A New Algorithm for Bottleneck Transportation Problem

Bhabani Mallia, Manjula Das, Chakradhar Das

Abstract: We have considered Bottleneck Transportation problem. These problem has significant importance in military operation and catastrophe situation, in both cases time factor very important for supplying amenities/ essential commodities to the destination point. Bottleneck Transportation Problem mathematically formulated with usual type of transportation constraints. We have designed an algorithm to find the optimal solution, numerical examples are illustrated and comparative study has been given. The paper has significant implication in other verity of Linear Programming Problem, for example minimax and maximin assigned problems. It has also significant implication in present day e-commerce, cargo loading and delivery in air transport.

Index Terms: Bottleneck Problems, Bottleneck Transportation Problem, Initial solution, Maxmin , Minimax, Pseudo cost, worse case analysis.

I. INTRODUCTION

Generally, allocation of a product from source to destination is known as "Transportation Problem" (TP). F.L Hitchcock is accredited to have developed TP in 1941 [1], [2]. It generally mean to the total transportation cost minimization [3]-[7]. We can set other objectives such as: the time minimization, profit maximization,etc. [9]-[11]. Transportation cost has significant impact on the cost and the pricing of raw materials and goods. Suppliers and producers try to control the cost of transportation. The way how the desirable transportation cost can be obtained is the subject matter of transportation problems in linear programming.

The time minimization transportation problem sometimes called Bottleneck Transportation Problem (BTP). It was first developed by Fulkerson, Glickberg and Goss in 1953. The theoretical and methodological development of the problem were made by P.L. Hammer in 1969.Time Minimization TP is significant for some special cases like military operation in combat zones, cyclone, earthquake affected people, natural calamities, fire service and ambulance service in healthcare. This paper proposes Bottleneck Transportation Problem (BTP) to get a better Basic Feasible Solution (BFS) . An algorithm has been developed to determine the optimal solution and it is illustrated in the following numerical examples.

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Bhabani Mallia, M.Phil., Utkal University, Bhubaneswar, Odisha.

Manjula Das, Professor, Centre for Applied Mathematics & Computing, SOA deemed to be University, Bhubaneswar.

C.Das, Professor, NIIT, Rourkela, Odisha.

This paper is designed as follows. The section I represents introduction, review of literature is given in section II. In section III mathematical formulation of transportation problem has been given. In Section IV we have proposed an algorithms for getting optimal solution . Illustration of numerical example has been given in section V. Section VI comprises outcomes and discussions respectively. Lastly, the conclusion has been given.

II. REVIEW OF LITERATURE

There are many methods developed for solving bottleneck transportation problem. Blocking method for getting optimal solution is one of the solution techniques proposed by Gaurav Sharma, S. H. Abbas, Vijay Kumar Gupta (2015). Similarly, Blocking Zero method is another method of solving bottleneck transportation problem, which gives very efficient solution to the decision maker. These methods mainly support while solving time minimization problems.The BTP (Garfinkel and Rao, 1971) provides a feasible solution which minimizes the maximum value of the entity costs. This is significant in situations where the c_{ij} represent transportation times and we transport all the items at the same time.

Process involved in solving bottleneck transportation problem aims at minimizing transportation time for shipping goods rather than the cost of shipping. However P.L. Hammer (1969) presented methods for solving time minimization problem. This classical version of problem was recognized as time minimization transportation problem as well as bottleneck transportation problem. After *Hammer* many subsequent works have been done by, Khanna, Bakhsi and Arora (1983), Szwarc (1971), Seshan and Tikekar (1980), Sharma and Swarup (1978), Garfinkel and Rao (1971). Isserman (1984) has further extended Hammer's problem, seeking to achieve better approaches.

M. Sharif Uddin, in his paper titled "Time Minimization Transportation Problem: An Algorithmic Approach " (2012), has discussed an Initial Basic Feasible Solution (IBFS) of transportation problem with equal constraints. IBFS obtained by this method is near to optimal solution.

An optimal algorithm of 2 x n BTP has been developed by R. Varadarajan (1991). Sonia and Puri (2004) have represented a hierarchical (two level) balanced time minimizing TP. I. Nikolic (2007) has also shown the total transportation time problem, considering the

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time of active transportation routes.

Two algorithms have been developed by Pandian and Natrajan (2011). The first one helps to find an optimal solution to BTP and the second is designed to find every efficient solution of a bottleneck cost transportation problem. Madhuri (2012) represented the Linear fractional time minimizing TP's solution procedure having adulteration in the goods to produce total transportation schedules. Jain and Saksena (2012) have talked about the time minimizing TP with fractional bottleneck objective function. Kolman (2015) has developed a new procedure for minimization of the transportation time from production centre to the processing unit.

III. MATHEMATICAL FORMULATION

If Linear programming problems with special type of objective functions are of two types :

(i) Bottleneck Problem (ii) Minimax or Maxmini problem. Fulkerson, Glickberg and Goss [1953] developed a particular case of bottleneck problems. Later on Hammer [1969] , Edmonds and Fulkerson [1970], Garfinkle and Rao [1976], Kaplan [1976] Posner and Wu [1981] developed theoretical and methodological aspect of the problem. A Bottleneck Linear Programming Problem (BLPP) mathematically stated as follows:

"Considering the problem,

P: Minimize Z = max $\{c_{ij}|x_{ij}>0\}$

such that

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,3, \ldots\ldots m$$

$$\sum_{j=1}^{n} x_{ij} = b_j, \qquad j = 1,2,3 \ldots. n$$

with $x_{ij} \geq 0$

where,

m - number of source;

n - number of destination;

x_{ij} - quantity of products transported from source *i* to destination *j*;

c_{ij} - time of shipping of products from source point *i* to destination point *j* ;

a_i - availability of goods at source *i* ;

b_j - required quantity of goods at demand point *j*;

The given model is the minimax bottleneck transportation problem. The minimax BTP has the same constraints and the same objective function as the above. Z = max $\{c_{ij}|x_{ij}>0\}$, a minimax BTP has a classical application:

Suppose the origin represents military depot in which certain supplies, say ammunitions, are stored and let the destinations represent combat zones, at which there are specified demands for ammunitions. To every depot to combat zone pair, a coefficient c_{ij} is assigned indicating the amount of time required to ship any number of items from the i^{th} source to the j^{th} destination. A military operation will start in all combat zones simultaneously at the earliest possible instant for which it is necessary that the requested amount of ammunitions are available in all combat zones. In other words, the operation can't start before the last shipment of ammunition arrives and the problem is to schedule the shipments so that the operation can start as soon as possible.

IV. PROPOSED ALGORITHM

We start with a cost or time matrix C (where c_{ij} =0 whenever the i^{th} origin or j^{th} destination is a dummy) and a feasible transportation plan T=(x_{ij}) which may be obtain by any of the phase 1 methods, such as NWC, VAM,LCM

The Bottleneck Transportation method (Minimax)

Step-1:Determine Z = max $\{c_{ij}|x_{ij}>0\}$, and set up the pseudo cost matrix $\hat{C} = (\hat{c}_{ij})$,with

	0	**0**	**M**	**0**
$\hat{C}_1$ =	**0**	**1**	**0**	**0**
	0	**0**	**0**	**0**

$$\begin{cases} M \square\ 0 & if\ c_{ij} > Z \\ 1 & if\ c_{ij} = Z \\ 0 & if\ c_{ij} < Z \end{cases}$$

Step-2: Solving the transportation problem with cost matrix $\hat{c}_{ij}$ .The result is a new T.P.

T = (x_{ij}); determine its pseudo cost

$$\hat{Z} = \sum_i \sum_j \hat{c}_{ij} x_{ij}$$

Step-3: $Is\ \hat{Z} = 0?$

If yes : Go to step 1.

If no: Stop, the current transportation plan is optimal.

V. NUMERICAL EXAMPLE

Example-1

Considering the problem with three origins having supplies of 20,30 and 50 units respectively and four destinations with demands of 10,20,40 and 30 units respectively. The time requirements for the various shipments are given matrix

$$c = \begin{bmatrix} 5 & 7 & 9 & 0 \\ 3 & 8 & 3 & 0 \\ 4 & 6 & 6 & 0 \end{bmatrix}$$

indicating that the fourth destination is dummy .Let the initial transportation plan be obtained by the NWC Rule

S_i/d_j	10	20	40	30
20	10	10	0	0
30	0	10	20	0
50	0	0	20	30

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The required time for shipments according to T^1 is Z= max {5,7,8,3,6,6,0}=8 ,so that the pseudo cost matrix is

The pseudo cost $\hat{Z}_1$=0

The shipment in T^2 require Z= max{5,7,3,6,6,0}=7 times unit and the new pseudo cost matrix is established as

	0	1	M	0
$\hat{C}_2$=	0	M	0	0
	0	0	0	0

Based on $\hat{c}_2$ an optimal transportation plan is given by

		10	20	40	30
T_3	20	10	0	0	10
	30	0	0	30	0
	50	0	20	10	20

With pseudo cost $\hat{Z}_2$=0 , with a time requirement of Z= max{5,0,3,6,6,0}=6, For this new value of the objective function the pseudo cost matrix is

	0	M	M	0
$\hat{C}_3$=	0	M	0	0
	0	1	1	0

A new optimal solution based on $\hat{C}_3$ is

		10	20	40	30
T^4	20	10	0	0	10
	30	0	0	30	0
	50	0	20	10	20

With pseudo cost $\hat{Z}_2$>0 ,hence the pseudo cost in T^4 is optimal; after z = max (5,0,3,6,6,0)=6 time units all shipments are made.

The optimal solution is $Z = \sum_i \sum_j \hat{c}_{ij} x_{ij} = 320$

Example-2

A company has 4 warehouses and 6 stores, The requirements of six stores are 15,20,15,25,20 and 10 quantities at the warehouses are 37,22,32, and 14 units respectively . The shipping time of single unit from i^{th} warehouse to j^{th} store is t_{ij} .

		12	20	15	25	20	10
	35	25	30	20	40	45	0
T	22	30	25	20	30	40	0
	32	40	20	40	35	45	0
	13	25	24	50	27	30	0

Solution:

		12	20	15	25	20	10
T^1	35	25	30	20	40	45	0
	22	30	25	20	30	40	0
	32	40	20	40	35	45	0
	13	25	24	50	27	30	0

The required time for shipments according to T^1 is Z= max {25,30,20,20,30,35,45,30,10}=45 ,so that the pseudo cost matrix is

	0	0	0	0	1	0
$\hat{c}_1$	0	0	0	0	0	0
	0	0	0	0	1	0
	0	0	M	0	0	0

With pseudo cost $\hat{Z}_2$=0 , with a time requirement of Z= max{25,30,20,20,35,40,0,30,0}=40, For this The shipment in T^2 require

		12	20	15	25	20	10
	35	25	30	20	40	45	0
T^2	22	30	25	20	30	40	0
	32	40	20	40	35	45	0
	13	25	24	50	27	30	0

,so that the pseudo cost matrix is

	0	0	0	1	M	0
$\hat{c}_2$=	0	0	0	0	1	0
	1	0	1	0	M	0
	0	0	M	0	0	0

A new optimal solution based on c_3 is

		12	20	15	25	20	10
	35	25	30	20	40	45	0
T^3	22	30	25	20	30	40	0
	32	40	20	40	35	45	0
	13	25	24	50	27	30	0

With pseudo cost $\hat{Z}_2$>0 ,hence the pseudo cost in T^3 is optimal

The optimal solution is $Z = \sum_i \sum_j \hat{c}_{ij} x_{ij} = 2775$

Example-3

Given the TP with m = 5 sources A_i , $i \in I = \{1, 2, 3, 4\}$ and n = 5 destination B_j, $j \in J = \{1, 2, 3, 4, 5\}$ in TABLE-I. Each row and each column corresponds to a supply and demand points respectively. Clearly the given problem is a BTP as both total supply and demand are 65.

	Destination						Supplies
		B_1	B_2	B_3	B_4	B_5	
	A_1	810	105	9	8	0	15
	A_2	6	310	410	7	0	20
T^1=	A_3	12	6	810	410	010	30
	A_4	9	5	6	7	010	10

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A_5	0	0	0	0	010	10
	10	15	20	10	30	

The required time for the shipment according to T^1 is Z= max{8,10,3,4,8,4,0,0,0}=10, so the pseudo cost matrix is

	0	1	0	0	0
	0	0	0	0	0
$\hat{C}_1$=	M	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0

One of the optimal solutions, based on $\hat{C}_1$ is ,

	8 10	10	9 5	8	0	15
	6	3 15	4 5	7	0	20
T^2=	12	6	8 10	4 10	0 10	30
	9	5	6	7	0 10	10
	0	0	0	0	0 10	10
	10	15	20	10	30	

The pseudo cost $\hat{Z}_1$ =0

The required time for the shipment according to T^1 is Z= max{8,9,3,4,8,4,0,0,0}=9, so the pseudo cost matrix is

	0	M	1	0	0
	0	0	0	0	0
$\hat{C}_2$=	M	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0

One of the optimal solutions, based on $\hat{C}_2$ is

	8 10	10	9	8 5	0	15
	6	3 15	4 5	7	0	20
T^3=	12	6	8	4 5	0 25	30
	9	5	6 5	7	0 5	10
	0	0	0 10	0	0	10
	10	15	20	10	30	

The pseudo cost $\hat{Z}_2$ =0

The required time for the shipment according to T^3 is Z= max{8,8,3,4,4,6,0,0,0}=8, so the pseudo cost matrix is

	1	M	M	1	0
	0	0	0	0	0
$\hat{C}_3$ =	M	0	1	0	0
	M	0	0	0	0
	0	0	0	0	0

One of the optimal solutions, based on $\hat{C}_3$ is,

	8	10	9	8	0 **15**	**15**
	6 **10**	3 **10**	4	7	0	**20**
T^4=	12	6 **5**	8	4 **10**	0 **15**	**30**
	9	5	6 **10**	7	0	**10**
	0	0	0 **10**	0	0 **0**	**10**
	10	**15**	**20**	**10**	**30**	

The pseudo cost $\hat{Z}_3$ =0

The required time for the shipment according to T^4 is Z= max{0,6,3,6,4,0,6,0,0}=6, so the pseudo cost matrix is

	M	M	M	M	0
	1	0	0	0	0
$\hat{C}_4$ =	M	1	M	0	0
	M	0	1	M	0
	0	0	0	0	0

One of the optimal solutions, based on $\hat{C}_3$ is,

	8	10	9	8	0 **15**	**15**
	6	3	4 **20**	7	0	**20**
T^4=	12	6 **5**	8	4 **10**	0 **15**	**30**
	9	5 **10**	6	7	0	**10**
	0 **10**	0	0	0 **0**	0 **0**	**10**
	10	**15**	**20**	**10**	**30**	

The pseudo cost $\hat{Z}_4$ > 0

Hence the optimal solution is Z=200

VI. COMPARISON OF RESULT

The new algorithm in current work gives optimal or near optimal solution. However a comparison of current method with existing conventional method is presented.

Methods	**Solution**		
	Example-1	**Example-2**	**Example-3**
Current Method	**320**	**2775**	**200**
NWC Method	**380**	**2775**	**320**
VAM	**310**	**2725**	**210**
Optimal Solution	**310**	**2090**	**210**

VII. CONCLUSION

Number At present different type of

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transportation problem are formulated and new algorithms are given for the solution of the problem .Four problems are found application in different areas of importance .From the numerical example it is evident that the objective function value is strictly decreasing at each iteration. This indicate the number of necessary iteration is bounded from above by the number of different value in the cost matrix 'c'. In the worst case no more a man (Transportation Problem) have to be solved.

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AUTHORS PROFILE

First Author Mrs. Bhabani Mallia , M.Phil. in Statistics from Utkal University, Bhubaneswar, Odisha. And continuing Ph.D at SOA deemed to be University, Bhubaneswar. She is working under the joint guidance of Prof. Manjula Das, Prof. C. Das in the field of transportation problem and goal programming. She has completed some paper on the topic and carrying on her research for Ph.D. degree under SOA deemed to be University.

Second Author Dr. Manjula Das is presently working as a Professor in the Centre for Applied Mathematics & Computing at SOA deemed to be University, Bhubaneswar. She has a teaching experience of 25 years in Mathematics & Statistics and 15 years of research experience in the area of Statistical Analysis of data. Presently working in the field of Transportation Engineering in Operations Research. Under her guidance two students are continuing their research. She has published six number of paper in different national / international journals and also presented some papers in national conferences.

Third Author Dr.C.Das, a retired Professor of Mathematics, from NIIT, Rourkela, Odisha a well-known figure in Operations Research, particularly in the area of Mathematical Programming and its applications to Transportation , assignment, networking system, petrinet modelling , graph theory, fuzzy set etc. He has a lots of contributions in polynomial (interior point) algorithms for linear and non- linear mathematical programming problems. he has more than hundreds published papers in national and International journals of repute. he is also in editorial board of national and international journals and reviewer of some international journals. After retirements, he is also active in research in operation research and some allied topics.