An Efficient Hardware Implementation of Finite Field Inversion for Elliptic Curve Cryptography

Thirumalesu Kudithi, Sakthivel R

Abstract: The modular inversion operation is an essential hardware design for computing speed when we use it in cryptography applications. Through this work, we present a FSM based design methodology to achieve speed, area and high-performance modular binary inversion algorithm over 256-bit prime field. The proposed architecture implemented using Xilinx Virtex-7 FPGA device, it achieves 37% reduction in area-delay product and 15% and 16% of improvement in speed and throughput respectively, when compared with existing designs. Also, ASIC based implementation is done using TSMC 65nm CMOS technology, the synthesis results achieved the maximum operating clock frequency is 833 MHz; and throughput of 626Mbp/s, which makes it suitable for speed-critical crypto-applications.

Index Terms: Modular inversion, Field-programmable gate-array (FPGA), Elliptic curve cryptography (ECC), Application specific integrated chip (ASIC).

I. INTRODUCTION

Cryptography aims to hide the information message through a public communication channel, which applies principles of mathematical algorithms. These algorithms extensively involve modular arithmetic operations in a large Galois Fields. Such operations are modular inversion, multiplication, subtraction and addition. In recent years, elliptic curve cryptography (ECC) has been adopted by growing Internet of Things (IoT) based applications in industries, that work on innovative security technologies. The modular inversion is one of the essential operations required by cryptographic algorithms or this operation cannot be avoided in practical cryptographic applications and also it is time consuming operation. It is important to ensure that the modular inversion logic is implemented to achieve optimized area and performance while using in affine coordinate systems. FPGA technology has a greater flexibility to implement cryptographic algorithms.

The goal is to implement high speed modular binary inversion algorithm over 256-bit prime field using architectural optimization techniques and algorithmic reformulations for ECC using FPGA / ASIC-based design technology. This paper is organized as follows: literature review is presented in section. II. A brief mathematical background about ECC and modular inversion is described in section. III. Register-Transfer Logic (RTL) model for inversion algorithm implementation or Finite-state machine with data-path (FSMD) for modular inversion and its components presented in Section. IV. The improved results achieved through FPGA and ASIC implementation in comparison with other available designs shown in Section. V. The final section concludes the paper by summarizing the advantages of the proposed architecture.

II. LITERATURE SURVEY

In literature, three well-known methods that are used to compute modular inversion/division: Extended Euclidean Algorithm (EEA), Montgomery modular inversion method and Fermat's Little Theorem (FLT). One of the efficient algorithm Binary Inversion Algorithm to compute modular inversion, which is easier to implement RTL based hardware perspective and also more area-efficient with high-speed operation. The modular inversion/division algorithm for ECC over $F_p$ was developed by Hossain [1, 2], Javeed [3]–[4], Mrabet [5], Kudithi [6], Liu [7], Lee [8], Vliegen [9], Ghosh [10], McIvor [11], and Daly [12]–[13]. Among these McIvor described different hardware architectures for modular inversion algorithm and remaining authors are used binary inversion algorithm or extended Euclidean algorithm. To implement hardware, Xilinx platform with Virtex-7 FPGA is used.

III. BACKGROUND

A. Definition of elliptic curve

The simplified Weierstrass elliptic curve $E$ is given in form:

$$E : y^2 = x^3 + ax + b$$  \hspace{1cm} (1)

Above equation exists in prime field $F_p (p > 3)$ , constant curve parameters are $a$ and $b$ . The points represented in curve using affine coordinates are $(x, y)$ . Equation (1) satisfies cyclic abelian group operations over prime field, $F_p$ and it need to satisfy sufficient condition $4a^2 + 27b^3 \neq 0$.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points represented on curve, then the group operation of point addition operation gives another point $R(x_3, y_3)$ on curve, i.e., $(R = P + Q)$ would be calculated as follows.
Hardware design for division or modular inversion requires at least one modular inversion operation. This operation can be represented as

\[ R = a^{-1} \mod p \]  

where \( a \) an integer and the prime is \( p \) with maximum length of bit \( n \). This inverse exists if and only if \( a \) and \( p \) must be relatively prime. Moreover the computation of division or inversion takes the same amount. An example of a modular inverter:

Let \( a = 13 \) and modulus \( p = 17 \) then

\[ R = a^{-1} \mod p = 13^{-1} \mod 17 = 4 \]  

because \( 4 \times 13 \mod 17 = 1 \) or \( \text{GCD}(4, 17) = 1 \)

The modular inversion/division can be computed in several ways based on the architecture used with the size and type of the field over a finite field, \( F_p \), [15]–[16]. The following are the list of algorithms used for computing modular inversion.

- Fermat’s little theorem,
- Dimitrov-Järvenin (DJ) method,
- Montgomery multiplication method,
- Extended Euclidean algorithm,
- Itoh-Tsujii multiplicative inverse algorithm,
- Almost Inverse Algorithm,
- Kaliski’s Montgomery inversion algorithm.

Among these algorithms, the Extended Euclidean Algorithm or Binary Euclidean Algorithm is efficient way and simple to compute modular inversion. The other method is the Fermat’s little theorem is useful to create constant time modularity to avoid attack in ECC like power attacks.

C. Introduction to high level state machine model

Most of the designers starts with C-code and can be implemented in HLSM (high level state machine). This HLSM is used to describe FSMD (finite state machine with data-path). The basic information of FSMD based has been provided in the book [17]–[19]. To convert any algorithm to RTL, Some rules need to be followed while converting to HLSM from basic types of C-code, the templates of which are shown in Fig. 1. It consists different basic constructs of C-code to translate into HLSM states representation, this process are mostly self-explanatory [19].

