacity go to step 14 else go to step 11.
- Step 14: Connect active switch across both nodes according to Dijkstra's algorithm in order to ensure line is energized and load is connected to generator.
- Step 15: If frequency limits are matched go to step 16 else go to step 17.
- Step 16: Change load/generation values based on their characteristics.
- Step 17: If other constraints are matched change the end node and go to step 7, else go to step 19.
- Step 18: Change connected load and generation values with respect to their characteristics.
- Step 19: Increment the value of  $\min$ .
- Step 20: If all loads have reached their maximum value and all branches are energized go to next step else do respective changes and go to step 9.
- Step 21: Stop.

**V. OBTAINED RESULTS**

**Restoration Plan for WSCC 9 Bus System**

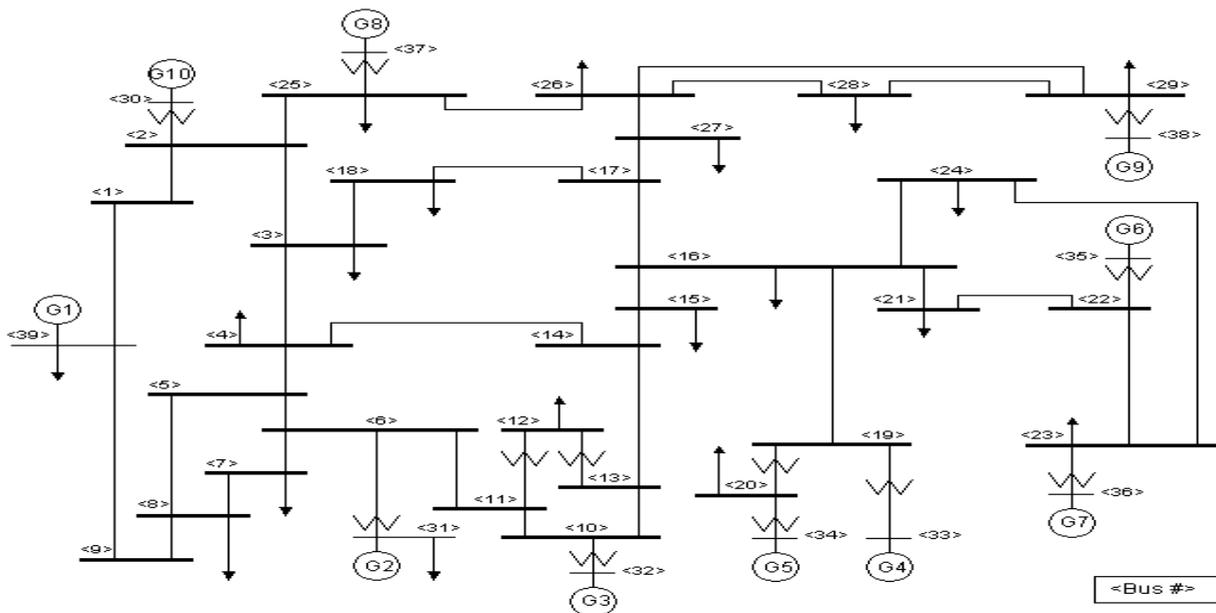
Based on the generator ramping rates and the load characteristics, the restoration plan for WSCC 9 bus system as per the algorithm is given as follows:

**Table. 1 Restoration Plan for IEEE 9 Bus System**

Minute	Restoration Action	Power System Frequency		
		PS 1(bus 4)	PS 2(bus 7)	PS 3(bus 9)
1	Start G1, G2, turn on L5, energize circuits 2_7, 7_5, 1_4, 4_5	50.068	50	0
3	Turn on L6, energize circuit 4_6	49.98	50	0
6	Start G3, energize circuits 3_9, 9_6	49.97	50	50
8	Turn on L8, energize circuit 7_8	50.005	50	50
15	Increase in pickup of loads	50.024	50	50
20	Increase in pickup of loads	50.029	50	50
24	Increase in pickup of loads	50.018	50	50
25	Energize circuit 8_9	49.988	50	50

**Observations for New England 39 Bus System**

The New England 39-bus system used for finding optimal path for restoring the system from a partial blackout/ complete blackout is as follows:



**Fig. 6 Network Diagram of New England 39 Bus System**

This network consists of 10 generating stations and 11 loads. The analysis progresses in the following way:

A partial blackout is considered Power is now supplied from each Black Start (BS) generator to the nearest load.

The identification of nearest load/ reaching load with minimum losses is found out by using the Shortest Path

Finding (SPF) algorithm. Here Dijkstra’s algorithm is being used. All the constraints are checked.

The process progresses until all the loads are connected.

After all the process has been completed the network is synchronized with the already energized network.

