Solving Game Problems involving Heptagonal and Hendecagonal Fuzzy Payoffs

Namarta, Umesh Chandra Gupta, Neha Ishesh Thakur

Abstract: ***The main aim of this paper is to deal with a two person zero sum game involving fuzzy payoff matrix comprising of heptagonal and hendecagonal fuzzy numbers. Ranking of fuzzy numbers is a hard task. Many methods have been proposed to rank different fuzzy numbers such as triangular, trapezoidal, hexagonal, octagonal etc. In this paper, a matrix game is considered whose payoffs are heptagonal and hendecagonal fuzzy numbers and ranking method is used to solve the matrix game. By using this proposed approach the fuzzy game problem is converted into crisp problem and then solved by applying the usual game problem techniques. The validity of proposed method is illustrated with the help of two different practical examples; one where the two companies*** **are venturing into online restaurant business** ***and the other where the two political parties with conflicting interests during elections are competing with each other.***

Index Terms: ***Heptagonal fuzzy numbers, Hendecagonal fuzzy numbers, Ranking function, Fuzzy game theory.***

I. INTRODUCTION

Game theory deals with the study of decision making in situations where two or more rational opponents attempts to maximise their gain in competitive situations. In this decision theory each player determines his course of action after considering the alternatives available to the opponent player. The objective of this theory is to know how the players optimize their payoffs by selecting their respective strategies. Game theory has wider applications in situations involving decision making viz. economics, management, defence, political science, etc. The technique of game theory was originated by John von Neumann, a mathematician and the Economist Oskar Morgenstren. Neumann‘s (1947) approach is based on the principle of best out of worst, that is, he described the principle of minimisation of the maximum losses. Nash (1949) proved that a finite game problem always has a point of equilibrium at which all players choose their best alternatives, when the opponent's strategies are given. However due to the lack of certainty in environment the availability of exact payoffs is the obvious problem. Such an uncertain situation can be modelled using fuzzy set theory.

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Zadeh (1965) was the first to introduce fuzzy set theory. This concept of fuzzy decision that may be viewed as the intersection of given fuzzy goals and / or fuzzy constraints was illustrated by Bellman and Zadeh (1970). Jain (1976) presented a decision procedure to give the best alternative and the rating of other alternatives as compared to optimal alternative in the presence of fuzzy variables. Cevikel and Ahlatcioglu (2010) presented two models for studying two persons zero sum matrix games in which the payoffs and goals are fuzzy and obtained that the fuzzy relation approach and and the max-min solution are equivalent. Kumar et al.(2010) proposed an approach to rank the generalized trapezoidal fuzzy numbers. Li (2012) developed an effective method for solving games with payoffs of triangular fuzzy numbers. Li and Hong (2012) introduced the concept of alpha to solve constrained matrix games in which payoffs are triangular fuzzy numbers. Kumar et al. (2013) proposed a minimax principle to get an imprecise game and the concept of fuzzy ranking. Nan et al. (2014) proposed a ranking technique to solve matrix game in terms of a value and ambiguity of TIFNs. Rathi and Balamohan (2014) introduced a new form of non normal heptagonal fuzzy number and its arithmetic operation is defined. Selvakumari and Lavanya (2014) described an approach for solving fuzzy game problem by using octagonal fuzzy numbers. Dhanalaxmi and Kennedy (2014) proposed area based method for ranking of octagonal fuzzy numbers. Sharma and Kumar (2015) described an algorithm to get lower and upper bounds on fuzzy payoffs so that the value of the game in fuzzy environment can be obtained. Selvakumari and Lavanya (2015) gave a methodology for all fuzzy game matrix problems where all the payoffs are trapezoidal and triangular fuzzy number. Bhaumik et al. (2017) used robust ranking technique of fuzzy numbers to solve matrix game whose payoffs are triangular intuitionistic fuzzy numbers. Kumar and Kumaraghuru (2015) considered solution of fuzzy game problem with triangular fuzzy numbers. Hussain and Priya (2016) proposed hexagonal fuzzy numbers to solve fuzzy game problem. Namarta et al. (2017) proposed ranking of heptagonal fuzzy numbers by using incentre of centroids. Selvam et al. (2017) introduced a parametric method by using ranking of fuzzy numbers where parameters are dodecagonal and octagonal normal uncertain numbers having the trapezoidal shape.

