A three rates of EOQ/EPQ Model for Instantaneous Deteriorating Items Involving Fuzzy Parameter Under Shortages

Swagatika Sahoo, Milu Acharya, Mitali Madhusmita Nayak

***Abstract:** This paper emphasizes on developing both crisp and fuzzy (Economic ordered quantity (EOQ)/Economic production quantity(EPQ)) single commodity models with a three rates of production inventory for deteriorating items in which the demand rate is a function of both advertisement and selling price. The objective of the crisp model is to determine the optimum values of advertising cost, cycle time and selling price with the aim of maximizing the total profit. Again, in the fuzzy inventory model, fuzziness is introduced for the price rate, Triangular fuzzy number is considered to represent the fuzziness of the price rate and the total profit function is defuzzified by using the signed distance method. We develop some useful theorems for each type of models to establish the formulas for advertising cost, selling price, replenishment schedule and optimal order quantity and algorithm is designed to find the optimum solutions of both the model. Numerical examples are provided for both the models and sensitivity analyses are conducted to know the effect of changes made the values of different parameters.*

***Keywords:** Defuzzification, Production inventory model, Price and advertising cost, Shortages, Triangular fuzzy number.*

I. INTRODUCTION

At present, researchers are investigating on inventory models for deteriorating items. Deterioration of inventory is a very common problem in case of the production inventories like medicines, volatile liquids, agricultural products, electronic goods, blood, gasoline, etc. Therefore for any business organization, it is a major concern to control and maintain the inventories of deteriorating items.

Ghare and Schrader [25] proposed the idea of deterioration in inventory models by taking an exponentially decaying inventory model with a constant demand. Convert and Philip [28] extended the model of Ghare and Scharder [25] by introducing variable rate of deterioration with two parameter Weibull distribution. Shah and Jaiswal [42] studied on deteriorating inventory model with constant demand by taking a finite as well as infinite planning horizon. A two rates of production inventory model with shortages and deterioration was proposed by Perumal and Arivarignan [40]. Sana et al. [32] reformulated the model of Permual and Arivarignan [40] by introducing a finite rate of production with Weibull demand under one-rate of production. Bhowmick and Samanta [15] proposed a two rates of deteriorating production inventory model with shortages under variable production cycle. Min, Jhou, and Jhao [18] also proposed the inventory model on deteriorating items under stock dependent demand and two level trade credit. Qin and Liu[19] and Shah [23] developed inventory models with trade credit financing under credit period-dependent demand and deterioration. Mishra [38] developed a waiting time deterministic inventory model for perishable items in stock and time dependent demand. Tabatabaei, Sadjadi, and Makui [33] studied on the market planning of a under pricing deteriorating model. Chanda and Kumar [35] optimized a fuzzy inventory model with advertisement and dynamic selling price dependent demand. Mishra [39] proposed a three-rates-of-production based deteriorating inventory model with demand function depending upon market selling price and advertising cost. We know the demand of any product depends on unit selling price. Generally it is observed that a lower selling price of products increases the selling rate, but when the selling price increases it has a reverse impact. Other than the unit selling price, the parameter which affects demand is advertisement and that to the frequency of advertisement. In the present paper, the main purpose of introducing advertisement and the frequency of advertisement is to raise the demand of the product. To attract the attention of customers, advertisement as well as its frequency of the product is made through popular newspapers, television, radio, magazines, social networks etc. Hence, the demand for an item becomes a function of both selling price and frequency of advertisement.

Several researchers have proposed different fuzzy inventory models to meet the unstable market situations. In crisp inventory models all the cost parameters involved in the total cost function are known and have definite values. But in real life problems due to the unstable market, disturbances occur in cost parameters. Hence fuzzy inventory model fulfils the gap. Different fuzzy inventory models occur due to various fuzzy numbers used in case of cost parameters involved in total cost. Researchers related to this area are: Zimmermann [[13],[14]], Kaufmann and Gupta [2], Kacprzyk and Staniewski [17], Bellman and Zadeh [27], Yao and Su [16], Mahata and Goswamy [12], Vijayan and Kumaran [34], etc. This paper develops both crisp as well as fuzzy

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deteriorating inventory models with the demand to be a power function of unit price and frequency of advertisement. In the present work, to restrict the holding cost, we have introduced three rates of productions. In case of the fuzzy model, a triangular fuzzy number (the most primary type of fuzzy number with high rate of usage than the others because of the computational efficiency mentioned by Shekarian et al.[[9],[10],[11]]) is used for the unit selling price of the product and an optimized fuzzy total profit function is formulated, and signed distance method is applied for defuzzification.

The novelties of the deteriorating inventory models are mentioned as follows, which are known from table-1: (i)three different rates of production inventory models are developed (both for crisp and fuzzy cases) (ii) rates of production are finite and proportional to the rate of demand (both for crisp and fuzzy models) (iii) demand is a power function of both advertisement and selling price(for both models) (iv) shortages are allowed (for both models) (v)unit selling price is a fuzzy parameter (for fuzzy models). The contribution of the present work in comparison with some other research works related to Economic ordered quantity (EOQ)/ Economic production quantity (EPQ) inventory models are mentioned in Table 1.

II. NOTATIONS AND ASSUMPTIONS

In this article, the following assumptions and notations are used in formulating the Mathematical models.

2.1 Notations for Crisp Model

- A_c: Cycle wise frequency of advertisements.
- T_{c6}: Cycle time for the inventory.
- U_p: Unit wise selling price.
- T_{c1}, T_{c2}, T_{c3}, T_{c4} and T_{c5}: Cycle times for different rates of productions, for no production, for shortage and for recovery.
- L_1: Lot size replenished at time T_{c1}.
- L_2: Lot size replenished at time T_{c2}.
- L_3: Lot size replenished at time T_{c3}.
- L: Lot size replenished at time T_{c6}.

Table I: Informations evolved from some leading research articles including the present models.

Author	EPQ	Assumption on demand rate	Production level	Shortage	Fuzzy cost parameter	Profit
Sana et al. (2004)	Yes	Linear time demand	One	Yes	No	No
Ghosh, Goyal, and Chaudhuri (2006)	Yes	Demand rate $R(t) = \alpha t^{\beta-1}$ $\alpha, \beta > 0$	One	Yes	No	No
Bhowmick and Samanta (2011)	Yes	Constant demand	Two	Yes	No	No
Min et al. (2012)	Yes	$R(t) = D + \alpha I(t)$ I(t) = instantaneous stock level	One	No	No	Yes
Qiu and Liu(2014)	Yes	No	One	No	No	No
Molamohamadi et al.(2014)	Yes	$D = Kv^{-\alpha}$ v=manufactures selling price, α=price elasticity K=scaling factor	One	Yes	No	Yes
Pal et al. (2015)	No	$D(p, \theta, \rho) = Ap^{-\lambda}e^{\theta} e^{\mu}$ $A(> 0)$ is the scaling factor $\rho(> 1)$, $\lambda > 0$, $\mu > 0$, $k > 0$ $\theta_{min} < \theta < \theta_{max}$, where θ_{min}, $\theta_{max} \geq 0$	one	No	No	Yes
Sivashankari and Panayappan (2015)	Yes	Constant rate of demand	Two	Yes	No	No
Sarkar et al. and Samanta (2015)	Yes	Constant rate of demand	One	No	No	No
Rada et al. (2016)	Yes	$D(p, e) = \alpha p^{-\beta} e^{\theta}$ $\alpha(>) = 0$ is a scaling factor, $\beta > 1$= price elasticity of demand, e=frequency of advertisement in a year	One	No	No	Yes
Mishra (2016)	Yes	Constant rate of demand	Three	Yes	No	No
Tsao et al. (2017)	Yes	constant rate of demand	One	No	No	No
Mishra (2017)	Yes	$D(A, p) = (r - lp)A^{\eta}$ $r(>) = 0$ is a scaling factor, l=index of price elasticity, η=Shape parameter	Thee	Yes	No	Yes

This article	Yes	$D(A,p)=\alpha U_p^{-b}A_c^{\eta}$ $a(>)=0$ is a scaling factor, b=index of price elasticity, η=Shape parameter	Thee	Yes	Yes	Yes

- B_o: Backorder size at time T_{c5}.
- $T_{c1}^*, T_{c2}^*, T_{c3}^*, T_{c4}^*, T_{c5}^*$ and T_{c6}^* : Cycle times for the inventory (optimum).
- L_1^*, L_2^* and L_3^*: Replenishment of lot size (optimum).
- B_0^*: Backorder size at time T_{c5}^* (optimum).
- L^*: Replenishment of lot-size at time T_{c6}^* (optimum).
- T_{Ap}^* (A_c, T_{c6}, U_p): Total average profit/ production cycle (optimum).
- $D(A_c, U_p)$: Rate of Demand (no. of units per unit time).
- C_p: Production cost in dollars (per unit time).
- C_0: Cycle wise per run set up cost in dollars.
- C_A: Cost in dollars for each advertisement

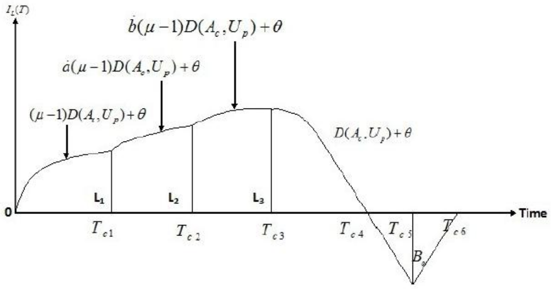

Figure I: Three-rates of production inventory model with shortages.

- C_H: Unit wise holding cost in dollars per unit time.
- C_s: Unit wise shortage cost in dollars per unit time.
- S_p: Yearly selling price in dollars.
- P_c: Cost incurred due to production in a year.
- O_c: Cost incurred due to set up in a year.
- AC: Cost incurred due to cycle wise advertisement in a year.
- H_c: Cost incurred due to holding the inventory in a year.
- D_c: Cost incurred due to deterioration in a year.
- S_c: Cost incurred due to shortage in a year.
- $I_L(T)$: Level of inventory at time T.
- $T_{AP}(A_c, T_{c6}, U_p)$: Per production cycle wise total average profit.

2.2 Assumptions for Crisp Model

(1) The planning horizon is finite.

(2) Instantaneous replenishment of inventory i.e., lead time is zero.

(3) Demand rate $D(A_c, U_p)$ depends on frequency of advertisement A_c and selling price U_p. Here demand is a power function of selling price and frequency of advertisement; i.e., $D(A_c, U_p) = A^{\eta} a U_p^{-b}$ where $a(>0)$ is the scaling factor, $b(>0)$ is the index of price elasticity, and η is the shape parameter, where, $0 \le \eta < 1$.

(4) Shortages are allowed.

(5) The level of inventory at both initial and final points of every time cycle are zero.

(6) $\mu D(A_c, U_p)$ is the production rate denoted as P_r, is greater than $D(A_c, U_p)$ as μ is always greater than 1.

(7) The on-hand inventory deteriorate at a constant rate $\theta(0<\theta<<1)$, where during the time period $[0, T_{c4}]$ there is no replacement of the deteriorated items.

2.3 Notations for Fuzzy Model

- $\tilde{D}(A_c, U_p)$: Fuzzy demand rate.
- $\widetilde{T_{AP}}(A_c, I_{c6})$: Unit time wise profit in fuzzy sense.
- $d(\widetilde{U_p}, \tilde{0}_1)$: Signed distance of fuzzy number.

III. FUZZY PRELIMINARIES

Following are the definitions used to defuzzify by signed distance method.

Definition 3.1. Let $\tilde{F}$ be a triangular fuzzy number presented by the triplet $\tilde{F} = (a_1, a_2, a_3)$ where $a_1 = a - \Delta_1$, $a_2 = a$ and $a_3 = a + \Delta_3$. The fuzzy number $\tilde{F}$ is said to be a triangular fuzzy number if it is fully determined by (a_1, a_2, a_3) are three crisp numbers such that $a_1<a_2<a_3$ and whose membership function, representing triangle, is denoted by

$$\mu_F(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Definition 3.2. Let $\tilde{F}$ be a fuzzy set defined on R. The signed distance of $\tilde{F}$ is defined as

$$d(\tilde{F}, 0) = \frac{1}{2}\int_0^1 [F_L(a) + F_R(a)]\,da$$

where, $F_a = \frac{1}{2}\int_0^1 [F_L(a) + F_R(a)] = [a_1 + (a_2 - a_1)a, a_3 - (a_3 - a_2)a], a \in [0,1]$ is an α cut of a fuzzy set $\tilde{F}$.

IV. MATHEMATICAL MODEL FORMULATION AND SOLUTION PROCEDURE

4.1 Crisp Model

In this paper, we have formulated a three rates of production inventory model under market selling price and advertising cost with deteriorating items. Here the production started at time

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interval $[0,T_{c1}]$, where both rate of production and the demand rate are $\mu D(A_c,U_p)$ and $D(A_c,U_p)$ respectively. Once the level of inventory reaches L_1 at $[T_{c1}, T_{c2}]$, the rate of production switches to a rate $\dot{a}(\mu-1)D(A_c,U_p)$. When the inventory level becomes L_2 at time $[T_{c2}, T_{c3}]$ the rate of production becomes $\dot{b}(\mu - 1)$D(Ac; Up). At time $[T_{c3}; T_{c4}]$, the inventory level starts to decrease and is depleted at a constant rate θ.

Shortages start to grow at a rate B_o in the shortage period and the inventory reaches to zero level at time T_{c6}, but shortages grows at a rate of $D(A_c; U_p)$ upto time T_{c5}. Once again the production starts at time $[T_{c5}; T_{c6}]$ at a rate of $(\mu-1)D(A_c,U_p)$ to recover the earlier shortages occurred in the period T_{c5}. To consume all units, the required time is taken to be T_{c6} with the demand rate is $L = D(A_c; U_p)T_{c6}$. The inventory level of the system is taken to be $I_L(t)$ at time T. Geometrically the behavior of the inventory of the present model is shown in fig.I.

The following are the differential equations obtained with reference to the chosen inventory system in the interval $(0; T_{c6})$:

$$\frac{dI_L(T)}{dT}+\theta I_L(T)=(\mu-1)D(A_c,U_p),\quad 0\le T\le T_{c1}. \qquad (1a)$$

$$\frac{dI_L(T)}{dT}+\theta I_L(T)=\dot{a}(\mu-1)D(A_c,U_p),\quad T_{c1}\le T\le T_{c2} \qquad (1b)$$

$$\frac{dI_L(T)}{dT}+\theta I_L(T)=\dot{b}(\mu-1)D(A_c,U_p),\quad T_{c2}\le T\le T_{c3} \qquad (1c)$$

$$\frac{dI_L(T)}{dT}+\theta I_L(T)=-D(A_c,U_p),\quad T_{c3}\le T\le T_{c4} \qquad (1d)$$

$$\frac{dI_L(T)}{dT}=-D(A_c,U_p),\quad T_{c4}\le T\le T_{c5} \qquad (1e)$$

$$\frac{dI_L(T)}{dT}=(\mu-1)D(A_c,U_p),\quad T_{c5}\le T\le T_{c6}. \qquad (1f)$$

The boundary conditions are

$I_L(0) = 0, I_L(T_{c1}) = L_1, I_L(T_{c2}) = L_2, I_L(T_{c3}) = L_3, I_L(T_{c4}) = 0,$

$I_L(T_{c5}) = -B_o, I_L(T_{c6}) = 0.$ (2)

Solving equation (1a)-(1f), we get

$$I_L(T)=\frac{[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{\theta T}\right],\quad 0\le T\le T_{c1}. \qquad (3a)$$

$$I_L(T)=\frac{\dot{a}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{\theta T}\right],\quad T_{c1}\le T\le T_{c2}. \qquad (3b)$$

$$I_L(T)=\frac{\dot{b}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{\theta T}\right],\quad T_{c2}\le T\le T_{c3}. \qquad (3c)$$

$$I_L(T)=\frac{D(A_c,U_p)}{\theta}\left[e^{\theta(T_{c4}-T)}-1\right],\quad T_{c3}\le T\le T_{c4}. \qquad (3d)$$

$$I_L(T)=D(A_c,U_p)[T_{c4}-T],\quad T_{c4}\le T\le T_{c5}. \qquad (3e)$$

$$I_L(T)=[\mu-1]D(A_c,U_p)[T_{c6}-T],\quad T_{c5}\le T\le T_{c6}. \qquad (3f)$$

From Equation (2)-(3a), the formula for the maximum inventory L_1 during time T_{c1} is given as follows.

$$I_L(T_{c1})=L_1$$

$$\Rightarrow L_1=\frac{[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T_{c1}}\right]. \qquad (4)$$

With the expansion of the exponential function and neglecting the higher powers of $\theta(0<\theta<1)$, in the above equation we find

$$L_1=[\mu-1]D(A_c,U_p)T_{c1}. \qquad (5)$$

From Equation (2)-(3b), the formula for the maximum inventory L_2 during time T_{c2} is given as follows.

$$I_L(T_{c2})=L_2$$

$$\Rightarrow L_2=\frac{\dot{a}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T_{c2}}\right]. \qquad (6)$$

With the expansion of the exponential function and neglecting the higher powers of $\theta(0 < \theta << 1)$, in the above equation we find

$$L_2=\dot{a}[\mu-1]D(A_c,U_p)T_{c2}.$$

From Equation (2)-(3c), the formula for the maximum inventory L_3 during time T_{c3} is given as follows.

$$\Rightarrow L_3=\frac{\dot{b}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T_{c3}}\right]. \qquad (8)$$

With the expansion of the exponential function and neglecting the higher powers of $\theta(0 < \theta << 1)$, in the above equation we find

$$L_3=\dot{b}[\mu-1]D(A_c,U_p)T_{c3}. \qquad (9)$$

From Equations (2)-(3e), the level of shortage S is derived and given as follows

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$$L(T_{c5}) = -B_0 \Rightarrow D(A_c,U_p)(T_{c5}-T_{c4})$$
$$= -B_0. \qquad (10)$$

From Equations (2)- (3f), we have $L(T_{c5}) = -B_0 \Rightarrow D(A_c,U_p)(T_{c6}-T_{c5})$

$$= -B_0. \qquad (11)$$

Now from equations (10) and (11), we find the formula for T_{c5} and is given as follows.

$$T_{c5} = \frac{D(A_c,U_p)T_{c4} + [\mu-1]D(A_c,U_p)T_{c6}}{\mu D(A_c,U_p)}. \quad (12)$$

The following formulas are used to write the total profit per unit time.