The restoration plan of New England 39 bus system is given as follows:

Table. 2 Restoration Plan for New England 39 Bus System

Time (Minute)	Restoration Action	Power System Frequency										Total System Generation (MW)
		PS 33	PS 34	PS 35	PS 36	PS 38	PS 30	PS 31	PS 32	PS 37	PS 39	
1	Start 34 33 36 35 38, Turn on 20, 21, 23, 29, Energize circuits 20_34, 20_19, 19_33, 38_29, 36_23, 35_22, 22_21	50.069	50	50.007	50	50.005	-	-	-	-	-	70
8	Turn on 28, Energize circuit 28_29	49.992	50	49.999	50.012	49.999	-	-	-	-	-	560
10	Energize circuit 22_23	50.004	50	49.998	50	50.002	-	-	-	-	-	700
14	Turn on 16, Energize circuit 19_16	49.995	50	50	50	49.994	-	-	-	-	-	980
16	Turn on 24, Energize circuits 21_16, 16_24	50	50	49.996	50	50.004	-	-	-	-	-	1120
19	Turn on 26, Energize circuit 28_26	50	50	50	50	50.004	-	-	-	-	-	1330
22	Turn on 15, Energize circuit 16_15	50	50	50.001	50	49.999	-	-	-	-	-	1540
26	Turn on 18, 27, Energize circuits 26_27, 16_17 & 17_18	50	50	50.001	50	50	-	-	-	-	-	1820
40	Energize circuit 17_27	50	50	49.999	50	50.001	-	-	-	-	-	2800
45	Increase in pickup of load	50	50	50	50	49.998	-	-	-	-	-	3012
50	Increase in pickup of load	50	50	50	50	49.998	-	-	-	-	-	3112
56	Increase in load to full value	50	50	50	50	50.001	-	-	-	-	-	3180
57	Synchronize area 1 and area 2	50	50	50	50	49.998	50	49.9	50	49.9	50	6191.8

VI. CONCLUSION

Power supply should be provided through an alternate path through available transmission line when a fault occurs. This modified power flow during fault should pass through lines containing minimum impedance, which ultimately contributes to reduction in losses in the power system network subject to the constraints mentioned in Section III. The frequency constraint as mentioned in Section III shows that the condition that the power network should be radial can be eliminated, which further reduces the bulk power restoration problem and hence provides an even simpler approach. The extended Dijkstra’s algorithm can also work in dynamic environment.

REFERENCES

1. N. Ganganath, J. V. Wang, X. Xu, C. Cheng and C. K. Tse, "Agglomerative Clustering-Based Network Partitioning for Parallel Power System Restoration," in *IEEE Transactions on Industrial Informatics*, vol. 14, no. 8, pp. 3325-3333, Aug. 2018.
2. M. M. R. Ibrahim, H. A. Mostafa, M. M. A. Salama, R. El-Shatshat and K. B. Shaban, "A graph-theoretic service restoration algorithm for power distribution systems," *2018 International Conference on Innovative Trends in Computer Engineering (ITCE)*, Aswan, 2018, pp. 338-343.
3. F. Qiu and P. Li, "An Integrated Approach for Power System Restoration Planning," in *Proceedings of the IEEE*, vol. 105, no. 7, pp. 1234-1252, July 2017.
4. Yutian Liu, Rui Fan, Vladimir Terzija, "Power system restoration: a literature review from 2006 to 2016", *Journal of Modern Power Systems and Clean Energy*, July 2016, volume 4, Issue 3, pp 332-341.
5. Xue YS, Xiao SJ (2013) Generalized congestion of power systems: insights from the massive blackouts in India. *J Mod Power Syst Clean Energy* 1(2):91–100. doi:10.1007/s40565-013-0014-2

6. Atputharajah and T. K. Saha, "Power system blackouts - literature review," *2009 International Conference on Industrial and Information Systems (ICIIS)*, Sri Lanka, 2009, pp. 460-465.
7. C.Y.Teo and Wei Shen, "Development of an interactive rule-based system for bulk power system restoration", in *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 646-653, May 2000.
8. C. Grigg *et al.*, "The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee," in *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1010-1020, Aug. 1999.
9. Y. Hain and I. Schweitzer, "Analysis of the power blackout of June 8, 1995 in the Israel Electric Corporation," in *IEEE Transactions on Power Systems*, vol. 12, no. 4, pp. 1752-1758, Nov. 1997.
10. K. Matsumoto, T. Sakaguchi, R. J. Kafka and M. M. Adibi, "Knowledge-based systems as operational aids in power system restoration," in *Proceedings of the IEEE*, vol. 80, no. 5, pp. 689-697, May 1992.
11. M. M. Adibi and R. J. Kafka, "Power system restoration issues," in *IEEE Computer Applications in Power*, vol. 4, no. 2, pp. 19-24, April 1991.
12. M. M. Adibi, J. N. Borkoski and R. J. Kafka, "Power System Restoration - The Second Task Force Report," in *IEEE Transactions on Power Systems*, vol. 2, no. 4, pp. 927-932, Nov. 1987.
13. M. Adibi *et al.*, "Power System Restoration - A Task Force Report," in *IEEE Transactions on Power Systems*, vol. 2, no. 2, pp. 271-277, May 1987.