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Majority of the existing research is available on triangular and trapezoidal fuzzy numbers and the different ranking techniques to use convert these fuzzy numbers into crisp numbers for solving optimizations problems. But not enough literature is available on heptagonal and hendecagonal fuzzy numbers and more specifically rare research is available on solving game problems where payoffs are expressed in fuzzy heptagonal or hendecagonal numbers.

The paper emphasizes different sections which consists of basic definitions of fuzzy set and fuzzy numbers, describes the proposed ranking method with examples.

II. PRELIMINARIES

In this section, definitions of fuzzy set, fuzzy numbers, heptagonal and hendecagonal fuzzy numbers are presented.

Definition 2.1

(Zadeh L.A (1965)) Let X = {x} is a collection of objects denoted generally by x. Then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A}$ = {(x, $\mu_{\tilde{A}}$ (x)); x ∈ X} where $\mu_{\tilde{A}}$(x) is termed as the grade of membership of x in A and $\mu_{\tilde{A}}$: X → M is a function from X to a space M which is called membership space. When M contains only two points, 0 and 1 then A is non fuzzy and its membership function becomes identical with the characteristic function of a non fuzzy set.

Definition 2.2

(Zadeh L.A (1965)) A Fuzzy set $\tilde{A}$ of universe set X is normal if and only if $Sup_{x\epsilon X}\ \mu_{\tilde{A}}$ (x) = 1.

Definition 2.3

(Zadeh L.A (1965)) A fuzzy set $\tilde{A}$ in universal set X is called convex iff

$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$ for all x_1, $x_2 \in$ X and $\lambda \in [0,1]$.

Definition 2.4

(Zadeh L.A (1965)) A fuzzy set $\tilde{A}$ of universal set is a fuzzy number if it has the following properties:

- $\tilde{A}$ is convex.
- $\tilde{A}$ is normal.
- $\tilde{A}$ is piecewise continuous.

Definition 2.5

Heptagonal fuzzy number [HFN]

A generalised fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is a heptagonal fuzzy number if its membership function is defined by

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & x \leq a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ \left(\frac{w}{2}\right) & a_2 \leq x \leq a_3 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_3}{a_3-a_2}\right) & a_3 \leq x \leq a_4 \\ w & x = a_4 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & a_4 \leq x \leq a_5 \\ \left(\frac{w}{2}\right) & a_5 \leq x \leq a_6 \\ \frac{w}{2}\left(\frac{a_7-x}{a_7-a_6}\right) & a_6 \leq x \leq a_7 \\ 0 & x \geq a_7 \end{cases}$$

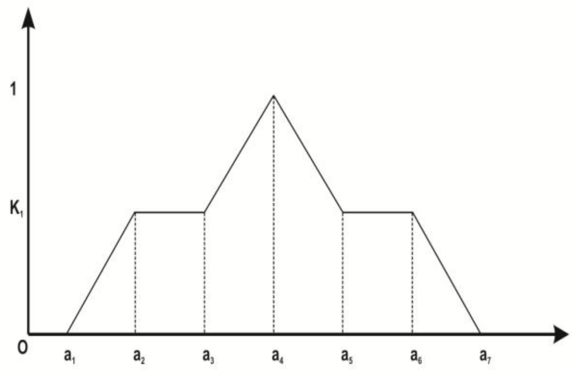

Figure 1

Definition 2.6

The α-cut of a normal heptagonal fuzzy number is defined by

$$\tilde{A} = [A_\alpha{}^L, A_\alpha{}^R] = \begin{cases} \left(A_\alpha{}^L\right)_1, \left(A_\alpha{}^R\right)_1 & \alpha \in [0, k_1] \\ \left(A_\alpha{}^L\right)_2, \left(A_\alpha{}^R\right)_2 & \alpha \in [k_1, 1] \end{cases}$$