$$S_p = \frac{LD(A_c,U_p)}{T_{c6}} = D(A_c,U_p)U_p. \quad (13)$$

$$P_c = \frac{LC_1}{T_{c6}} = D(A_c,U_p)C_1. \qquad (14)$$

$$O_c = \frac{C_0}{T_{c6}} \qquad (15)$$

$$AC = \frac{C_A \times A_c}{T_{c6}} \qquad (16)$$

$$H_c = \frac{C_H}{T_{c6}}\Big[\int_0^{T_{c1}} I_L(T)dT + \int_{T_{c1}}^{T_{c2}}(T)dT$$
$$+\int_{T_{c2}}^{T_{c3}} I_L(T)dT + \int_{T_{c3}}^{T_{c4}} I_L(T)dT\Big]$$
$$= \frac{C_H}{T_{c6}}\left[\int_0^{T_{c1}} \frac{[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T}\right]dT\right.$$
$$+\int_{T_{c1}}^{T_{c2}} \frac{\dot{a}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T}\right]dT$$
$$+\int_{T_{c2}}^{T_{c3}} \frac{\dot{b}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T}\right]dT$$
$$\left.+\int_{T_{c3}}^{T_{c4}} \frac{\dot{b}[\mu-1]D(A_c,U_p)}{\theta}\left[e^{\theta(T_4-T)}-1\right]dT\right]$$
$$= \frac{C_H}{T_{c6}}\left[\frac{[\mu-1]D(A_c,U_p)}{\theta}\left[T+\frac{e^{-\theta T}}{\theta}\right]_0^{T_{c1}}\right.$$
$$+\frac{\dot{a}[\mu-1]D(A_c,U_p)}{\theta}+\left[T+\frac{e^{-\theta T}}{\theta}\right]_{T_{c1}}^{T_{c2}}$$
$$+\frac{\dot{b}[\mu-1]D(A_c,U_p)}{\theta}+\left[T+\frac{e^{-\theta T}}{\theta}\right]_{T_{c2}}^{T_{c3}}$$
$$\left.+\frac{D(A_c,U_p)}{\theta}+\left[\frac{e^{-\theta T(T_{c4}-T)}}{\theta}-T\right]_{T_{c3}}^{T_{c4}}\right]$$

The following is the reduced form of holding cost, which is obtained by removing the terms involving the second and higher powers of θ as $0 < \theta << 1$:

$$\Rightarrow H_c = \frac{C_H}{2T_{c6}}\Big[[\mu-1]D(A_c,U_p)T_{c1}^2$$
$$+\dot{a}[\mu-1]D(A_c,U_p)(T_{c2}^2-T_{c1}^2)$$
$$+\dot{b}[\mu-1]D(A_c,U_p)(T_{c3}^2-T_{c2}^2)$$
$$+D(A_c,U_p)(T_{c4}^2-T_{c3}^2)\Big]$$

$$D_c = \frac{\theta C_p}{T_{c6}}\Big[\int_0^{T_{c1}} I_L(T)dT + \int_{T_{c1}}^{T_{c2}} I_L(T) \qquad (17)$$
$$dT + \int_{Tc2}^{T_{c3}} I_L(T)dT + \int_{T_{c3}}^{T_{c4}} I_L(T)dT\Big]$$
$$= \frac{\theta C_p}{T_{c6}}\left[\int_0^{T_{c1}} \frac{[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{-\theta T}\right]dT\right.$$
$$+\int_{T_{c1}}^{T_{c2}} \frac{\dot{a}[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{\theta T}\right]dT$$
$$+\int_{T_{c2}}^{T_{c3}} \frac{[\mu-1]D(A_c,U_p)}{\theta}\left[1-e^{\theta T}\right]dT$$
$$\left.+\int_{T_{c3}}^{T_{c4}} \frac{D(A_c,U_p)}{\theta}\left[e^{\theta(T_4-T)}-1\right]dT\right]$$
$$= \frac{\theta C_p}{T_{c6}}\left[\frac{[\mu-1]D(A_c,U_p)}{\theta}\left[T+\frac{e^{-\theta T}}{\theta}\right]_0^{T_{c1}}\right.$$
$$+\frac{\dot{a}[\mu-1]D(A_c,U_p)}{\theta}\left[T+\frac{e^{-\theta T}}{\theta}\right]_{T_{c1}}^{T_{c2}}$$
$$+\frac{\dot{b}[\mu-1]D(A_c,U_p)}{\theta}\left[T+\frac{e^{-\theta T}}{\theta}\right]_{T_{c2}}^{T_{c3}}$$
$$\left.+\frac{D(A_c,U_p)}{\theta}\left[-\frac{e^{\theta(T_{c4}-T)}}{\theta}-T\right]_{Tc3}^{T_{c4}}\right]$$

The following is the reduced form of deteriorating cost, which is obtained by removing the terms involving the second and higher powers of θ as $0 < \theta << 1$:

$$\Rightarrow D_c = \frac{\theta C_p}{T_{c6}}\Big[[\mu-1]D(A_c,U_p)T_{c1}^2$$

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$$+\dot{a}\left[\mu-1\right]D\left(A_c,U_p\right)\left(T_{c2}^2-T_{c1}^2\right)$$
$$+\dot{b}\left[\mu-1\right]D\left(A_c,U_p\right)\left(T_{c3}^2-T_{c2}^2\right)$$
$$+D\left(A_c,U_p\right)\left(T_{c4}^2-T_{c3}^2\right)\Big] \quad (18)$$

Cost incurred due to shortage:

$$S_c=\frac{C_s}{T_{c6}}\left[\int_{T_{c4}}^{T_{c5}}L\left(T\right)dT+\int_{T_{c5}}^{T_{c6}}L\left(T\right)dT\right]$$
$$=\frac{C_s}{T_{c6}}\left[\int_{T_{c4}}^{T_{c5}}D\left(A_c,U_p\right)\left[T-T_{c4}\right]dT\right.$$
$$\left.+\int_{T_{c5}}^{T_{c6}}D\left(A_c,U_p\right)\left[T_{c6}-T\right]dT\right]$$
$$=\frac{D\left(A_c,U_p\right)C_s\left[\mu-1\right]}{T_{c6}}\left[T_{c6}-T_{c4}\right]^2. \quad (19)$$

Hence, for the present models the following mathematical form is to be used to write the total profit per unit time.

$$T_{Ap}\left(A_c,T_{c1},T_{c2},T_{c3},T_{c4},T_{c5},T_{c6},U_p\right)$$
$$=S_p-P_c-C_0-AC-H_c-D_c-S_c \quad (20)$$

Using (20), we get

$$T_{Ap}\left(A_c,T_{c1},T_{c2},T_{c3},T_{c4},T_{c5},T_{c6},U_p\right)$$
$$=D\left(A_c,U_p\right)U_p-D\left(A_c,U_p\right)C_p-$$
$$\frac{\left(C_0+A_cC_A\right)}{T_{c6}}-\frac{\left(C_H+\theta C_p\right)}{2T_{c6}}$$
$$\Big[\left[\mu-1\right]D\left(A_c,U_p\right)T_{c1}^2$$
$$+\dot{a}\left[\mu-1\right]D\left(A_c,U_p\right)\left(T_{c2}^2-T_{c1}^2\right)$$
$$+\dot{b}\left[\mu-1\right]D\left(A_c,U_p\right)\left(T_{c3}^2-T_{c2}^2\right)$$
$$+D\left(A_c,U_p\right)\left(T_{c4}-T_{c3}\right)^2\Big]$$
$$-\frac{D\left(A_c,U_p\right)C_s\left[\mu-1\right]}{\mu T_{c6}}\left[T_{c6}-T_{c4}\right]^2 \quad (21)$$

Setting $T_{c1}=\kappa_1T_{c4},T_{c2}=k_2T_{c4}$, and $T_{c3}=\kappa_3T_{c4}$ in (21), the obtained total profit is given as follows.

$$T_{AP}\left(A_c,T_{c4},T_{c6},U_p\right)=D\left(A_c,U_p\right)U_p$$
$$-D\left(A_c,U_p\right)C_p-\frac{\left(C_0+A_cC_A\right)}{T_{c6}}$$
$$-\frac{\left(C_H+\theta C_p\right)}{2T_{c6}}\Big[\left[\mu-1\right]D\left(A_c,U_p\right)\kappa_1^2T_{c4}^2$$
$$+\dot{a}\left[\mu-1\right]D\left(A_c,U_p\right)\left(\kappa_2^2-\kappa_1^2\right)T_{c4}^2$$
$$+\dot{b}\left[\mu-1\right]D\left(A_c,U_p\right)\left(\kappa_3^2-\kappa_2^2\right)T_{c4}^2$$
$$+D\left(A_c,U_p\right)\left(T_{c4}-\kappa_3T_{c4}\right)^2\Big]$$
$$-\frac{D\left(A_c,U_p\right)C_s\left[\mu-1\right]}{\mu T_{c6}}\left[T_{c6}-T_{c4}\right]^2 \quad (22)$$

4.1.1 Necessary Conditions for Optimum Solutions

Theorem 4.1 For constant values of cycle times T_{c4}, T_{c6} and market price U_p, the total profit per unit time relating to advertisement A_c is concave.

Proof: The first order partial derivative of the total profit T_{AP} (A_c, T_{c4}, T_{c6}, U_p) with respect to A_c is given below:

$$\frac{\partial T_{AP}\left[A_c,T_{c4},T_{c6},U_p\right]}{\partial A_c}=\eta A_c^{\eta-1}aU_p^{-b}U_p$$
$$-\eta A_c^{\eta-1}aU_p^{-b}U_pC_p-\frac{C_A}{T_{c6}}$$
$$-\frac{\left(C_H+\theta C_p\right)}{2T_{c6}}\Big[\left[\mu-1\right]\eta A_c^{\eta-1}aU_p^{-b}\kappa_1^2T_{c4}^2$$
$$+\dot{a}\left[\mu-1\right]\eta A_c^{\eta-1}aU_p^{-b}\left(\kappa_2^2-\kappa_1^2\right)T_{c4}^2$$
$$+\dot{b}\left[\mu-1\right]\eta A_c^{\eta-1}aU_p^{-b}\left(\kappa_3^2-\kappa_2^2\right)T_{c4}^2$$
$$+\eta A_c^{\eta-1}aU_p^{-b}\left(T_{c4}-\kappa_3T_{c4}\right)^2\Big]$$
$$-\frac{\eta A^{\eta-1}aU_p^{-b}C_s\left[\mu-1\right]}{\mu T_{c6}}\left[T_{c6}-T_{c4}\right]^2. \quad (23)$$

The first partial derivative of the total profit $T_{AP}\left(A_c,T_{c4},T_{c6},U_p\right)$ with respect to A_c is given as follows.