IV. HARDWARE DESIGN FOR INVERTER OVER \( F_p \)

This section presents FSM based hardware architecture implementations for modular inversion algorithm. The National Institute of Standards and Technology (NIST) standard chosen for elliptic curves over \( F_{2^{56}} \) to implement modular inversion algorithm [20]–[21].

Algorithm 1 is a well-known method to compute modular inversion over the prime field. This is used four intermediate variables, i.e., \( u, v, x, y \). The even or odd numbers represented by the single bit variables \( u_0 \) and \( v_0 \) and which is present in least-significant-bit (LSB) variables \( u \) and \( v \). The representation of flow chart is shown in Fig. 2.
The hardware architecture of inversion algorithm works iteratively and it consists a series of additions/subtractions, and shift operations. At least one bit length is reduced either \( u \) or \( v \) variable at each loop iteration. The result of modular inversion represented \( R \) over prime field \( F_p \).

In FSM method, the approach to design HLSM for modular inversion algorithm is described. The basic C-language to HLSM template or FSM based constructs are discussed in section IIIC [17]–[19]. These constructs help in easily implementing RTL based architecture and enables us to generate HDL code for any algorithm. By using this constructs, we translated into basic HLSM for Algorithm 1 as shown in Fig. 2.

**Algorithm 1** Binary inversion algorithm over \( F_p \) [1, 10]
This FSM consists of “states” and “arcs (arrows)” each state represents either an operational statement (i.e., arithmetic or shift operations) or null operation and each arc represents a conditional statement or passing of a signal value.

Fig. 2, consists of total 23 states and after synthesizing this FSM using synthesis tool, it might be reduces upto 10 states. The “Go” signal indicates the start of modular inversion process. The result R in state $S_{21}$ or $S_{22}$, requires one modulo reduction operation. The modulo reduction hardware can be implemented using modular addition by tying one input to zero [12].

The FSM method, the implementation of inversion hardware requires four essential intermediate registers $u, v, x$ and $y$ and needs to be initialized to $u = 1; v = 1; x = 1$ and $y = 0$ along with the control signal “$\text{sig}_{\text{inv}}$” assigned to 0 in state $S_2$. The LSB bits of these intermediate registers are indicated by variables $u_0, v_0, x_0$ and $y_0$.

These intermediate register values will change while the state transitions from state $S_i$ to state $S_j$, and it repeats until the condition $u = 1$ or $v = 1$ becomes true. In state $S_{50}$ the control signal “$\text{sig}_{\text{inv}}$” is assigned to 1 and moves to next state $S_{21}$ or $S_{22}$ to compute the modular reduction operation, which gets the result R, and assigns the control signal “$\text{sig}_{\text{inv}}$” to zero. Finally from state $S_{21}$ or $S_{22}$ FSM moves to the initial state $S_0$ to perform modular inversion for another set of input values $a$ and $p$. The name of the dashed block ‘PSM’ indicates parallel state machine, the values of $u_i$ and $v_i$ are computed in parallel when the condition $u \neq 1$ is satisfied in state $S_2$. When coding RTL for PSM block, using the same variables $u, v, x$ and $y$ for parallel computation in states $S_5$ and $S_8$ might cause a problem during synthesis. This can be avoided by taking different register variables for computation in states $S_5$ and $S_8$. Which might impact the area in ASIC implementation, but not in FPGA implementation. All intermediate signals or variables are bounded by $(n + 2)$-bits throughout the calculation.

The computation time for one modular inversion using FSM based implementation (proposed method), takes maximum $2n + 1$ clock cycles (CCs) or an average $1.33n + 10$ CCs; where $n$ is maximum length of bits $p$ and $a$ [1-2].

This design provides low hardware resources (low area) but the computation time is more. The performance metrics of AT is decreased around 90% and throughput is increased around 294%. So, our design provide better results in terms of AT and throughput with respect to recent designs are Hossain et al. [2], Javeed [3-4] and Mrabet et al. [5].

Our proposed methods for implementing the modular inverter achieves a computation time and frequency of 2.013µs and 168.56MHz respectively using Xilinx Virtex-7 FPGA over F_{256}. Our proposed design provides 15% improved frequency of operation as compared with previous designs [1-2]. The proposed method uses 1069 slices without DSP slices. We observed an area improvement of around 38% when compared with previous designs [1-2, 9]. The comparison of our results with available designs is not straight forward as most of the listed authors have implemented using different FPGA technology except Hossain et al. [1-2] and Kudithi et al. [9].

Table I, gives the summary of device utilization in FPGA. The major comparison is utilization of area and it is defined the total number of slices utilized in FPGA. The occupied area is 27.8% improvement using our FSM method when compared with [1-2, 9].

Most of the authors presented the hardware implementation for modular inversion using different FPGA platforms which makes the comparison is difficult. So, we used throughput rate parameter and area-delay product (AT) parameter for comparison purposes.

Table II, gives the simulated results and performance from various papers presented by Hossain et al. [2], Liu et al. [3], Lee et al. [4], Vliegen [5], Ghosh et al. [6], McIvor et al. [7], Daly et al. [8] and Kudithi et al. [9]. We can observe that the results from Hossain et al. [2] and Kudithi [9] are better in terms of area and computation time when compared with other literature results. