

Observations of Blood Flow in an Inclined Improved Generalized Multiple Constricted Artery Influenced by a Magnetic Field

Archana Dixit, Agraj Gupta, N. R. Garg

Abstract This mathematically designed model explores the facts about the velocity of blood flow in an inclined artery as well as in the presence of a magnetic field. This model conjointly reveals regarding core velocity, shear stress on the wall of artery and volumetric rate of flow under constricted condition. Here we are describing blood as Herschel-Bulkley fluid through an inclined artery which is multiple stenosed at equally spaced and distanced. By using appropriate boundary conditions, analytic expressions for these flow characteristics are administered. The results are shown numerically and diagrammatically still as compared with the prevailing results. This data could also be valuable within the progress of the most recent diagnosing tools for solidifying hardening of the arteries.

Keywords: Herschel-Bulkley Fluid, Magnetic porosity, wall shear stress, Magnetization

I. INTRODUCTION

In today's scenario atherosclerosis is a well-known cardiovascular disease across the globe and these diseases are very obvious cause of death and disability (Nichols and O'Rourke, 2005) [1]. It conjointly standard that induration of the arteries is principally the tissue layer disease of these arteries that are massive and medium in size. This unwellness is represented by the development of tissue layer plaques entailing of super molecule deposition persuasive muscles with seditious cells animal tissue fibers and metallic element deposits (Nichols and O'Rourke, 2005) [1]. Acclimation of the arteries is known as coronary artery disease. It brings up to lipids or fats referred to as steroid alcohol, creating the artery wall rigid being deposited on the inner wall of it. This Plaque is the explanation for blockage of an artery downstream. Stenosis could be a pattern that shows that arteries contraction, the plaques and fats are the chargeable for contraction within the blood vessel wall. This contraction is the explanation of reduced blood flow to the area where blood is supplied by that artery. Its main effects are narrowing of the artery and diminution of blood flow within the downward tissues. It conjointly shrinks the wall domestically, alters epithelium perform, tube-shaped structure tone, cell adhesion molecules, and platelets. Butt smoking cardiovascular disease of male sex, diabetes, physical laziness,

adiposeness race and, symptom are the foremost risk factors for coronary artery disease. Presently the golden approach for examining the complexness of a coronary stenosis coronary arthrography whereas X-ray photography is sometimes examined in percent. However, different lesion characteristics like form, length, and eccentricity don't seem to be enclosed here by this approach they will conjointly have an effect on the electrical phenomenon to blood flow. Consequently, Gould, Kirkeeide, and colleagues counseled and substantiated the utilization of one life of coronary flow spare on quantitative arthrography as a parameter that replicates all the many anatomic factors influencing stenosis acuteness [2]. Similarly, different investigators conjointly turned up a detailed association between coronary flow reserve assessed with intracoronary Christian Johann Doppler probes and purpleheart stenosis by quantitative arthrography. A. Sinha and J. C. Mishra [3] studied the model of magnetic fields and perceived the numerical study of flow and heat transfer during an unhealthy artery throughout oscillating blood flow. During this model, they numerically investigated the results of heat transfer as well as mass transfer under unsteady hydro dynamic flow in a vessel once the lumen of the vessel has rapt as a porous structure. All numerical results displayed diagrammatically for velocity, effect of temperature along with concentration fields for different values of skin friction constant. This study delineates the flow that is prejudiced by the presence of a flux and also the value of the Grashof range. S. Chakravarty et al. [4] designed the mathematical model about overlapped stenosed blood vessel. The artery is assumed to be elastic cylindrical tube full of elastic fluid representing blood. Exploitation of the Finite distinction technique they found many results associated with the physiological character of the blood. S.U Siddiqui et al [5] mathematically deliberate the pulsatile flow of Herschel-Bulkley fluid via constricted arteries. This model mentioned regarding velocity, rate of flow, shear stress at the wall of artery and longitudinal electrical phenomenon because of the pulsatile and non-Newtonian nature of blood. The results outline the velocity and flux decline whereas wall shear stress and longitudinal electrical phenomenon incline for increasing values of yield stress with remaining parameters stand mounted. For the opposite state of affairs, because the radius of the artery will increase the volumetric flow, rate and shear stress at the wall of artery decline, however, resistance to flow incline with remaining parameters stand mounted. Azar Mirza et al. [6] mentioned regarding the 2-dimensional steady flow by taking a cosine formed constricted (stenosed) axisymmetric artery with heat transfer. It's conjointly thought-