$$\frac{\partial^2T_{AP}\left[A_c,T_{c4},T_{c6},U_p\right]}{\partial A_c^2}=\eta\left(\eta-1\right)A_c^{\eta-2}$$
$$aU_p^{-b}U_p-\eta\left[\eta-1\right]A_c^{\eta-2}aU_p^{-b}U_pC_p$$
$$-\frac{C_A}{T_{c6}}-\frac{\left[C_H+\theta C_p\right]}{2T_{c6}}$$
$$\Big[\left[\mu-1\right]\eta\left[\eta-1\right]A_c^{\eta-2}aU_p^{-b}\kappa_1^2T_{c4}^2$$
$$+\dot{a}\left[\mu-1\right]\eta\left[\eta-1\right]A_c^{\eta-2}aU_p^{-b}\left(\kappa_2^2-\kappa_1^2\right)$$
$$T_{c4}^2+\dot{b}\left[\mu-1\right]\eta\left[\eta-1\right]A_c^{\eta-2}aU_p^{-b}\left(\kappa_3^2-\kappa_2^2\right)T_{c4}^2$$
$$+\eta\left[\eta-1\right]A^{\eta-2}aU_p^{-b}\left(T_{c4}-\kappa_3T_{c4}\right)^2\Big]$$
$$-\frac{\eta\left[\eta-1\right]A^{\eta-2}aU_p^{-b}C_s\left[\mu-1\right]}{\mu T_{c6}}$$
$$\times\left[T_{c6}-T_{c4}\right]^2<0. \quad (24)$$

According to the equation (25), the total profit function is concave with respect to A_c and the frequency of advertisement is a positive integer with $0<\eta<1$.

Theorem 4.2 For constant values of advertising cost A_c, cycle time T_{c6} and market price

U_p, the total profit per unit time relating to the cycle time T_{c4} where ($T_{c4} = \kappa_4 T_{c6}$) is concave.

Proof: The first order partial derivative of the total profit T_{AP} (A_c, T_{c4}, T_{c6}, U_p) with respect to T_{c4} is given as follows.

$$\frac{\partial T_{AP}\left[A_c,T_{c4},T_{c6},U_p\right]}{\partial T_{c4}} = -\frac{\left(C_H+\theta C_p\right)}{T_{c6}} \Big[\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\kappa_1^2T_{c4} +\dot{a}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_2^2-\kappa_1^2\right)T_{c4} +\dot{b}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_3^2-\kappa_2^2\right)T_{c4} +A^{\eta}aU_p^{-b}\left(1-\kappa_3\right)T_{c4}\Big] +\frac{A^{\eta}aU_p^{-b}C_s\left[\mu-1\right]}{\mu T_{c6}}\left[T_{c6}-T_{c4}\right]<0. \quad (25)$$

The second order partial derivative of the total profit T_{AP} (A_c, T_{c4}, T_{c6}, U_p) with respect to T_{c4} is given as follows.

$$\frac{\partial^2 T_{AP}\left[A_c,T_{c4},T_{c6},U_p\right]}{\partial T_{c4}^2} = -\frac{\left(C_H+\theta C_p\right)}{T_{c6}} \Big[\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\kappa_1^2+\dot{a}\left[\mu-1\right]A_c^{\eta}aU_p^{-b} \left(\kappa_2^2-\kappa_1^2\right)+\dot{b}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_3^2-\kappa_2^2\right) T_{c4}+A^{\eta}aU_p^{-b}\left(1-\kappa_3\right)^2\Big] -\frac{A^{\eta}aU_p^{-b}C_s\left[\mu-1\right]}{\mu}. \quad (26)$$

Therefore,

$$\frac{\partial T_{AP}\left[A_c,T_{c4},T_{c6},U_p\right]}{\partial T_{c4}}=0$$

$$\Rightarrow T_{c4}=\kappa_4T_{c6}$$

$$\Rightarrow \frac{\partial T_{AP}\left[A_c,T_{c4},T_{c6},U_p\right]}{\partial T_{c4}} = -\frac{\left(C_H+\theta C_p\right)}{T_{c6}} \Big[\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\kappa_1^2T_{c4}^2 +\dot{a}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_2^2-\kappa_1^2\right)T_{c4}^2 +\dot{b}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_3^2-\kappa_2^2\right)T_{c4} +\frac{A^{\eta}aU_p^{-b}C_s\left[\mu-1\right]}{\mu T_{c6}}\left[T_{c6}-T_{c4}\right]. \quad (27)$$

Where κ_4

$$= \frac{C_s}{\mu\Big[\left(C_H+\theta C_p\right)\Big[\kappa_1^2+\dot{a}\left(\kappa_2^2-\kappa_1^2\right) +\dot{b}\left(\kappa_3^2-\kappa_2^2\right)+\left(1-\kappa_3\right)^2\Big]\Big]+C_s}$$

Putting the value of $\kappa_4 T_{c6}$ in place of T_{c4} in equation (22), the obtained profit function is given as follows.

$$T_{AP}\left(A_c,T_{c6},U_p\right)=D\left(A_c,U_p\right)U_p - D\left(A_c,U_p\right)C_p-\frac{\left(C_0+A_cC_A\right)}{T_{c6}} - \frac{\left(C_H+\theta C_p\right)}{2T_{c6}}\Big[\left[\mu-1\right]D\left(A_c,U_p\right)\kappa_1^2\kappa_4^2T_{c6}^2 +\dot{a}\left[\mu-1\right]D\left(A_c,U_p\right)\left(\kappa_2^2-\kappa_1^2\right)\kappa_4^2T_{c6}^2 +\dot{b}\left[\mu-1\right]D\left(A_c,U_p\right)\left(\kappa_3^2-\kappa_2^2\right)\kappa_4^2T_{c6}^2 +D\left(A_c,U_p\right)\left(\kappa_4T_{c6}-\kappa_3\kappa_4T_{c6}\right)^2\Big] -\frac{D\left(A_c,U_p\right)C_s\left[\mu-1\right]\left[T_{c6}-\kappa_4T_{c6}\right]^2}{\mu T_{c6}} \quad (28)$$

Theorem 4.3 For constant values of advertising cost A_c, cycle time T_{c6} and market price U_p, the total profit per unit time in terms of cycle time T_{c6} is concave.

Proof: The first order partial derivative of the total profit T_{AP} (A, T_{c4}, T_{c6}, U_p) with respect to T_{c6} is given as follows.

$$\frac{\partial T_{AP}\left[A_c,T_{c6},U_p\right]}{\partial T_{c6}}=\frac{\left(C_0+A_cC_A\right)}{T_{c6}^2} -\frac{\left(C_H+\theta C_p\right)}{2}\Big[\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\kappa_1^2\kappa_4^2 +\dot{a}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_2^2-\kappa_1^2\right)\kappa_4^2 +\dot{b}\left[\mu-1\right]A_c^{\eta}aU_p^{-b}\left(\kappa_3^2-\kappa_2^2\right)\kappa_4^2 +A^{\eta}aU_p^{-b}\left(\kappa_3-1\right)^2\Big] -\frac{A^{\eta}aU_p^{-b}C_s\left[\mu-1\right]\left(\kappa_4-1\right)^2}{\mu}. \quad (29)$$

and the second order partial derivative of the total profit T_{AP} (A_c, T_{c4}, T_{c6}, U_p) with respect to T_{c6} is found to be negative and is given as follows.

$$\frac{\partial^2 T_{AP}\left[A_c,T_{c6},U_p\right]}{\partial T_{c6}^2}=-\frac{2\left[C_0+A_cC_A\right]}{T_{c6}^3}<0.$$

Now equating

$$\frac{\partial T_{AP}\left[A_c,T_{c6},U_p\right]}{\partial T_{c6}}$$

to zero ad solving it for

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T_{c6} we find:

$$T_{c6}=\sqrt{\frac{2\left(C_0+A_cC_A\right)}{\left(C_H+\theta C_p\right)D\left(A_c,U_p\right)\left(\mu-1\right)\left(\kappa_1^2\kappa_4^2+\dot{a}\left(\kappa_2^2-\kappa_1^2\right)\kappa_4^2+\dot{b}\left(\kappa_3^2-\kappa_2^2\right)\right)+\left(C_H+\theta C_p\right)\kappa_4^2\left(\kappa_3-1\right)^2+\left[\frac{\mu\left(C_0+A_cC_A\right)}{C_s\left(\mu-1\right)D\left(A_c,U_p\right)\left(\kappa_4-1\right)^2}\right]}}$$

(30)

Theorem 4.4 If the frequency of advertisement is fixed, there exists unique U_p^* which maximizes the total profit function T_{AP} (A_c, T_{c6}, U_p) .

Proof: The first order partial derivative of T_{AP} (A_c, T_{c6}, U_p) with respect to U_p is given as follows.

$$\frac{\partial T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial U_p}=a\left(1-b\right)U_p^{-b}A_c^{\eta}+abU_p^{-b-1}A_c^{\eta}C_p-\frac{1}{2T_{c6}}\Big[\left(C_H+\theta C_P\right)\Big[-A_c^{\eta}ab\kappa_1^2\kappa_4^2T_{c6}^2\left(\mu-1\right)-abU_p^{-b-1}\dot{a}A_c^{\eta}\left(\kappa_2^2-\kappa_1^2\right)\kappa_4^2T_{c6}^2\left(\mu-1\right)-abU_p^{-b-1}\dot{b}A_c^{\eta}\left(\kappa_3^2-\kappa_2^2\right)\kappa_4^2T_{c6}^2\left(\mu-1\right)-abU_p^{-b-1}A^{\eta}\left(\kappa_4T_{c6}-\kappa_4\kappa_3T_{c6}\right)^2\Big]\Big]+\frac{C_s\left(\mu-1\right)abU_p^{-b-1}A_c^{\eta}\left(1-\kappa_4\right)^2T_{c6}}{\mu}\quad(31)$$

Equating $\frac{\partial T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial U_p}$ to zero, we obtain the optimal value of U_p.

$$U_p=\left(\frac{b}{b-1}\right)\Bigg[C_p+\frac{C_s\left(\mu-1\right)\left(1-\kappa_4\right)^2T_{c6}}{\mu}+\frac{\kappa_4^3}{2}\left(\kappa_1^2\left(\mu-1\right)+\dot{a}\left(\kappa_2^2-\kappa_1^2\right)\left(\mu-1\right)+\dot{b}\left(\kappa_3^2-\kappa_2^2\right)\left(\mu-1\right)+\left(1-\kappa_3\right)^2\right.\left(\theta C_p+C_H\right)T_{c6}\Bigg]\quad(32)$$

Putting $U_p = U^*_p$, in the second derivative of the profit function obtained with respect to the U_p is found to be negative i.e.,

$$\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial U_p^2}=-a\left(b-1\right)U_p^{*(-b-1)}$$

$$=\frac{-a\left(b-1\right)}{U_p^{*(-b-1)}}<0\quad(33)$$

Therefore, from (33) we observe that the average profit is concave with respect to U_p.

4.2.1 Sufficient Condition for Optimum Results

Theorem 4.5 The optimum solution of A_c^*,T_{c6}^*, and U_p^* exist uniquely, which maximizes the total profit function $T_{AP}\left(A_{c,}T_{c6},U_p\right)$.

$$H_M=\begin{pmatrix}\frac{\partial^2T_{AP}}{\partial A_c^2} & \frac{\partial^2T_{AP}}{\partial T_{c6}\partial A_c} & \frac{\partial^2T_{AP}}{\partial U_p\partial A_c^2}\\ \frac{\partial^2T_{AP}}{\partial T_{c6}\partial A_c} & \frac{\partial^2T_{AP}}{\partial T_6^2} & \frac{\partial^2T_{AP}}{\partial T_{c6}\partial U_p}\\ \frac{\partial^2T_{AP}}{\partial U_p\partial A_c} & \frac{\partial^2T_{AP}}{\partial U_p\partial T_{c6}} & \frac{\partial^2T_{AP}}{\partial U_p^2}\end{pmatrix}$$

be the Hessian matrix of the profit function $T_{AP}\left(A_c,T_{c6},U_p\right)$, where the following are satisfied by the different partial derivatives used in H_M.

$$\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial T_6^2}=-\frac{2\left[C_0+A_cC_A\right]}{T_{c6}^2}$$

and

$$\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial U_p^2}=\frac{-a\left(b-1\right)}{U_p^{*(-b-1)}}$$

$$\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial T_{c6}\partial A_c}=\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial A_c\partial T_{c6}}$$

$$=\left(\frac{A_c}{T_{c6}^2}-\frac{\left(C_H+\theta C_p\right)}{2}\left(\mu-1\right)aU_p^{-b}\eta A^{\eta-1}\kappa_4^2\left[\kappa_1^2+\dot{a}\left(k_2^2-\kappa_1^2\right)+\dot{b}\left(\kappa_3^2-\kappa_2^2\right)+\left(1-\kappa_4\right)^2\right]-\frac{C_s\left(\mu-1\right)aU_p^{-b}\eta A^{\eta-1}\left(1-\kappa_4\right)^2}{\mu}\right)$$

$$\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial U_p\partial A_c}=\frac{\partial^2T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial A_c\partial U_p}$$

$$=\left(a\left(1-b\right)U_p^{-b}\eta A_c^{\eta-1}+abU_p^{-b-1}\right.$$

$$\eta A_c^{\eta-1}C_p+\frac{C_H+\theta C_p}{2}\left(\eta A_c^{\eta-1}abU_p^{-b-1}\right.$$

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$$(\mu-1)\kappa_4^2T_{c6}\Big)\Big[\dot{a}\left(\kappa_2^2-\kappa_1^2\right)$$
$$+\dot{b}\left(\kappa_3^2-\kappa_2^2\right)+\left(1-\kappa_4\right)^2\Big]\Big)$$
$$-\frac{C_s\left(\mu-1\right)aU_p^{-b-1}\eta A^{\eta-1}\left(1-\kappa_4T_{c6}\right)^2}{\mu}$$

$$\frac{\partial^2 T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial T_{c6}\partial U_p}$$
$$=\Bigg(\frac{C_s\left(\mu-1\right)abU_p^{-b-1}A^{\eta}\left(1-\kappa_4\right)^2}{\mu}$$
$$+\frac{\left(C_H+\theta C_p\right)}{2}\left(\mu-1\right)abU_p^{-b-1}A^{\eta}\kappa_4^2$$
$$\left[\kappa_1^2+\dot{a}\left(\kappa_2^2-\kappa_1^2\right)-\dot{b}\left(\kappa_3^2-\kappa_2^2\right)\right]$$
$$+abU_p^{-b-1}A_c^{\eta}\left(1-\kappa_3\right)^2\kappa_4^2\Bigg)$$

$$\frac{\partial^2 T_{AP}\left(A_c,T_{c6},U_p\right)}{\partial A_c^2}=\eta\left(\eta-1\right)A_c^{\eta-2}$$
$$aU_p^{-b}\Bigg(U_p-C_pU_p-\frac{\left(C_H+\theta C_p\right)}{2T_{c6}}$$
$$\left[\left(\mu-1\right)\kappa_1^2\kappa_4^2T_{c6}^2+\dot{a}\left(\mu-1\right)\left(\kappa_2^2-\kappa_1^2\right)\right.$$
$$\left.\left(1-\kappa_2^3\right)\right]-\frac{C_s\left(\mu-1\right)\left(1-\kappa_4\right)^2T_{c6}}{\mu}\Bigg)$$

Based on the above conditions, it is observed that the concavity behavior of objective function is satisfied with respect to the decision variables T_{AP} (A_c, T_{c6}, U_p) and the following algorithm used to identify the global optimum solutions for the decision variables.

- **Algorithm:-**

Step 1. Set $A_c = 1$.

Step 2. Set $k = 1$ and initialize the value $U_p = C$.

Step3. Obtain the values of κ_4 and T_{c6} from equation (28) and (31). Substitute the values of κ_4 and T_{c6} into equation (33), and obtain the corresponding value of $U_p^{(k)}$

Step 4. If $\left|U_p^{k+1}-U_p^{k}\right|\le 10^{-3}$, then set $\left(U_p^*,T_{c6}^*\right)=\left(U_p^{(k+1)},T_{c6}^{k}\right)$ and go to step 3.

Step 5. Find $T_{AP}\left(A_c,U_p^*,T_{c6}^*\right)$ and $\left(U_p^*,T_{c6}^*\right)$ is the optimal solution and $T_{AP}\left(A_c,U_p^*,T_{c6}^*\right)$ is the maximum value of the objective function for A_c.

Step 6. Set $A_c' = A_c + 1$ and repeat (3), (4), and (5) and find $T_{AP}\left(A_c,U_p^*,T_{c6}^*\right)$ and go to step(7).

Step7. If $T_{AP}\left(A_c',U_p^*,T_{c6}^*\right)\ge T_{AP}\left(A_c,U_p^*,T_{c6}^*\right)$ then set $A_c = A_c'$ and go to step (8).

Step8. Set $\left(A_c',U_p^*,T_{c6}^*\right)\ge\left(A_c^*,U_p^*,T_{c6}^*\right)$ then $\left(A_c',U_p^*,T_{c6}^*\right)$ is the optimal solution.

Step 9. Calculate the corresponding $T_{c1}^*,T_{c2}^*,T_{c3}^*,T_{c4}^*,T_{c5}^*,L_1^*,L_2^*,L_3^*,B_0^*$ and L^*.

4.2 Fuzzy Model

Considering unstable market situations, we introduce fuzziness, used triangular fuzzy number for the formulation of the present model, for the unit price U_p i.e., $\widetilde{U_p}$ and also assume that the parameter $\widetilde{U_p}$ changes within some limits $\widetilde{U_p}$ = (U_p - Δ_1, U_p, U_p - Δ_2). Now, the total profit per unit time, for fuzzy unit price, is given as follows.

$$\widetilde{T_{AP}}\left(A_c,T_{c6},U_p\right)=D\left(A_c,\widetilde{U_p}\right)\widetilde{U_p}$$
$$-D\left(A_c,\widetilde{U_p}\right)C_p-\frac{\left(C_0+A_cC_A\right)}{T_{c6}}$$
$$-\frac{\left(C_H+\theta C_p\right)}{2T_{c6}}\Big[\left(\mu-1\right)D\left(A_c,\widetilde{U_p}\right)\kappa_1^2\kappa_4^2T_{c6}^2$$
$$+\dot{a}\left(\mu-1\right)D\left(A_c,\widetilde{U_p}\right)\left(\kappa_2^2-\kappa_1^2\right)$$
$$\kappa_4^2T_{c6}^2+\dot{b}\left(\mu-1\right)D\left(A_c,\widetilde{U_p}\right)\left(\kappa_3^2-\kappa_2^2\right)$$
$$\kappa_4^2T_{c6}^2+D\left(A_c,\widetilde{U_p}\right)\left(\kappa_4T_{c6}-\kappa_4\kappa_3T_{c6}\right)^2\Big]$$
$$-\frac{C_s\left(\mu-1\right)D\left(A_c,\widetilde{U_p}\right)\left(T_{c6}-\kappa_4T_{c6}\right)^2}{T_{c6}\mu}\quad(34)$$

Now we defuzzify $\widetilde{T_{AP}}\left(A_c,T_{c6},U_p\right)$ using the signed distance method. The signed distance of $\widetilde{T_{AP}}\left(A_c,T_{c6},U_p\right)$ to $\widetilde{0}_1$ is given as follows:

$$d\left(\widetilde{T_{PU}},\widetilde{0}_1\right)=D\left(A_c,d\left(\widetilde{U_p},\widetilde{0}_1\right)\right)d\left(\widetilde{U_p},\widetilde{0}_1\right)$$
$$-D\left(A_c,d\left(\widetilde{U_p},\widetilde{0}_1\right)\right)C_p-\frac{\left(C_0+A_cC_A\right)}{T_{c6}}$$
$$-\frac{\left(C_H+\theta C_p\right)}{2T_{c6}}\Big[\left(\mu-1\right)D\left(A_c,\widetilde{U_p}\right)\kappa_1^2\kappa_4^2$$
$$T_{c6}^2+\dot{a}\left(\mu-1\right)D\left(A_c,d\left(\widetilde{U_p},\widetilde{0}_1\right)\right)\left(\kappa_2^2-\kappa_1^2\right)$$
$$\kappa_4^2T_{c6}^2+\dot{b}\left(\mu-1\right)D\left(A_c,d\left(\widetilde{U_p},\widetilde{0}_1\right)\right)\left(\kappa_3^2-\kappa_2^2\right)$$

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$$\kappa_4^2 T_{c6}^2 + D\left(A_c, d\left(\widetilde{U_p}, \widetilde{0_1}\right)\right)\left(k_4 T_{c6} - \kappa_4 \kappa_3 T_{c6}\right)^2 \Big]$$
$$-\frac{C_s(\mu-1) D\left(A_c, \widetilde{U_p}\right)\left(T_{c6} - \kappa_4 T_{c6}\right)^2}{T_{c6}\mu} \quad (35)$$

where $d\left(\widetilde{U_p}, \widetilde{0_1}\right)$ is measured as follows:

$$d\left(\widetilde{U_p}, \widetilde{0_1}\right) = U_p + \frac{\Delta_2 - \Delta_1}{4}$$
$$\widetilde{T_{PU}}\left(A_c, T_{c6}, U_p\right) \cong \left(\widetilde{U_p}, \widetilde{0_1}\right)$$
$$\widetilde{T_{PU}}\left(A_c, T_{c6}, U_p\right) = A_c^{\eta} a$$
$$\Bigg[\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b+1} - \left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b} C_p$$
$$-\frac{\left(C_H + \theta C_p\right)}{2T_{c6}} \kappa_4^2 T_{c6}^2 \Big((\mu-1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b} \kappa_1^2 + \dot{a}(\mu-1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b} \left(\kappa_2^2 - \kappa_1^2\right) + \dot{b}(\mu-1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b} \left(\kappa_3^2 - \kappa_2^2\right)$$
$$+\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b} (1-\kappa_3)^2\Big)\Bigg]$$
$$-\frac{C_s(\mu-1)\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b}}{\mu}$$
$$\times \frac{(1-\kappa_4)^2 T_{c6}}{\mu} \quad (36)$$

The first order partial derivative of $\widetilde{T_{AP}}\left(A_c, T_{c6}, U_p\right)$ with respect to U_p is given as follows.

$$\frac{\partial \widetilde{T_{PU}}\left(A_c, T_{c6}, U_p\right)}{\partial U_p} = A_c^{\eta} a$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-1} \Bigg[(1-b)\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)$$
$$+bC_p + \frac{\left(C_H + \theta C_p\right)}{2} \kappa_4^2 T_{c6}$$
$$\left(b(\mu-1)\kappa_1^2 + \dot{a}(\mu-1) b\left(\kappa_2^2 - \kappa_1^2\right)\right.$$
$$\left.+\dot{b}b(\mu-1)\left(\kappa_3^2 - \kappa_2^2\right) + b(1-\kappa_3)^2\right)$$
$$+\frac{C_s(\mu-1)(1-\kappa_4)^2 T_{c6}}{\mu}\Bigg] \quad (37)$$

Equating $\frac{\partial \widetilde{T_{PU}}\left(A_c, T_{c6}, U_p\right)}{\partial U_p}$ to zero and solving it, we obtain the following value of U_p.

$$\widetilde{U_p} = \frac{1}{b-1}\Bigg[\Bigg(bC_p + \frac{\left(C_H + \theta C_p\right)}{2} \kappa_4^2 T_{c6}$$
$$\left(b(\mu-1)\kappa_1^2 + \dot{a}(\mu-1) b\left(\kappa_2^2 - \kappa_1^2\right)\right.$$
$$\left.+\dot{b}b(\mu-1)\left(\kappa_3^2 - \kappa_2^2\right) + b(1-\kappa_3)^2\right)$$
$$+\frac{C_s(\mu-1)(1-\kappa_4)^2 T_{c6}}{\mu}\Bigg) - \frac{\Delta_2 - \Delta_1}{4}\Bigg] \quad (38)$$

The second order partial derivative of $\widetilde{T_{AP}}\left(A_c, T_{c6}, U_p\right)$ with respect to $\widetilde{U_p}$ is given as follows.

$$\frac{\partial^2 \widetilde{T_{PU}}\left(A_c, T_{c6}, U_p\right)}{\partial U_p^2} = A_c^{\eta} a\Big[-b(1-b)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-1} - b(b+1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-2} C_p - \frac{\left(C_H + \theta C_p\right)}{2T_{c6}}$$
$$\kappa_4^2 T_{c6}^2 \Big((\mu-1) b(b+1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-2} \kappa_1^2 + \dot{a}(\mu-1) \quad (39)$$
$$b(b+1)\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-2}$$
$$\left(\kappa_2^2 - \kappa_1^2\right) + \dot{b}b(b+1)(\mu-1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-2} \left(\kappa_3^2 - \kappa_2^2\right) + b(b+1)$$
$$\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-2} (1-\kappa_3)^2\Big)\Big]$$
$$-\frac{C_s(\mu-1)\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-2}}{\mu}$$
$$\times \frac{(1-\kappa_4)^2 T_{c6}}{\mu}$$

(40)

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At point $\widetilde{U_p} = \widetilde{U_p^*}$, we have

$$\frac{\partial^2 \widetilde{T_{PU}}\left(A_c, T_{c6}, U_p\right)}{\partial U_p^2} = -2A_c^{\eta} ab\left(U_p + \frac{\Delta_2 - \Delta_1}{4}\right)^{-b-1} < 0 \quad (41)$$

From equation (41), the average profit function $\widetilde{T_{AP}}\left(A_c, T_{c6}, U_p\right)$ is obtained concave with respect to $\widetilde{U_p}$.

V. NUMERICAL EXAMPLE

To illustrate the proposed model, consider the following example.

Example-1

Setting values as C_0= \$250/set-up. year, $\mu = 5, C_p = \$3/$ unit/year, $C_s = \$0.02$ /unit/time unit, $C_H = \$0.4$ / unit/year, $\theta = 0.08, C_A = \$40$ /advertisement/year $\kappa_1 = 0.2, \kappa_2 = 0.4, \kappa_3 = 0.6,$ and the demand function $D\left(A_c, U_p\right) = 400000 U_p^{-b} A^{0.04}$, $\Delta_2 = 5.15$, $\Delta_2 = 5.50$. The values of different parameters considered here are realistic, although these are not taken from any case study. Using the algorithm and MATLAB- R2015a, starting with U_p= 5, it is easy to deduce that the optimal values of the decision variables are

$$A_c^* = 24, T_{c1}^* = 0.002928, T_{c2}^* = 0.005857,$$
$$T_{c3}^* = 0.008785, T_{c4}^* = 0.014642,$$
$$T_{c5}^* = 3.463279, T_{c6}^* = 4.325438, B_0^* = 28021.4697$$
$$L_1^* = 95.1816, L_2^* = 380.8072$$
$$L_3^* = 856.6346, L^* = 140583.5674$$
$$U_{p1}^* = 5.1433, T_{AP}\left(A_c^*, T_{c6}^*, U_p^*\right) = 15691.2698.$$

The above results are optimal as the eigen values of the Hessian Matrix is H_M are

$$\frac{\partial^2 T_{AP}\left(A_c, T_{c6}, U_p\right)}{\partial A_c^2} = -0.0000028,$$
$$\frac{\partial^2 T_{AP}\left(A_c, T_{c6}, U_p\right)}{\partial T_6^2} = -27.780720,$$
$$\frac{\partial^2 T_{AP}\left(A_c, T_{c6}, U_p\right)}{\partial U_p^2} = -2823.0830.$$

So the profit function is concave.

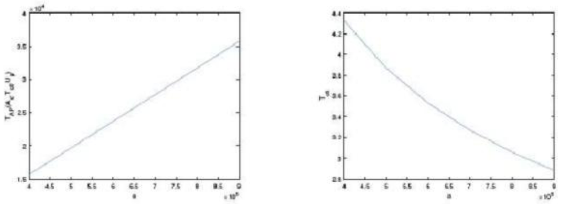

Figure II: T_{AP} (A_c, T_{c6}, U_p) w.r.t a and T_{c6} w.r.t a .

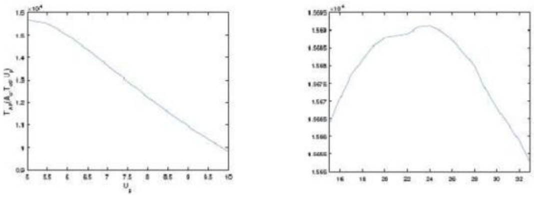

Figure III: T_{AP} (A_c, T_{c6}, U_p) w.r.t U_p and A_c.

Table II: Optimum value of A_c w.r.t the maximum total profit in crisp case.

A_c	κ_4	T_{c1}^*	T_{c2}^*	T_{c3}^*	T_{c4}^*	T_{c5}^*	T_{c6}^*
23	0.003385	0.002882	0.005764	0.008646	0.014410	3.408450	4.256965
24	0.003385	0.002928	0.005857	0.008785	0.014642	3.463279	4.325438
25	0.003385	0.002974	0.005948	0.008922	0.014870	3.517185	4.392764

Table III: Continuing Table II.

B_0^*	L_1^*	L_2^*	L_3^*	L^*	U_p^*	$T_{AP}^*(A_c, T_{c6}^*, U_p^*)$
27530.9683	93.5155	374.0620	841.6396	138122.4195	5.1410	15673.4217
28021.4697	95.1816	380.7264	856.6346	140583.5674	5.1433	15691.2698
28504.1311	96.8210	387.2843	871.3898	143004.7610	5.1455	15688.1901

Table IV: Optimum value of A_c w.r.t the maximum total profit in fuzzy sense.

A_c	κ_4	T_{c1}^*	T_{c2}^*	T_{c3}^*	T_{c4}^*	T_{c5}^*	T_{c6}^*
23	0.003385	0.003098	0.006196	0.009294	0.015490	3.663758	4.575825
24	0.003385	0.003149	0.006299	0.009449	0.015748	3.724912	4.652203
25	0.003385	0.003200	0.006401	0.009602	0.016003	3.785114	4.727391

Table V: Continuing Table IV.

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B_0^*	L_1^*	L_2^*	L_3^*	L^*	U_p^*	$T_{AP}^*(A_c, T_{c6}^*, U_p^*)$
25612.5117	86.9990	347.9960	782.9911	128497.5540	5.2974	15617.5530
26053.2759	88.4961	353.9847	796.4656	130708.8604	5.2999	15634.7245
26486.4738	89.9676	359.8705	809.7087	132882.2207	5.3024	15631.9774

Table VI: Sensitivity analysis with respect to different parameters in crisp case.

Parameter	Changes	A_c	κ_4	T_{c1}^*	T_{c2}^*	T_{c3}^*	T_{c4}^*	T_{c5}^*	T_{c6}^*	B_0^*	L_1^*	L_2^*	L_3^*	L^*	U_p^*	$T_{AP}^*(A_c, T_{c6}^*, U_p^*)$
C_0	-20	24	0.003385	0.00286	0.00573	0.00860	0.01433	3.3909	4.2351	27436.40	93.1943	372.77	838.74	137648.00	5.1403	15702.95
	-10	24	0.003385	0.00289	0.00579	0.00869	0.01449	3.4273	4.2805	27730.48	94.1932	376.77	847.73	139123.37	5.1418	15697.08
	0	24	0.003385	0.00293	0.00586	0.00879	0.01464	3.4633	4.3254	28021.47	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003385	0.00295	0.00592	0.00887	0.01474	3.4988	4.3698	28309.46	96.1598	384.63	865.43	142028.13	5.1447	15685.51
	20	24	0.003385	0.00298	0.00597	0.00896	0.01494	3.5341	4.4139	28594.56	97.1282	388.51	874.15	143458.46	5.1462	15679.82
μ	-20	24	0.004167	0.00372	0.00745	0.01117	0.01862	3.3571	4.4699	21726.33	90.8247	363.29	817.42	108959.45	5.1386	15709.35
	-10	24	0.003736	0.00327	0.00655	0.00983	0.01639	3.4164	4.3874	27627.10	93.2627	373.05	839.36	124788.26	5.1412	15699.24
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28012.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003093	0.00264	0.00529	0.00793	0.01322	3.5010	4.2761	28340.18	96.7315	386.92	870.58	156354.67	5.1449	15684.82
	20	24	0.002847	0.00241	0.00482	0.00723	0.01206	3.5322	4.2363	28603.15	98.0097	392.03	882.08	172108.89	5.1463	15679.50
a	-20	24	0.003570	0.00308	0.00617	0.00926	0.01544	3.4639	4.3260	28020.16	100.4162	321.33	903.74	14002.89	5.1432	15691.34
	-10	24	0.003475	0.00300	0.00601	0.00902	0.01503	3.4635	4.3257	28020.83	97.7289	351.82	879.56	140592.81	5.1432	15691.30
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3253	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003299	0.00285	0.00570	0.00856	0.01427	3.4629	4.3251	28022.07	92.7637	408.16	834.87	140571.87	5.1433	15691.23
	20	24	0.003217	0.00278	0.00556	0.00835	0.01391	3.4626	4.3248	28022.64	90.4656	434.23	814.91	140565.56	5.1433	15691.19
b	-20	24	0.003891	0.00336	0.00673	0.01010	0.01683	3.4650	4.3270	28017.90	109.4449	437.77	788.00	140636.76	5.1432	15691.48
	-10	24	0.003620	0.00313	0.00626	0.00939	0.01566	3.4640	4.3262	28019.81	101.8161	407.26	824.71	140608.14	5.1432	15691.36
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003178	0.00274	0.00549	0.00824	0.01374	3.4625	4.3247	28022.92	89.3588	357.43	884.65	140561.41	5.1433	15691.18
	20	24	0.002995	0.00259	0.00518	0.00777	0.01295	3.4619	4.3241	28024.21	84.2073	336.82	909.43	140542.09	5.1433	15691.10
C_p	-20	24	0.003658	0.00316	0.00633	0.00949	0.01582	3.4642	4.3263	28019.54	102.8119	411.56	926.02	140612.21	4.1432	20566.61
	-10	24	0.003516	0.00304	0.00608	0.00912	0.01521	3.4637	4.3258	28020.54	98.8896	395.54	889.98	140597.15	4.6432	18128.93
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.726	856.63	140583.25	5.1433	15691.26
	10	24	0.003263	0.00282	0.00564	0.00846	0.01411	3.4628	4.3250	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	20	24	0.003149	0.00272	0.00544	0.00817	0.01362	3.4624	4.3467	28023.12	88.5462	354.18	796.91	140558.36	6.1434	10815.94
C_s	-20	24	0.002710	0.00261	0.00523	0.00785	0.01309	3.8694	4.8335	31334.26	85.1474	340.58	766.32	157097.08	5.1282	15750.08
	-10	24	0.003047	0.00277	0.00555	0.00833	0.01389	3.64938	4.5582	29539.72	90.3048	361.21	812.74	148150.15	5.1359	15719.84
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003722	0.00307	0.00614	0.00921	0.01535	3.3032	4.1251	26715.15	99.8188	399.27	898.36	134074.88	5.1502	15664.11
	20	24	0.004059	0.00320	0.00641	0.00962	0.01603	3.1636	3.9505	25575.64	104.2485	416.99	938.23	128399.48	5.1568	15638.18
C_H	-20	24	0.003866	0.00334	0.00669	0.01003	0.01673	3.4649	4.3270	28018.07	108.7658	435.06	978.89	140634.21	5.1432	15691.47
	-10	24	0.003610	0.00312	0.00624	0.00937	0.01561	3.4640	4.3261	28019.88	101.5213	406.08	913.69	140607.03	5.1432	15691.36
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003186	0.00275	0.00551	0.00826	0.01378	3.4625	4.3247	28022.86	89.5871	358.34	806.28	140562.17	5.1433	15691.18
	20	24	0.003010	0.00260	0.00520	0.00781	0.01301	3.4619	4.3242	28024.10	84.6138	338.45	761.52	140543.61	5.1433	15691.11
θ	-20	24	0.003658	0.00316	0.00633	0.00949	0.01582	3.4642	4.3263	28019.54	102.8919	411.56	926.02	140612.17	5.1432	15691.38
	-10	24	0.003516	0.00304	0.00608	0.00912	0.01521	3.4637	4.3258	28020.54	98.8867	395.54	889.98	140597.15	5.1432	15691.32
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003263	0.00282	0.00564	0.00846	0.01411	3.4628	4.3250	28022.32	91.7441	366.97	825.69	140570.36	5.1433	15691.21
	20	24	0.003149	0.00272	0.00544	0.00817	0.01362	3.4624	4.3246	28023.12	88.5462	354.18	796.91	140558.36	5.1433	15691.17
C_A	-20	24	0.003385	0.00268	0.00537	0.00805	0.01343	3.1766	3.9674	25702.30	87.3040	349.21	785.73	128948.05	5.1314	15737.57
	-10	24	0.003385	0.00280	0.00561	0.00842	0.01404	3.3230	4.1503	26886.90	91.3278	365.31	821.95	134891.16	5.1375	15713.92
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003385	0.00304	0.00608	0.00912	0.01521	3.5980	4.4937	29111.85	98.8853	395.54	889.96	146053.67	5.1488	15669.49
	20	24	0.003385	0.00630	0.00945	0.00945	0.01576	3.7279	4.6559	30162.84	102.4552	409.82	922.09	151326.47	5.1542	15648.51
η	-20	24	0.003385	0.00296	0.00593	0.00889	0.01482	3.5075	4.3807	27843.92	93.9793	375.91	845.81	138807.44	5.1451	15290.37
	-10	24	0.003385	0.00294	0.00589	0.00884	0.01473	3.4853	4.3530	27667.50	94.5785	378.31	851.20	139692.52	5.1442	15489.53
	0	24	0.003385	0.00292	0.00585	0.00878	0.01464	3.4632	4.3254	28021.46	95.1816	380.72	856.63	140583.25	5.1433	15691.26
	10	24	0.003385	0.00290	0.00581	0.00872	0.01454	3.4413	4.2980	28200.14	95.7885	383.15	862.09	141479.66	5.1423	15895.60
	20	24	0.003385	0.00289	0.00578	0.00867	0.01445	3.4195	4.2708	28379.95	96.3993	385.59	867.59	142381.78	5.1414	16102.57

Table VII: Sensitivity analysis with respect to different parameters in fuzzy sense.

Parameter	Changes	A_c	κ_4	T_{c1}^*	T_{c2}^*	T_{c3}^*	T_{c4}^*	T_{c5}^*	T_{c6}^*	B_0^*	L_1^*	L_2^*	L_3^*	L^*	U_p^*	$T_{AP}^*(A_c, T_{c6}^*, U_p^*)$
C_0	-20	24	0.003385	0.00308	0.00616	0.00924	0.01540	3.6442	4.5514	25529.39	86.7166	346.86	780.45	128080.56	5.2966	15647.26
	-10	24	0.003385	0.00311	0.00623	0.00934	0.01557	3.6847	4.6020	25792.82	87.6114	350.44	788.50	129402.18	5.2983	15640.91
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003385	0.00318	0.00636	0.00955	0.01591	3.7646	4.7018	26310.84	89.3710	357.48	804.33	132001.05	5.3016	15628.59
	20	24	0.003385	0.00321	0.00643	0.00965	0.01608	3.8040	4.7510	26565.60	90.2364	360.94	812.12	133279.20	5.3032	15622.52
μ	-20	24	0.004167	0.00400	0.00800	0.01200	0.02001	3.6062	4.8017	25251.97	84.5490	338.19	760.94	101430.66	5.2947	15654.01
	-10	24	0.003736	0.00352	0.00705	0.01057	0. 01762	3.6726	4.7168	25700.51	86.7590	347.03	780.83	116086.10	5.2976	15643.22
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003040	0.00279	0.00558	0.00837	0.01396	3.7747	4.5923	26388.56	90.1462	360.58	811.31	148226.69	5.3021	15626.62
	20	24	0.002847	0.00259	0.00519	0.00779	0.01298	3.8021	4.5600	26572.83	91.0527	364.21	819.47	159892.28	5.3033	15622.16
a	-20	24	0.003570	0.00332	0.00664	0.00996	0.01661	3.7255	4.6528	26052.52	93.36386	298.76	840.27	130728.08	5.2999	15634.90
	-10	24	0.003475	0.00323	0.00646	0.00970	0.01617	3.7552	4.6525	26052.77	90.8648	327.11	817.78	130718.21	5.2999	15634.76
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003299	0.00306	0.00613	0.00920	0.01534	3.7246	4.6519	26053.74	86.2478	379.49	776.23	130699.98	5.2999	15634.68
	20	24	0.003217	0.00299	0.00598	0.00898	0.01496	3.7243	4.6516	26054.19	84.1109	403.73	756.99	130691.54	5.2999	15634.64
b	-20	24	0.003891	0.00362	0.00724	0.01086	0.01810	3.7267	4.6050	26050.48	101.7597	407.03	732.66	130761.23	5.2998	15634.96
	-10	24	0.003620	0.00336	0.00673	0.01010	0.01684	3.7257	4.6529	26015.97	94.6656	378.66	766.79	130733.22	5.2999	15634.83
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003178	0.00295	0.00591	0.00887	0.01478	3.7241	4.6515	26054.41	83.0816	332.32	822.50	130687.48	5.2999	15634.62
	20	24	0.002995	0.00278	0.00557	0.00835	0.01393	3.7235	4.6509	26055.42	78.2195	313.16	845.48	130668.53	5.3000	15634.54
C_p	-20	24	0.003658	0.00259	0.00518	0.00771	0.01296	2.8383	3.5446	34198.12	125.5806	502.32	1130.22	171618.53	4.2632	21880.25
	-10	24	0.003516	0.00287	0.00575	0.00863	0.01438	3.2754	4.0906	29631.48	104.5718	418.28	941.14	148680.28	4.7813	18320.50
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003263	0.00366	0.00732	0.01099	0.01832	4.6576	5.8175	23181.87	75.8966	303.58	683.07	116288.87	5.8190	13544.83
	20	24	0.003149	0.00366	0.00732	0.01099	0.01832	4.6576	5.8175	20832.09	65.8243	263.29	592.41	104489.59	6.3386	11881.39
C_s	-20	24	0.002710	0.00280	0.00561	0.00841	0.01403	4.1453	5.1781	29248.66	79.4800	317.92	715.32	146640.72	5.2832	15697.40
	-10	24	0.003047	0.00298	0.00596	0.00894	0.01491	3.9175	4.8932	27517.69	84.1234	336.93	757.11	138009.11	5.2918	15665.16
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003722	0.00330	0.00661	0.00992	0.01654	3.5592	4.4449	24793.32	92.6381	370.55	833.74	124429.81	5.3077	15605.77
	20	24	0.004059	0.00346	0.00692	0.01038	0.01731	3.41486	4.2642	23694.29	96.5800	386.32	869.22	118954.38	5.3151	15578.11
C_H	-20	24	0.003866	0.00359	0.00719	0.01079	0.01799	3.7266	4.6538	26050.61	101.1281	404.51	910.15	130758.74	5.2998	15634.95
	-10	24	0.003610	0.00335	0.00671	0.01007	0.01679	3.7257	4.6529	26052.03	94.3914	377.56	849.52	130732.13	5.299	15634.83

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	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003186	0.00296	0.00592	0.00889	0.01482	3.7242	4.6515	26054.36	83.2940	333.17	749.64	130688.31	5.2999	15634.63
	20	24	0.003010	0.00280	0.00560	0.00840	0.01400	3.7235	4.6509	26055.34	78.6695	314.67	708.02	130670.05	5.3000	15634.54
θ	-20	24	0.003658	0.00404	0.00680	0.01021	0.01702	3.7258	4.6531	26051.76	95.6660	382.66	860.99	130737.17	5.2999	15634.85
	-10	24	0.003516	0.00327	0.00654	0.00981	0.01636	3.7253	4.6526	26052.55	91.9415	367.76	827.47	130722.46	5.2999	15634.78
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003263	0.00303	0.00607	0.00910	0.01518	3.7244	4.6517	26053.94	85.2997	341.19	767.69	130696.23	5.2999	15634.66
	20	24	0.003149	0.00293	0.00586	0.00879	0.01465	3.7240	4.6514	26054.57	82.3261	329.30	740.93	130684.49	5.3000	15634.61
C_A	-20	24	0.003685	0.00288	0.00576	0.00864	0.01440	3.4059	4.2538	23971.59	81.4252	325.70	732.827	120265.09	5.2867	15684.04
	-10	24	0.003385	0.00301	0.00603	0.00905	0.01508	3.5686	4.4570	25036.57	85.0427	340.17	765.38	125608.10	5.2934	15658.85
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003385	0.00327	0.00655	0.00983	0.01638	3.8755	4.8403	27027.33	91.8047	367.21	826.24	135595.68	5.3061	15611.51
	20	24	0.003835	0.00340	0.00680	0.01020	0.01700	4.0211	5.0222	27963.37	94.9842	379.93	854.85	140291.77	5.3122	15589.12
η	-20	24	0.003385	0.00319	0.00638	0.00957	0.01595	3.7743	4.7139	25117.62	87.3361	349.34	786.02	128995.49	5.3020	15234.79
	-10	24	0.003385	0.00317	0.00634	0.00951	0.01585	3.7495	4.6829	25881.97	87.9143	351.65	791.22	129849.45	5.3009	15433.47
	0	24	0.003385	0.00314	0.00629	0.00944	0.01574	3.7249	4.6522	26053.27	88.4961	353.98	796.46	130708.88	5.2999	15634.72
	10	24	0.003385	0.00312	0.00625	0.00938	0.01564	3.7004	4.6216	26225.66	89.0817	356.32	801.73	131573.74	5.2989	15838.66
	20	24	0.003835	0.00310	0.00621	0.00932	0.01554	3.6761	4.5912	26399.15	89.6710	358.68	807.03	132444.14	5.2979	16045.05

VI. SUB-CASES OF THE PRESENT MODEL

The following are the different cases leading from the present model:

(a) When P_r approaches to ∞, the model reduces to that of Bhunia and Shaikh (2011).

(b) When both P_r and C_s approach to ∞, the model reduces to Shah et al.(2013).

(c) When $\dot{b}$ approaches to 0(Zero), the model reduces to Sivashankari and Panayappan(2015).

(d) When $\dot{a}$ and $\dot{b}$ approach to 0 (zero), the model reduces to Rada et al.(2016).

When production is independent of the demand rate with demand rate is a constant function, the model reduces to that of Mishra (2016iv).

VII. MANAGERIAL APPLICATIONS

Basing on the tabular results, we comprehend the following for the managerial purpose:

- With the increasing values of C_0, the cycle wise total profit for each production run decreases. This analysis can help the manager to reduce the number of orders, so that the cost incurred due to high ordering cost can be checked.
- With the increasing values of C_H, profit related to each production run decreases.
- With the increasing value of the scaling factor a, profit related to each production run decreases.
- With the increasing value of per unit production cost C_p, in per unit time, the profit related to each production run decreases. This analysis can help the manager to have a proper ground- work on the cheapest production price of the products after the due negotiation of price with the suppliers of the raw materials.

With the increasing value of the shape parameter η, profit related to each production run decreases.

VIII. CONCLUSION

In this paper, we have developed three rates of production inventory models in both crisp and fuzzy sense. For both the models, three different production rates are taken with the initial rate to be the least one and the final rate to be the maximum among the three. Shortages are allowed for the models and are completely backlogged. The objective behind the three rates of production is to decrease the holding cost as compared to the costs generated corresponding to either one or two rates of production. In both the models, we have taken the demand in power form of frequency of advertisement and unit price of the item and particularly for the fuzzy model, due to the market unstable situation, unit price is taken to be a fuzzy number. We derived the concavity conditions for the profit functions and ensured the existence and uniqueness of the optimal solutions. Numerical examples are taken to illustrate the results. Sensitivity analysis is performed to study the effects of change (20% to +20%) in the parameters taking exactly one of the parameters at a time, keeping others to be fixed. It is observed from the analysis that with increase in the value of the shape parameter, the total profit also increases. In case of the fuzzy model, the cost parameter is taken as a triangular fuzzy number and signed distance method is used for defuzzification to evaluate the total profit.

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