Where

$$\left(A_\alpha{}^L\right)_1 = a_1 + \frac{r}{k}(a_2 - a_1),$$

$$\left(A_\alpha{}^L\right)_2 = a_3 + \frac{s-k}{1-k}(a_4 - a_3),$$

$$\left(A_\alpha{}^R\right)_1 = a_7 - \frac{r}{k}(a_7 - a_6),$$

$$\left(A_\alpha{}^R\right)_2 = a_5 - \frac{s-k}{1-k}(a_5 - a_4)$$

Definition 2.7

Hendecagonal fuzzy number (H_nFN)

A generalised fuzzy number

$\tilde{A}_{Hn} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}; w)$ is said to be Hendecagonal fuzzy number if its membership function $\mu_{\tilde{A}_{Hn}}$(x) is given below:

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$$\mu_{\widetilde{A_{Hn}}}(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (k_2-k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ k_2 + (1-k_2)\left(\frac{a_6-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ k_2 + (1-k_2)\left(\frac{a_7-x}{a_7-a_6}\right) & a_6 \le x \le a_7 \\ k_2 & a_7 \le x \le a_8 \\ k_1 + (k_2-k_1)\left(\frac{a_9-x}{a_9-a_8}\right) & a_8 \le x \le a_9 \\ k_1 & a_9 \le x \le a_{10} \\ k_1\left(\frac{a_{11}-x}{a_{11}-a_{10}}\right) & a_{10} \le x \le a_{11} \end{cases}$$

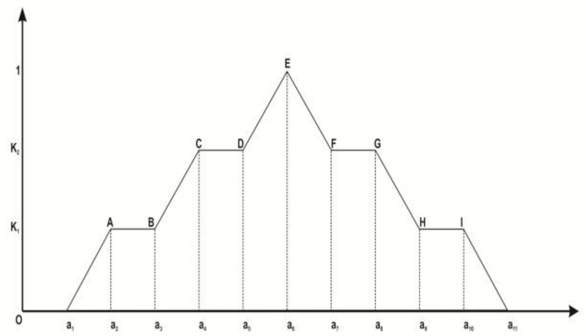

Figure 2

Definition 2.8

The α -cut of a normal hendecagonal fuzzy number $\widetilde{A_{Hn}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11})$ is

$$\tilde{A} = \begin{cases} f_1(r), f_2(r) & \alpha \in [0, k_1] \\ g_1(s), g_2(s) & \alpha \in [k_1, k_2] \\ h_1(t), h_2(t) & \alpha \in [k_2, 1] \end{cases} \text{ Where}$$

$$f_1(r) = a_1 + \frac{r}{k_1}(a_2 - a_1),$$

$$g_1(s) = a_3 + \frac{s-k_1}{k_2-k_1}(a_4 - a_3),$$

$$h_1(t) = a_5 + \frac{t-k_2}{1-k_2}(a_6 - a_5),$$

$$f_2(r) = a_{11} - \frac{r}{k_1}(a_{11} - a_{10})$$

$$g_2(s) = a_9 - \frac{s-k_1}{k_2-k_1}(a_9 - a_8),$$

$$h_2(t) = a_7 - \frac{t-k_2}{1-k_2}(a_7 - a_6)$$

III. PROPOSED RANKING METHOD

Lemma 3.1

Show that ranking of normal heptagonal fuzzy numbers by using α cut is defined as

$$R(\tilde{A}_H) = \frac{1}{4}\{(a_1 + a_2 - a_3 - 2a_4 - a_5 + a_6 + a_7)k + (a_3 + 2a_4 + a_5)\}$$

Proof:

$$R(\tilde{A}_H) = \frac{1}{2}\int_0^k \{l_1(r) + l_2(r)\}dr + \frac{1}{2}\int_k^1 \{s_1(t) + s_2(t)\}ds$$

$$R(\tilde{A}_H) = \frac{1}{2}\int_0^k [a_1 + \frac{r}{k}(a_2 - a_1) + a_7 - \frac{r}{k}(a_7 - a_6)]dr +$$

$$\frac{1}{2}\int_k^1 [a_3 + \frac{s-k}{1-k}(a_4 - a_3) + a_5 - \frac{s-k}{1-k}(a_5 - a_4)]ds$$

$$= \frac{1}{2}\int_0^k [(a_1 + a_7) + \frac{r}{k}(a_2 - a_1 - a_7 + a_6)]dr +$$

$$\frac{1}{2}\int_k^1 [(a_3 + a_5) + \frac{s-k}{1-k}(a_4 - a_3 - a_5 + a_4)]ds$$

$$= \frac{1}{2}\Big\{(a_1 + a_7)k + \frac{k}{2}(a_2 - a_1 - a_7 + a_6)\Big) + (a_3 + a_5)(1-k) + \frac{1-k^2}{2(1-k)}(a_4 - a_3) - \frac{k}{1-k}(a_4 - a_3)(1 - k) - \frac{1-k^2}{2(1-k)}(a_5 - a_4) + \frac{k}{1-k}(a_5 - a_4)(1-k)$$

$$R(\tilde{A}_H) = \frac{1}{4}\{(a_1 + a_2 - a_3 - 2a_4 - a_5 + a_6 + a_7)k + (a_3 + 2a_4 + a_5)\} = \frac{1}{4}\{(a_1 + a_2 - a_3 - 2a_4 - a_5 + a_6 + a_7)k + (a_3 + 2a_4 + a_5)\}$$

Lemma 3.2

Show that ranking of normal hendecagonal fuzzy numbers with α cut is defined by

$$R(\widetilde{A_{Hn}}) = \frac{1}{2}\{(a_1 + a_2 + a_3 - a_4 - a_8 - a_9 + a_{10} + a_{11})k_1 + (a_3 + a_4 - a_5 - 2a_6 - a_7 + a_8 + a_9)k_2 + \frac{(a_5 + 2a_6 + a_7)}{2}\}$$

Proof:

$$R(\widetilde{A_{Hn}}) = \frac{1}{2}\int_0^{k_1}\{f_1(r) + f_2(r)\}dr + \frac{1}{2}\int_{k_1}^{k_2}\{g_1(s) + g_2(s)\}ds + \frac{1}{2}\int_{k_2}^{1}\{h_1(t) + h_2(t)\}dt$$

$$R(\widetilde{A_{Hn}}) = \frac{1}{2}\int_0^{k} [a_1 + \frac{r}{k_1}(a_2 - a_1) + a_{11} - \frac{r}{k_1}(a_{11} - a_{10})]dr$$

$$+$$

$$\frac{1}{2}\int_{k_1}^{k_2} [a_3 + \frac{s-k_1}{k_1-k_2}(a_4 - a_3) + a_9 - \frac{s-k_1}{k_2-k_1}(a_9 - a_8)]ds + \frac{1}{2}\int_{k_2}^{1} [a_5 + \frac{t-k_2}{1-k_2}(a_6 - a_5) + a_7 - \frac{t-k_2}{1-k_2}(a_7 - a_6)]dt$$

$$= (a_1 + a_{11})k_1 + (a_2 - a_1 - a_{11} + a_{10})\frac{k_1}{2} + (a_3 + a_9 - a_5 - a_7)k_2 - (a_3 + a_9)k_1 + \frac{(a_4 - a_3 - a_9 + a_8)}{(k_2 - k_1)}\frac{(k_2{}^2 - k_1{}^2)}{2} - \frac{(a_4 - a_3 - a_9 + a_8)}{(k_2 - k_1)}\frac{(k_1{}^2 - k_1 k_2)}{2} + (a_6 - a_5 - a_7 + a_6)(\frac{1-k_2{}^2}{2(1-k_2)} + \frac{k_2(k_2-1)}{(1-k_2)})$$

$$= \frac{1}{2}\{(a_1 + a_2 - 2a_3 - 2a_9 + a_{10} + a_{11})k_1 + (a_3 + a_9)k_2 + (a_5 + a_7)(1 - k_2) + \frac{(a_4 - a_3 - a_9 + a_8)}{2}(k_2 - k_1) + \frac{(a_6 - a_5 - a_7 + a_6)}{2}(1 - k_2)$$

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$$R\left(\tilde{A_{H_n}}\right) = \frac{1}{2}\{(a_1 + a_2 + a_3 - a_4 - a_8 - a_9 + a_{10} + a_{11})\,k_1 + (a_3 + a_4 - a_5 - 2a_6 - a_7 + a_8 + a_9)k_2 + \frac{(a_5 + 2a_6 + a_7)}{2}\}$$

IV. ALGORITHM TO SOLVE GAME PROBLEMS

Consider a two person zero sum fuzzy game whose all the entries in the payoffs matrix are heptagonal or hendecagonal fuzzy numbers. Let player A is assumed to be the maximisation player with 'm' strategies and player B is assumed to be the minimisation player having 'n' strategies. Each player has to select a strategy from the set of pure strategies.

B

$$A = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \ldots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} & \ldots & \tilde{a}_{3n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3} & \ldots & \tilde{a}_{mn} \end{bmatrix}$$

By using principle of dominance, a game problem of any order matrix can be reduced in size. When there does not exist an equilibrium point (where Maxmin = Minimax), then the game solution has mixed strategies and arithmetic method is used to find the value of the game.

Step1. Let A = $(a_{ij})_{n \times n}$ a payoff matrix. Find the column matrices $c_1, c_2, c_{3 \ldots\ldots} c_{n-1}$ each of order $n \times 1$ by subtracting successive columns of matrix A from its proceeding columns. Obtain a new matrix $(C_{ij})_{n \times (n-1)}$ whose columns are $c_1, c_2, c_{3 \ldots\ldots} c_{n-1}$.

Step2. Find the row matrices $r_1, r_2, r_{3 \ldots\ldots} r_{n-1}$ each of order $1 \times n - 1$ by subtracting successive rows of matrix A from its proceeding rows. Obtain a new matrix $(R_{ij})_{(n-1) \times n}$ whose rows are $r_1, r_2, r_{3 \ldots\ldots} r_{n-1}$.

Step3. Determine the oddments corresponding to each row and is defined as the determinant $|C_\gamma|$, where C_γ is obtained from C_{ij} by deleting its γth row.

Similarly, find the oddments $|R_\delta|$, where R_δ is obtained from R_{ij} by deleting its δth column and write the oddments against their respective rows and columns.

Step4. Check whether the sum of row oddments is equal to sum of column oddments. If yes, go to step 5, otherwise method fails.

Step5. Obtain the probabilities of applying mixed optimum strategies by dividing the corresponding oddments with the oddments total.

Step6. Calculate the value of game corresponding to the optimum strategies by multiplying the probabilities of mixed strategies with their corresponding payoffs and obtaining the sum thereof.

V. FLOW CHART FOR SOLVING FUZZY GAME PROBLEMS

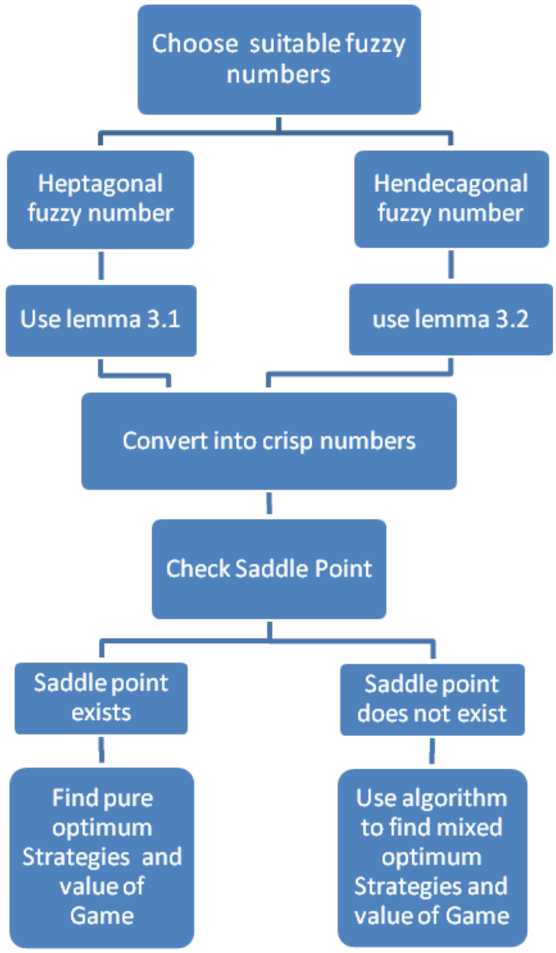

Figure 3

VI. APPLICATION OF PROPOSED METHOD TO GAME PROBLEMS

6.1 Solution of fuzzy game problems using ranking method of heptagonal fuzzy numbers:

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Example: *Online restaurant finder companies' marketing problem*

Let there are two companies namely, Zocato and Figgy, venturing into online restaurant finding business almost at the same time, each trying to garner maximum customers at the cost of other. Both the companies list down their three major strategies in response to the strategies adopted by the counter company.

Let Zocato has the following three strategies to boost its sales;

It will offer 50 per cent discount on first five orders to every customer subject to maximum discount of Rs. 200 per order.

It will give 100 per cent cash back to customers on their first ten orders which can be used by the customers in the subsequent month orders.

It will give a flat 20 percent off to customers and complete waiver of delivery charge throughout the month.

In contrast Figgy has the following three strategies:

It will give credit of Rs. 200 as a reference bonus for introducing the company to their friends which the customers may use during their subsequent purchases.

It will give a complimentary surprise gift on every order placed with the company.

It will provide a flat 35 percent off on customers' orders all throughout the first two months.

Let the resulting payoff to the player A is as follows

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} (2,4,5,6,7,8,9) & (-1,0,1,2,3,4,7) & (6,8,11,14,19,23,27) \\ (7,10,12,14,16,18,58) & (13,14,16,18,22,28,30) & (-5,-3,-1,1,6,8,10) \\ (-12,-8,-5,-4,-2,0,1) & (6,7,8,12,14,16,53) & (2,3,4,6,7,8,35) \end{bmatrix} \end{array}$$

By applying the proposed ranking technique as defined in lemma 3.1 the fuzzy pay off values are converted into crisp values and it is reduced into the crisp game problem as follows:

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5.88 & 2.25 & 15.25 \\ 2.13 & 19.88 & 18.62 \\ -4.25 & 16 & 8.87 \end{bmatrix} \end{array}$$

Minimum of 1st row = 2.25
Minimum of 2nd row = 2.13
Minimum of 3rd row = -4.25
Maximum of 1st column = 5.88
Maximum of 2nd column = 19.88
Maximum of 3rd column = 18.62
Max (min) = 2.25 and Min (max) = 5.88

Here Max (min) $\neq$ Min (max). Hence the saddle point does not exist.

The reduced crisp value problem is then solved by applying dominance principle. Since all the elements of third column are greater than first column thus the third column is dominated by the first column. Hence eliminating the third column, the crisp game problem is reduced to

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5.88 & 2.25 \\ 2.13 & 19.88 \\ -4.25 & 16 \end{bmatrix} \end{array}$$

Now, third row is dominated by the second row as all the elements of third row are less than the second row. Hence eliminating third row, the crisp game problem is further reduced into 2 X 2 payoff matrix as below:

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5.88 & 2.25 \\ 2.13 & 19.88 \end{bmatrix} \end{array}$$

The reduced matrix as above is solved further by applying oddment method. The augmented payoff matrix so obtained is

	B1	B2	Row Oddments	Probability
A1	5.88	2.25	17.75	17.75/21.38
A2	2.13	19.88	3.63	3.63/21.38
Column Oddments	17.63	3.75	21.38	
Probability	17.63/21.38	3.75/21.38		

Hence the player A chooses mix of two strategies to maximize their profits. So Player A will use his strategy of offering 50 per cent discount on first five orders to every customer subject to maximum discount of Rs. 200 with probability of 0.83 and the strategy of giving 100 per cash back to customers on their first ten orders with probability of 0.17. On the other hand the player B will use strategy of offering credit of Rs. 200 as reference bonus with probability 0.82 and also the strategy of giving a complimentary surprise gift on every order with probability 0.18. So finally when the Player A uses above said strategies, he will get the benefit of

$5.88 \times 0.83 + 2.13 \times 0.17$
$= 5.24$ units.

6.2 Solution of fuzzy game problems using ranking method of hendecagonal fuzzy numbers:

Example: *Election Problem:*

In a country having two party system, the gain of one political party is generally the loss of other political party during elections. Under such system political parties try a vast variety of strategies during elections to snap the vote share of the opponent party and also to retain its own vote share. Let there are two political parties namely Janshakti Lok Party and Devsamaj Party, each trying to increase their vote share at the cost of opponent to win the elections. Both the parties list down their three major strategies in response to the strategies adopted by the opponent party.

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Let Janshakti Lok Party (player A) has the following three strategies to increase its vote share;
It promises complete loan waiver of farmers having land size of less than two hectares.
It promises to provide guaranteed unemployment allowance of Rs. 5000/- p.m. to every unemployed youth between 25 years and 45 years of age.
It promises to provide completely free out-door and in-door treatment to every citizen up to the maximum of Rs. 5 lac per family in a year.
In contrast Devsmaj party (player B) plans to use the following three strategies to counter the moves of Janshakti Lok party:
It promises free crop insurance for every farmer having land size of less than five hectares.
It promises guaranteed employment to every unemployed youth between 20 years and 40 years of age.
It promises a free medical insurance for every family up to maximum Rs. 2 lac. per year.
In this situation the payoff matrix to Janshakti Lok Party in the form of Hendecagonal Fuzzy Numbers (HnFNs) is stated in matrix. The cell position (1,1) shows that when both the parties use their first strategy then the payoff to Janshakti Lok Party is (1,2,3,4,7,10,13,15,16,17,22) which implies a minimum gain of one percent and maximum gain of 22 percent to Janshakti Lok Party. The other cells in the pay off matrix represent gains to the Janshakti Lok Party under different possibilities. The resulting payoff matrix to the player A will be as follows:

$$\begin{array}{c} \quad B \\ A \begin{bmatrix} (1,2,3,4,7,10,13,15,16,17,22) & (3,7,11,13,17,21,22,25,29,32,40) & (2.3,2.9,3.5,4.5,4.9,5.2,5.7,6,6.9,7.4,8) \\ (3.3,3.6,3.9,4.2,4.5,4.8,5.2,5.5,5.8,6.1,6.3) & (-4,-3,-2,-1,0,1,2,3,4,5,6) & (-2,0,1,2,3,5,6,7,8,10,11) \\ (-1,3,4,6,9,15,18,23,27,32,35) & (2,3,5,6,8,9,10,11,12,13,15) & (6,7,8,9,10,11,12,13,14,15,16) \end{bmatrix} \end{array}$$

By applying the proposed ranking technique as defined in lemma 3.2 the fuzzy pay off values are converted into crisp values and the given fuzzy game is reduced into crisp game problem and then solved by maximin-minimax criterion as follows:

$$\begin{array}{c} \quad B \\ A \begin{bmatrix} 10.45 & 21.75 & 5.75 \\ 5.4 & 0.7 & 4.8 \\ 15.95 & 9.4 & 12.2 \end{bmatrix} \end{array}$$

Minimum of 1st row = 5.75
Minimum of 2nd row = 0.7
Minimum of 3rd row = 9.4
Maximum of 1st column = 15.95
Maximum of 2nd column = 21.75
Maximum of 3rd column = 12.2
Max (min) = 9.4 and Min (max) = 12.20

Here Max (min) $\neq$ Min (max). Hence the saddle point does not exist.

