

# Acceleration Control of Structures Based on An Optimized Tuned Mass Damper Utilizing A Computing Algebra System

Min-Ho Chey, Hyung-Jin Mun

**Abstract Background/Objectives:** Nowadays, the serviceability of building structures under earthquake excitations is an interesting issue and building acceleration responses are considered as a main factor producing the residents' uncomfortable conditions and the unstable fluctuating response of non-structural elements in the buildings.

**Methods/Statistical analysis:** In this study, the design requirements of an optimized and passively operating tuned mass damper (TMD) system used for absorbing earthquake force to suppress vibrational acceleration of structure is explored and its control effectiveness is also introduced. To control the dynamic characteristics of a structure, the parameters for an optimized TMD system are derived using a computing algebra system, @Mathematica. From the optimally derived design parameters, the seismic responses of total acceleration of the superstructure (target system) and the TMD are presented numerically.

**Findings:** Through the dynamic responses of the existing structure and the TMD under random excitations, the mass ratios' effectiveness on the control of the main system by TMD adopted are approved and the effects of the main system's damping on optimized TMD parameters is also verified. The controlling ability of the TMD is depends on the design optimization based on the natural frequency and damping ratios of the TMD designed. Especially, the larger TMD mass ratio makes more reliable acceleration control of the main target structure.

**Improvements/Applications:** It is approved that the optimized design parameters for reducing the acceleration of the structures against earthquake excitations can be clearly derived through the optimization process. Thus, these parameters can be finally utilized in the practical seismic design of the TMD.

**Keywords:** TMD, seismic, optimization, acceleration, control

## I. INTRODUCTION

Passive structural control is achieved by using passive control devices. The passive seismic absorbers are based on the seismic energy dissipating mechanism through the dynamic motion of the structure and the passive control involves damping materials dispersed throughout the structure [1-2].

Recently, some damping devices for seismic control have been newly suggested and some proposed designs offer the

innovative and robust applications and thus updating the efficiency of the approach. For slender structures, such as tall buildings, some types of supplemental damping devices have been efficiently adopted for reducing the building responses and serviceability purposes. Especially, the serviceability of a building which relies on the residents' comfortability is mainly resulted by severe acceleration at the building floors in the seismic excitations and it cause the severe damage on non-structural elements simultaneously.

One of the widely known and used vibrational absorbing system is the Tuned Mass Damper (TMD) and is consists of an additional mass (about 1% of the mass of the superstructure). The TMD is usually constructed by an amount of concrete blocks and positioning at the top floor of the building structure with an elastic spring and damper to control the vibration demands. Ideally, the utmost positive contribution of the TMD can be achieved when the TMD frequency is coincide with the one of dynamic forces such as earthquake and wind[3].

## II. TUNED MASS DAMPER SYSTEM

The original principle of the TMD goes back to the 1940s [4]. It is an additional concrete mass block with an elastic spring and damping devices, generating dynamic performances that improves damping ability in the main target system. The principle of reducing structural dynamic performance by making interacting forces between a TMD and the main structure is to take the seismic energy from the main structure to the TMD. Finally, the seismic energy is dissipated in the TMD system. To increase the energy dissipation in a TMD, it is necessary to find the tunable frequency of the TMD to that of the performance of the target system (superstructure). Moreover, the suitable TMD damping ratio is another crucial factor for the optimized TMD system. The successful adaptation of TMD to the high-rise buildings for reducing wind induced responses is now well known, whereas the efficient real cases of using TMD systems for the seismic strategy is relatively difficult to find.

Meanwhile, the above disadvantages of TMD application on the seismic control is due to some design limitations, such as detuning and fluctuation effects. The slight frequency change of the external force makes the large tuning effect of the TMD and it can even amplify the response of

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the structure. Also, the external excitation doesn't have a single frequency, but usually contains secondary harmonics. Thus, the main structure is being vibrated at multiple frequencies and it is difficult to make an

appropriate tuning. Furthermore, all structures have some internal damping and these hidden and crucial dynamic effects cannot be neglected. Although a TMD is likely to be installed to a small internal damping system, the evaluation of the damping ability in the target structure on the design tuning of the TMD is a necessary design perspective. Consequently, the designers must consider an appropriate

$$\begin{aligned} \sigma_{\ddot{x}_1}^2 &= E[\ddot{x}_1^2] = S_0 \int_{-\infty}^{\infty} |H_{\ddot{x}_1}(\omega)|^2 d\omega \\ &= \pi S_0 \frac{(c_2^2 k_1 + c_2^2 k_1 \mu + k_2^2 m_1 - 2k_1 k_2 \mu m_1 + 2k_2^2 \mu m_1 + k_1^2 \mu^2 m_1 - k_1 k_2 \mu^2 m_1 + k_2^2 \mu^2 m_1)}{c_2 k_1 \mu^2 m_1} \end{aligned} \tag{1}$$

$$\begin{aligned} \sigma_{\ddot{x}_2}^2 &= E[\ddot{x}_2^2] = S_0 \int_{-\infty}^{\infty} |H_{\ddot{x}_2}(\omega)|^2 d\omega \\ &= \pi S_0 \frac{(c_2^2 k_1 + c_2^2 k_1 \mu + k_2^2 m_1 + 2k_2^2 \mu m_1 + k_1 k_2 \mu^2 m_1 + k_2^2 \mu^2 m_1)}{c_2 k_1 \mu^2 m_1} \end{aligned} \tag{2}$$

in which,  $S_0$  is Gaussian white noise excitation with spectral density [10],  $m_1$  and  $m_2$  are masses of main system and TMD,  $k_1$  and  $k_2$  are stiffness coefficients of main system and TMD,  $c_1$  and  $c_2$  are damping coefficients of main system and TMD with a mass ratio of  $\mu$  to the main system, respectively.

The optimizing conditions in terms of accelerations are

$$\frac{\partial \sigma_{\ddot{x}_1}^2}{\partial k_2} = 0, \quad \frac{\partial \sigma_{\ddot{x}_1}^2}{\partial c_2} = 0 \tag{3}$$

The optimized parameters of the damping stiffness,  $k_{2aopt}$ , and damping coefficient,  $c_{2aopt}$ , of the TMD based on acceleration followed by the optimized tuning ratio frequency,  $f_{2aopt}$ , and damping ratio,  $\xi_{2aopt}$ , are derived to as,

$$f_{2aopt} = \frac{(1 + \frac{1}{2}\mu)^2}{1 + \mu}, \quad \xi_{2aopt} = \left( \frac{\mu(1 + \frac{3}{4}\mu)^2}{4(1 + \mu)(1 + \frac{1}{2}\mu)} \right)^{\frac{1}{2}} \tag{4}$$

$$k_{2aopt} = \frac{\mu(1 + \frac{1}{2}\mu)}{(1 + \mu)^2} k_1, \quad c_{2aopt} = \left( \frac{\mu^{\frac{3}{2}}(1 + \frac{3}{4}\mu)^{\frac{1}{2}}}{2\xi_1(1 + \mu)^{\frac{3}{2}}} \right) c_1 \tag{5}$$

#### IV. PERFORMANCE RESULTS

To represent the dynamic performances of the target system and the TMD, two acceleration root-mean-square (RMS) results are computed as  $\sigma_{\ddot{x}_1}$  and  $\sigma_{\ddot{x}_2}$ . To verify the effectiveness of the TMD used in controlling the performance of the target system, the normalized RMS responses are also defined as,  $\sigma_{\ddot{x}_1} / \sigma_{\ddot{x}}$  and  $\sigma_{\ddot{x}_2} / \sigma_{\ddot{x}}$ .

optimization procedure for the calculation of TMD design parameters and its combination which affects the response of the system [5-9].

### III. OPTIMIZATION AND CONTROL PARAMETERS

For the acceleration based optimal parameters, the mean square acceleration responses for the target existing system,  $\sigma_{\ddot{x}_2}$  and the TMD,  $\sigma_{\ddot{x}_2}$  become

These results are derived from the functioning calculation of the natural frequency ratio (uncoupled),  $f_2$ , and the damping ratio of the TMD,  $\xi_2$ . In this study, the parameter values of  $\xi_1 = 0.05$  and  $\omega_1$  (angular frequency ratio) = 3.343 rad/sec are used. The system is assumed to be subjected to a stationary Gaussian white noise with a spectral density of  $S_0 = 3.62 \times 10^{-4} \text{m}^2/\text{s}^3$ .

Figures 1 and 2 show that the acceleration RMS response results of the target system and TMD, when the mass ratio (TMD mass / main system's mass),  $\mu = 0.02$ . When the mass ratio of 0.02, the acceleration reduction of the main system is 0.93-1 for  $f_2$  and 0.05-0.12 for  $\xi_2$  respectively from the view point of RMS and normalized RMS responses.

The diagram of the frequency tuning represents a sharper shape and the optimal value of  $\xi_2$  is between 0.05 and 0.1. The numerical outcomes of Mathematica [11] describes the maximum reductions of the RMS response as 0.1578 and the normalized RMS response as 0.81 and this value means about 19% reduction when  $f_2 = 0.982$  and  $\xi_2 = 0.072$ .

The results for acceleration response for  $\mu = 0.05$  has a relatively broader range of frequency tuning ratios for the greatest response reductions. The ranges of  $f_2$  and  $\xi_2$  are 0.87-1.0 and 0.06-0.2 respectively as shown in Figures 3 and 4. The figures describe the responses in terms of optimal parameters and show that there are greater reductions of the accelerations of the target system than for the case of  $\mu = 0.02$ . When  $f_2$  and  $\xi_2$  have optimal values of 0.9594 and 0.1098 respectively, the two responses are 0.1414 and 0.7216 (28% reduction).

To decrease the response of the target system's acceleration, the TMD acceleration is amplified with the used acceleration parameters. The smaller mass ratio shows a sharper tuning shape and the larger mass ratio has a relatively small acceleration



response for the TMD.

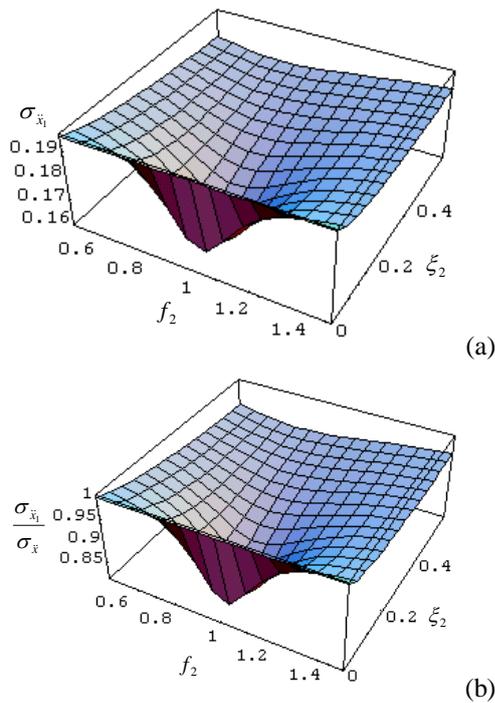


Fig. 1 RMS acceleration (a) and normalized RMS acceleration (b) of main system ( $\mu=0.02$ )

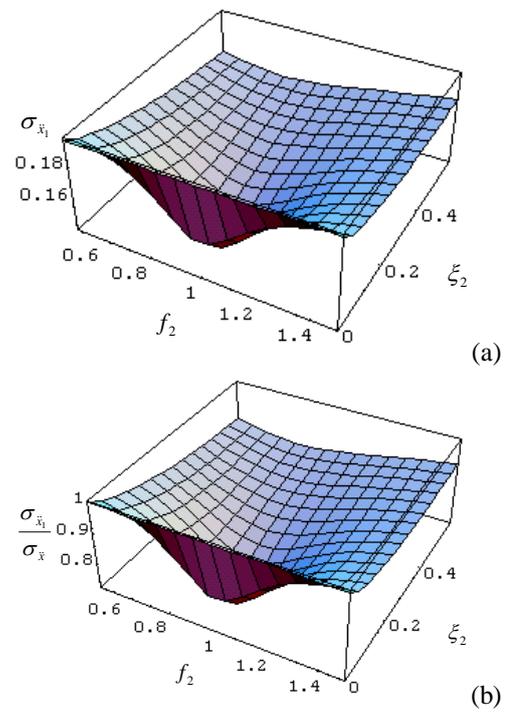


Fig. 3 RMS acceleration (a) and normalized RMS acceleration (b) of main system ( $\mu=0.05$ )

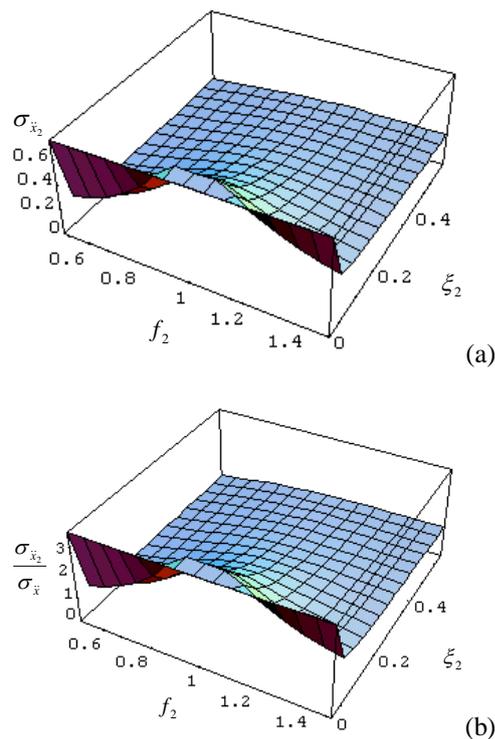


Fig. 2 RMS acceleration (a) and normalized RMS acceleration (b) of TMD ( $\mu=0.02$ )

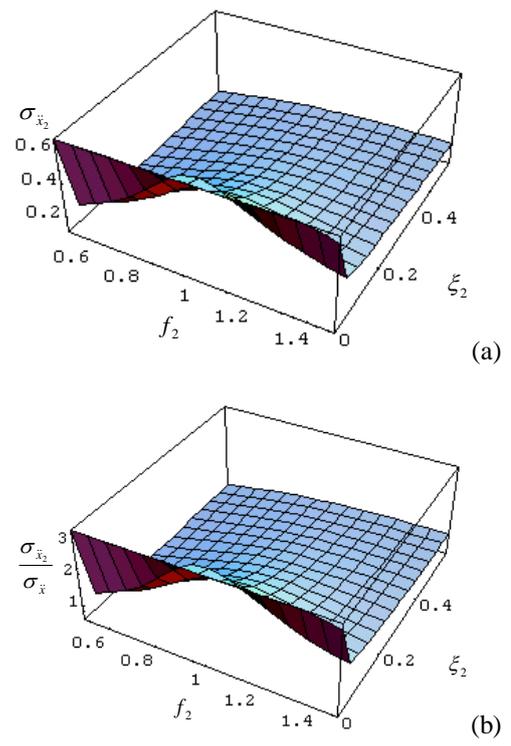


Fig. 4 RMS acceleration (a) and normalized RMS acceleration (b) of TMD ( $\mu=0.05$ )

In conclusion, Table 1 shows the lists of the final optimized acceleration results using optimal design parameters of  $f_2$  and  $\zeta_2$  for the various ratio cases of mass as computed by <sup>®</sup>Mathematica. The larger mass ratio ( $\mu$ ) decreases the accelerations of the target system and the



TMD with four design parameters of  $f_{2opt}$ ,  $\zeta_{2opt}$ ,  $k_{2opt}$  and  $c_{2opt}$ .

**Table 1. Numerical results of acceleration responses using optimal parameters ( $\xi_1 = 0.05$ )**

$\mu$	$f_{2opt}$	$\zeta_{2opt}$	$k_{2opt}$	$c_{2opt}$	$\sigma_{\ddot{x}_1}$	$\frac{\sigma_{\ddot{x}_1}}{\sigma_{\ddot{x}}}$	$\sigma_{\ddot{x}_2}$	$\frac{\sigma_{\ddot{x}_2}}{\sigma_{\ddot{x}}}$
0.003	0.9965	0.0274	50.4	0.8279	0.1834	0.9353	1.7209	8.7823
0.01	0.9902	0.0498	165.9	4.9937	0.1697	0.8658	0.8322	4.2472
0.02	0.9820	0.0702	326.3	13.957	0.1587	0.8100	0.7032	3.5886
0.05	0.9594	0.1098	778.5	53.323	0.1414	0.7216	0.4316	2.2027
0.1	0.9253	0.1527	1448.5	142.98	0.1269	0.6477	0.2963	1.5122

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**V. CONCLUSION AND FUTURE WORK**

To explore the TMD effectiveness, various parametric examinations exploring important response indices in terms of acceleration was conducted and some numerical results were obtained as follows.

- The mass ratio ( $\mu$ ), frequency tuning ratio ( $f_2$ ) and TMD damping ratio ( $\zeta_2$ ) are critical parameters in reducing the response of the target system.
- The parameters for optimal TMD design are derived through a programming of algebra system, <sup>®</sup>Mathematica, and the optimized frequency tuning ratio ( $f_2$ ) increases and the optimized TMD damping ratio ( $\zeta_2$ ), decreases with increasing mass ratio.
- For a given internal structure damping, the larger TMD mass ratio ( $\mu$ ) guaranties the more efficient TMD in decreasing structural acceleration performance. Thus, the similar level of acceleration reduction for the higher internal damping system, a larger TMD mass ratio needs to be provided.
- To avoid the discomfort to the building occupants and severe damage on non-structural elements, the above results could be referred efficiently, and the response spectrum of story acceleration need to be adopted.

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