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about that blood behavior like an incompressible Newtonian fluid. The outcomes within the sort of streamline, wall shear stress, separation purpose, pressure distribution, velocity elements and temperature distribution are displayed diagrammatically. Sapna Ratan Shah [7] designed a mathematical model for generalized Bingham plastic fluid with a constricted artery. An innovative answer shows that because the tube radius inclines wall shear stress resistance to flow inclines and declines as stenosis form parameter inclines Z . Abbas et. al. [8] analyzed the rheologic belongings of Herschel Bulkley fluid for the rhythmic flow of blood in w -shaped stenosis artery. During this article, they examined the temporal and axial distribution of velocity, wall shear stress, volumetric flow rate and resistance to flow and displayed diagrammatically. They conjointly found the blood flow comparison between single cruciform stenosis and w -formed constricted arteries. M.Y. Abdollahzadeh Jamalabadi et al., in [9] used bio-magnetic fluid dynamics and taken blood as non-Newtonian for small fix cosine stenosed arteries in the presence of transverse magnetic field and applied Carreau-Yasuda model. They also applied principals of ferro and magneto hydrodynamics. They found the results of wall temperature controlling on blood flow velocity and shear stress at the wall of arteries. R. Agujetas et al., [10] numerically examined effect of fractional flow reserve (FFR) on the pressure drop through the highly eccentric coronary arteries. They studied the errors made due to uniform velocity profile and laminar approximation and gave the result as there is negligible effect on FFR values near about threshold value taken 0.8. Mathematical modeling provides an efficient and non-invasive methodology to review the blood flow. Analytical models are best suited to exploring the original physics of the condition and producing real time results. The aim of the present study is to look at the result of improved multiple constricted with magnetization and flow is deliberate to be a non-Newtonian fluid representing Herschel Bulkley property on an indented tube throughout the cycle.

II. MATERIALS AND METHODS

In this mathematical model, we have a tendency to think about the axially symmetric flow of blood through an artery. An artery is deliberate to be cylindrical tube think about multiple pathologies. The pure mathematics of the stenosis is given by:

$$R^*(z^*) = R_0 \left[1 - \beta \left\{ (l_0^*)^{s-1} \left\{ \beta z^* - m d^* - (m-1) l_0^* \right\} - \left\{ \beta z^* - m d^* - (m-1) l_0^* \right\}^s \right\} \right]$$

$$m(d^* + l_0^*) - l_0^* \leq \beta z^* \leq m(d^* + l_0^*);$$

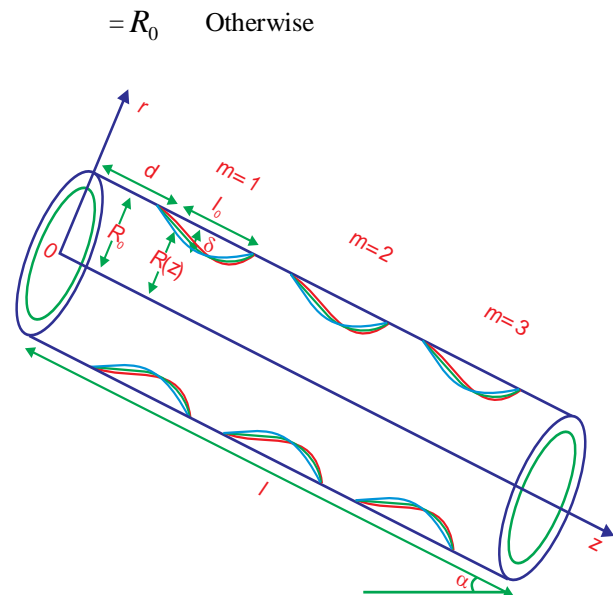


Figure 1: Geometry of the inclined generalized multiple constricted artery

R_0 = Radius of traditional artery

l = Length of the artery

l_0 = Length of stenosis

δ = height of the stenosis

m = number of stenosis in artery

s = shape parameter of the stenosis ($s \geq 2$)

d = distance between the points at the equal distance

(r, z) = Radial and axial co-ordinates

u = Axial speed element

K = Viscosity constant of blood

M = Magnetic intensity level

Where $\beta = \frac{\delta}{R_0 (l_0^*)^s} (s-1)$ and δ be the greatest

height of stenosis at

$$z^* = \frac{m d^* + (m-1) l_0^* + l_0^* / s^{1/s-1}}{\beta}$$

It is assumed here the blood flow is steady laminar and totally developed and think about as Herschel Bulkley fluid the fundamental momentum equation

governing the flow is

$$-\frac{\partial p^*}{\partial z^*} + \frac{1}{r^*} \frac{\partial}{\partial r^*} (r^* \tau^*) + \mu_0 M \frac{\partial H^*}{\partial z^*} + \frac{\sin \theta}{G'} = 0 \quad (2)$$

The constituent equation for Herschel-Bulkley fluid is given

$$\text{by } (\tau^* - \tau_0^*)^n = K \left(-\frac{\partial u^*}{\partial r^*} \right); \tau^* \geq \tau_0^* \quad (3)$$

$$\frac{\partial u^*}{\partial r^*} = 0; \tau^* \leq \tau_0^* \quad (4)$$

Where τ_0^* be the yield stress and κ be the viscosity coefficient of blood.

The boundary conditions set up the matter

$$u^* = 0 \text{ at } r^* = R^*(z) \quad (5a)$$

$$\tau^* \text{ is finite at } r^* = 0 \quad (5b)$$

$$\text{In the core region } u^* = u_c^* \text{ at } r^* = R_c^* \quad (5c)$$

Here u_c^* is the core velocity.

Now non-dimensional theme is as follows

$$r = \frac{r^*}{l}, z = \frac{z^*}{l}, R = \frac{R^*}{R_0}, P = \frac{P^*}{\rho u_0^2}, u = \frac{u^*}{u_0}, \delta = \frac{\delta^*}{R_0}, \tau = \frac{\tau^*}{\rho u_0^2},$$

$$d = \frac{d^*}{l}, l_0 = \frac{l_0^*}{l}, H = \frac{H^*}{H_0}, G = \frac{G}{\rho u_0^2/l} \quad (6)$$

The geometrical illustration of the stenosis in non-dimensional type is as follows

$$R(z) = R_0 \left[1 - B \left\{ \begin{array}{l} l_0^{s-1} \{ \beta z - md - (m-1)l_0 \} - \\ \{ \beta z - md - (m-1)l_0 \}^s \end{array} \right\} \right] \quad (7)$$

$; m(d+l_0) - l_0 \leq \beta z \leq m(d+l_0)$

$= 1 \quad \text{otherwise}$

Where $B = \frac{\delta}{R_0 l_0^s} \frac{s^{s/(s-1)}}{(s-1)}$ and δ be the greatest height of stenosis at

$$z = \frac{md + (m-1)l_0 + l_0/s^{1/s-1}}{\beta}$$

Equation (2), (3) and (4) reduces to

$$-\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r\tau) + c_1 \frac{\partial H}{\partial z} + \frac{\sin \theta}{G} = 0 \quad (8)$$

$$\text{Where } c_1 = \frac{\mu_0 M H_0}{\rho u_0^2}$$

$$(\tau - \tau_0)^n = c_2 \left(-\frac{\partial u}{\partial r} \right); \tau \geq \tau_0 \quad (9)$$

$$\text{Where } c_2 = \frac{K}{\rho^n u_0^{2n-1} R_0}$$

$$\frac{\partial u}{\partial r} = 0; \tau \leq \tau_0 \quad (10)$$

The boundary conditions (5a-5c) can currently become

$$u = 0 \text{ at } r = R(z) \quad (11)(a)$$

$$\tau \text{ is finite at } r = 0 \quad (11)(b)$$

Within the core region $u = u_c$ at $r = R_c$ (11) (c)

On exploitation analytical methodology in Equations (8-10) and exploitation boundary conditions (11) (a, b, c). The representations for velocity u and core velocity u_c are

$$u = -\frac{1}{2^n(n+1)c_2 \left(\frac{\partial p}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right)} \left[\left\{ \left(\frac{\partial p}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) r - 2\tau_0 \right\}^{n+1} - \left\{ \left(\frac{\partial p}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) R - 2\tau_0 \right\}^{n+1} \right] \quad (12)$$

If $u = u_c$ at $r = R_c$

$$u_c = -\frac{1}{2^n(n+1)c_2 \left(\frac{\partial p}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right)} \left[\left\{ \left(\frac{\partial p}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) R_c - 2\tau_0 \right\}^{n+1} - \left\{ \left(\frac{\partial p}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) R - 2\tau_0 \right\}^{n+1} \right] \quad (13)$$

The volumetrically flow Q rate is given by

$$Q = 2\pi \int_0^{R_c} r u_c dr + 2\pi \int_{R_c}^R r u dr = Q_c + Q_1 \quad (14)$$

Where Q_c the flow rate in core region and Q_1 is in the circular region of the constricted artery.

With u and u_c from equations (12) and (13) in equation (14), then flow rate

$$Q = \varepsilon \left[\omega (\phi^{n+2} \varphi - \chi^{n+2} \sigma) + \frac{1}{2} (\chi^{n+1} R^2 - \phi^{n+1} R_c^2) \right] \quad (15)$$

Where

$$\phi = \left[\left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) R_c - 2\tau_0 \right]$$

$$\chi = \left[\left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) R - 2\tau_0 \right]$$

$$\varepsilon = \frac{\pi}{2^n f_2(n+1) \left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right)}$$

$$\omega = \frac{1}{\left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right)^2 (n+2)(n+3)}$$

$$\varphi = \left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) (n+2) R_c - 2\tau_0$$

$$\sigma = \left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) (n+2) R - 2\tau_0$$

The wall shear stress

$$\tau_R = -K \left(\frac{\partial u}{\partial r} \right)_{r=R} \quad (16)$$



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Where $K = \mu$

Now differentiating Equation (12) with reference to r and substituting in Equation (16), then

$$\tau_R = \frac{\mu}{2^n c_2} \left[\left(\frac{\partial P}{\partial z} - c_1 \frac{\partial H}{\partial z} - \frac{\sin \theta}{G} \right) R - 2\tau_0 \right]^n \quad (17)$$

III. RESULT AND DISCUSSION

Fig. 2: Core velocity vs axial distance for different values of α

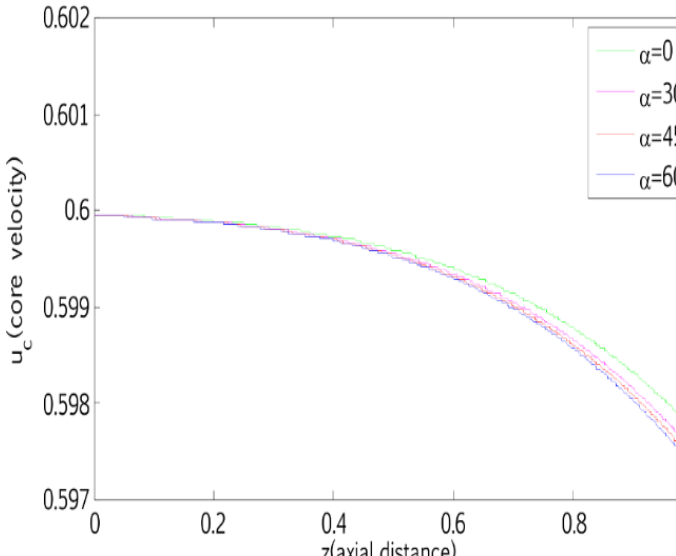


Fig. 3: Core velocity vs axial distance for different values of n

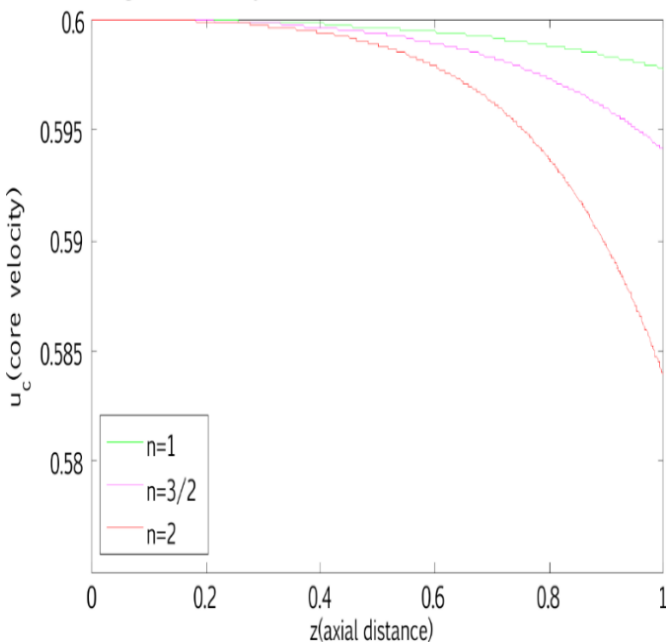


Fig. 4: core velocity vs axial distance for different values of β

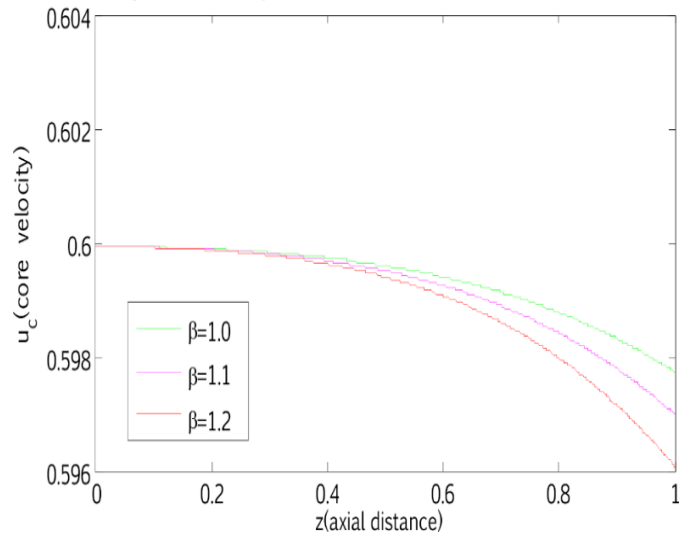


Fig. 5: Core velocity vs axial distance for different values of H

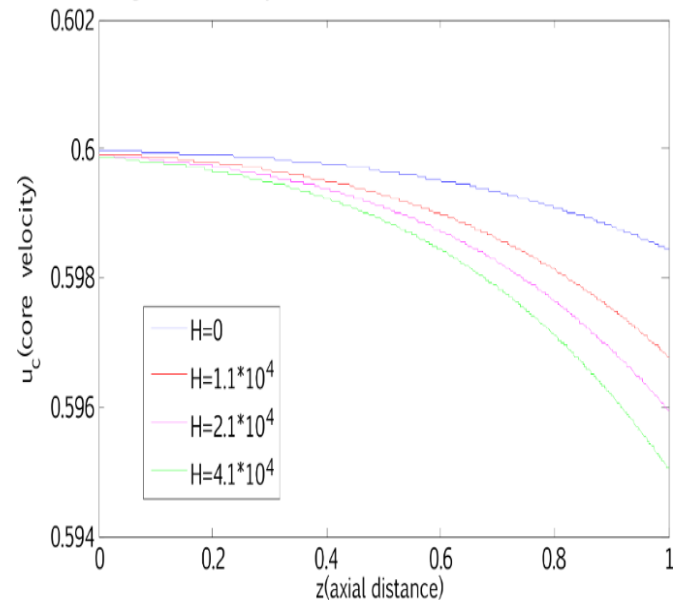


Fig. 6: Velocity of blood vs axial distance for different values of α

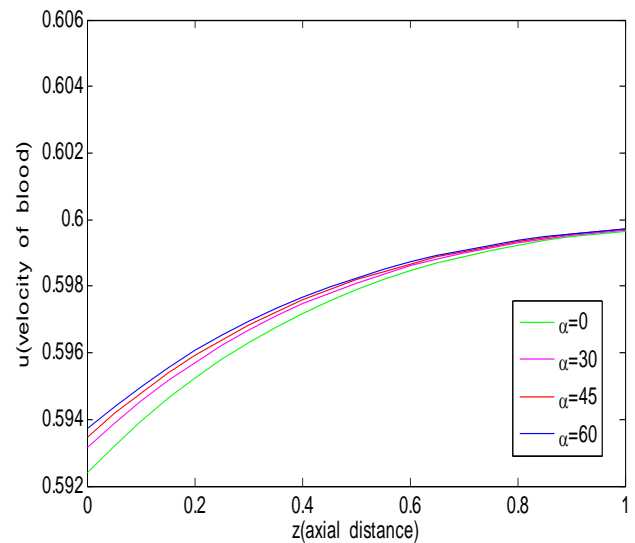


Fig. 7: Velocity of blood vs axial distance for different values of H

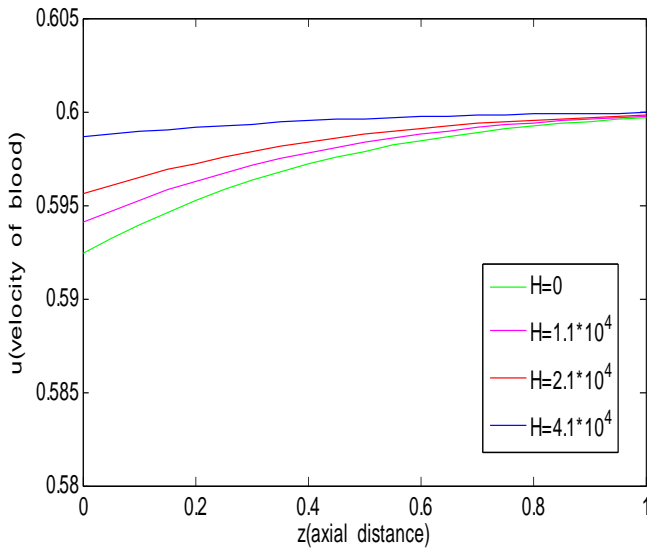


Fig. 10: Wall shear stress vs axial distance for different values of α

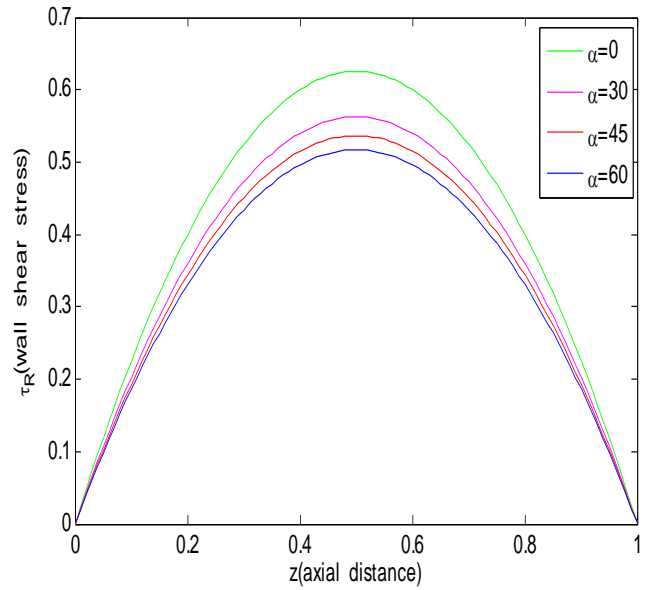


Fig. 8: Velocity of blood vs axial distance for different values of n

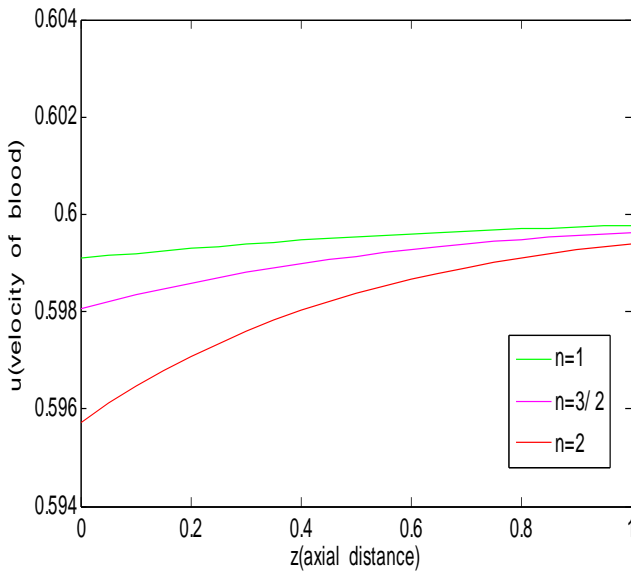


Fig. 11: Wall shear stress vs axial distance for different values of β

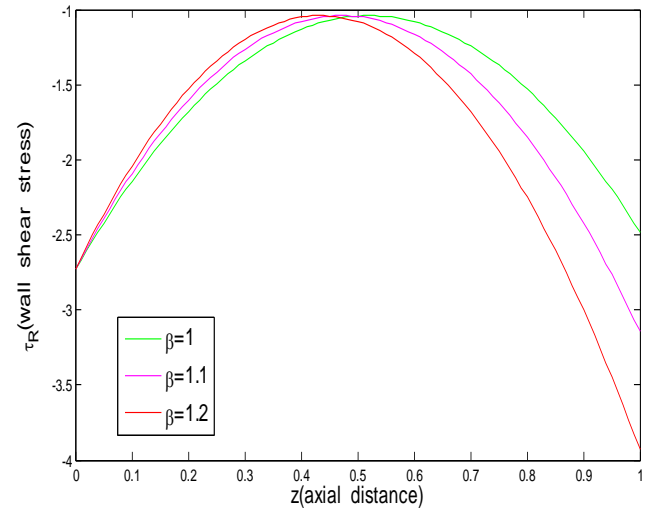


Fig. 9: Velocity of blood vs axial distance for different values of β

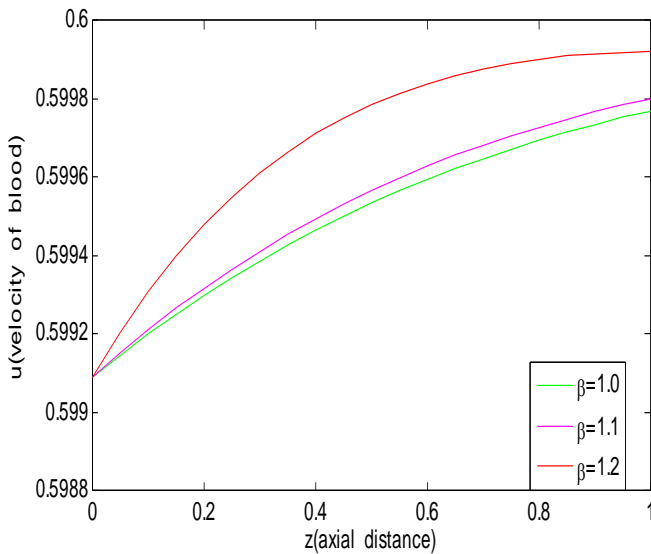
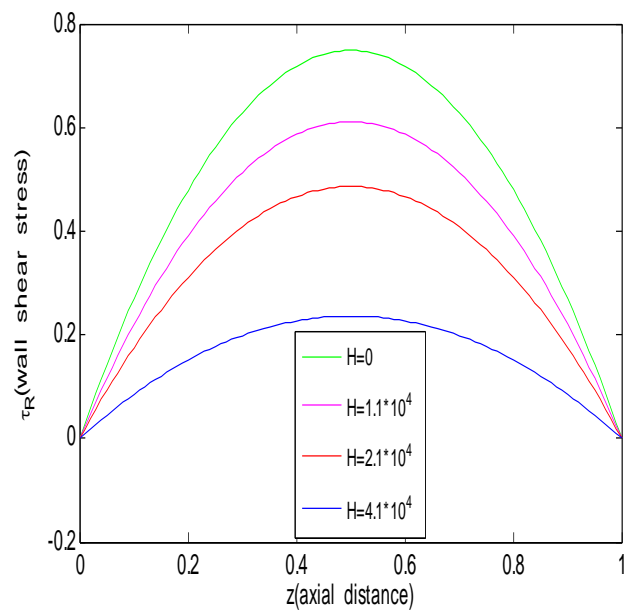


Fig. 12: wall shear stress vs axial distance for different values of H



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Fig. 13: Volumetric flow rate vs axial distance for different values of I_0

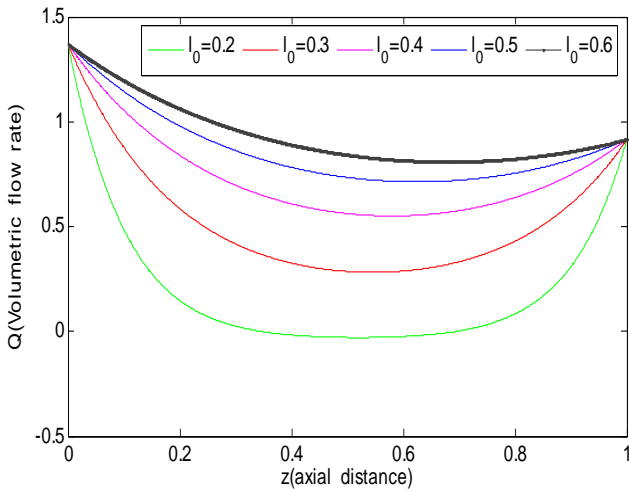


Fig. 16: Volumetric flow rate vs axial distance for different values of β

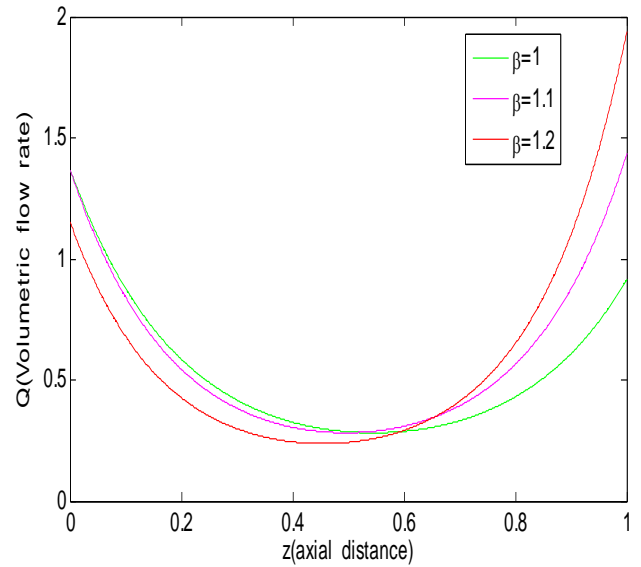


Fig. 14: Volumetric flow rate vs axial distance for different values of H at

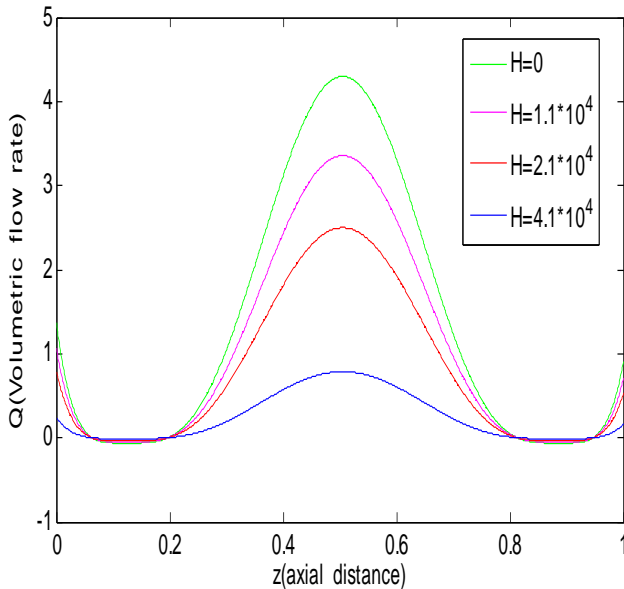


Fig. 17: Volumetric flow rate vs axial distance for different values of n

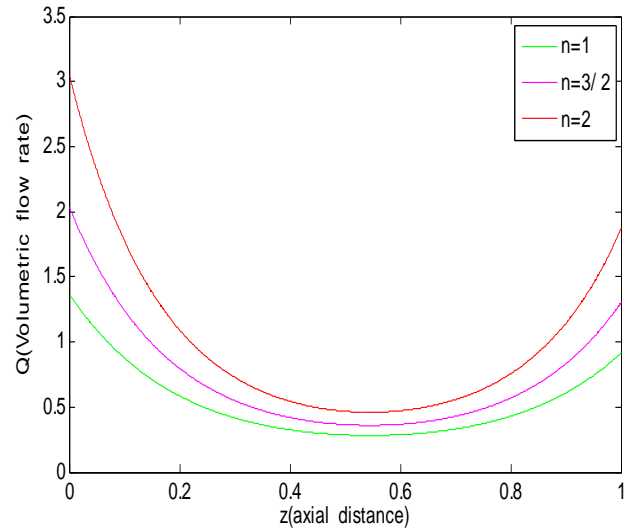
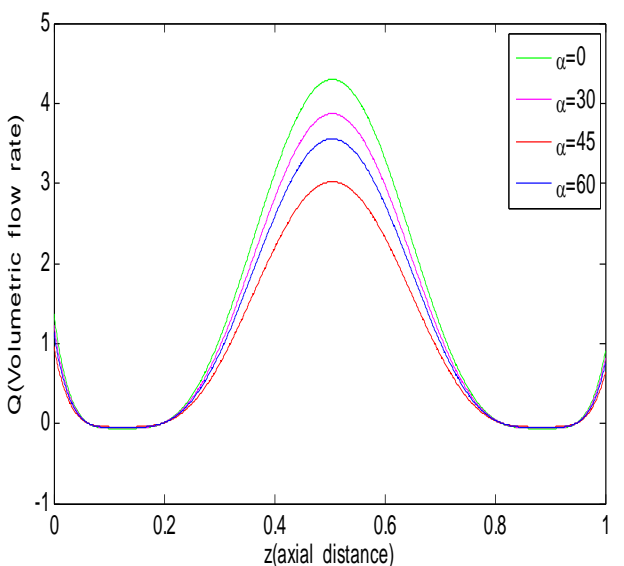


Fig. 15: Volumetric flow rate vs axial distance for different values of α at I_0



Many researchers ascertained that in unhealthy condition blood behaves like Newtonian fluid if the shear rate of blood is high enough. However, the non-Newtonian model of blood is additionally acceptable because the shear rate of blood is extremely little. Numerical calculations are finished varied combos of parameter. Figure 2 indicates the variant of core velocity of blood with axial distance for incline parameter $\alpha = 0, 30, 45, 60$. It shows that the flow velocity decreases with will increase the incline parameter α . Figure 3 depicts the variation of core velocity with the axial distance for various values of the flow behavior index $n = 1, 3/2, 2$. The computational results show the velocity decreases with will increase the flow behavior index. Figure 4 illustrates the variation of core velocity with axial distance for various values of parameter (β). The results show if the β will increases the core velocity decreases. Figure 5 Exhibits the variation of core velocity with axial distance for various values of the magnetic intensity (H) is shown. The graph shows that the axial velocity decreases with increases in magnetic intensity. Conjointly it's seen that within the nonexistence of the external magnetic field ($H=0$) the fluid



velocity is upper in its presence ($H > 0$). We have a tendency to conjointly ascertain the upper values of magnetic intensity reduced the axial velocity to a larger extent. We also found that here the fluid velocity is most at the axis of the tube and step by step decreases towards the walls. Here we have also obtained the variation by assuming the worth of $H = 0, 1.1 \times 10^4, 2.1 \times 10^4, 4.1 \times 10^4$. The graph represents the core velocity decreases with increases the worth of H . Figure 6-9 illustrate the velocity of blood with axial distance. If the parameters α, β, n, H increase, the velocity of blood additionally will increase. Within the analysis of blood flow, wall shear stress plays a very important role in blood vessel flow in the constricted region. Figure 10-17 exhibits the disparity of shear stress at the wall of artery for various values of α, β, n, H . We have observed to ascertain here: once these parameters increase the wall shear stress rate conjointly decreases. It's complete that the occurrence of magnetic field disturbs the wall shear stress and also the flow rate.

IV. CONCLUSION

The present model has been examined from the assorted aspects of blood flow in a constricted artery with the result of magnetic field. During this analysis, we've thought about the Herschel Bulkley fluid model for blood. During this multiple stenosis mode that the stenosis formed parameter, core velocity, blood flow velocity, wall shear stress, rate and intensity of magnetic field are the strongest parameters to influence the flow. The study shows that the rapid flow characteristics considerably laid low with the magnetic parameter and tapered angle. The most advantage of this study is that we have a tendency to calculate varied stenosis parameter and obtained graphical results. So, the mathematical expression could facilitate medical practitioners to regulate the blood flow of a patient up to normal range by applying an appropriate magnetic field

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