The reduced crisp value problem is then solved by applying dominance principle. Since all the elements of first column are greater than third column thus the first column is dominated by the third column. Hence eliminating the first column, the crisp game problem is reduced to

$$\begin{array}{c} \quad B \\ A \begin{bmatrix} 21.75 & 5.75 \\ 0.7 & 4.8 \\ 9.4 & 12.2 \end{bmatrix} \end{array}$$

Now, second row is dominated by the first row as all the elements of second row are less than the first row. Hence eliminating second row, the crisp game problem is further reduced into 2 X 2 payoff matrix as below:

$$\begin{array}{c} \quad B \\ A \begin{bmatrix} 21.75 & 5.75 \\ 9.4 & 12.2 \end{bmatrix} \end{array}$$

The reduced matrix as above is solved further by applying oddment method. The augmented payoff matrix so obtained is:

	B2	B3	Row Oddments	Probability
A1	21.75	5.75	2.8	2.8/18.8
A3	9.4	12.2	16	16/18.8
Column Oddments	6.45	12.35	18.8	
Probability	6.45/18.8	12.35/18.8		

Hence the player A chooses mix of two strategies to maximize his profits. So Player A will use his strategy of offering to complete loan waiver of farmers having land size of less than two hectares with probability of 0.15 and the strategy of giving completely free out-door and in-door treatment up to Maximum Rs. 5 lac to every citizen with probability 0.85 . On the other hand player B will use strategy of offering employment to every unemployed youth between age 20 years to 40 years with probability 0.34 and also the strategy of giving a free medical insurance for every family up to maximum Rs. 2 lac. per year with probability 0.66. So finally when the Player A uses above said strategies, he will get the benefit of 11.25 units (21.75 × 0.15 + 9.4 × 0.85).

VII. RESULTS AND DISCUSSION

The environment in the real life competing situations is always uncertain. Under such fuzzy environment when two opponents compete with each other the resulting pay offs may be fuzzy numbers of any order. The proposed ranking method for converting heptagonal and hendecagonal fuzzy numbers into crisp numbers is applied to two different real life competing situations. Results of first example reveal that when the two online restaurant finder companies namely Zocato and Figgy compete with each other to win more market share in environment where the payoffs are heptagonal fuzzy numbers the optimum strategies are obtained by first converting the fuzzy payoffs into crisp payoffs by following the proposed method in lemma 3.1.

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The optimum result so obtained reveals that the Zocato company will use its two of the three strategies, that is, A1 and A2 with probability of 0.83 and 0.17, Figgy company will also use its two of three strategies, that is B1 and B2 with probabilities 0.82 and 0.18 and in this optimum solution of fuzzy game situation Zocato company will gain a market share of 5.24 units. In the second example where the political parties compete with each other in situation where the resulting payoffs are hendecagonal fuzzy numbers the solution to competing situation is obtained by converting the hendecagonal numbers into crisp numbers using lemma 3.2. The result so obtained reveals that the Janshakti lok party (Player A) will use its strategy A1 and A3 with probabilities 0.15 and 0.85 respectively. On the other hand the opponent Devsmaj party (Player B) will use its two strategies B2 and B3 with probabilities 0.34 and 0.66 respectively. In this competing situation Player A will finally get 11.25 units.

VIII. CONCLUSION

In this paper, an algorithm is developed for ranking of heptagonal and hendecagonal fuzzy numbers and a solution procedure is proposed to get the value of game. This paper integrates the concept of fuzzy ranking and the maximin-minimax principle to solve fuzzy game problems for heptagonal and hendecagonal fuzzy numbers. In future this proposed ranking technique shall be very helpful in solving many practical problems involving situations having heptagonal and hendecagonal fuzzy numbers.

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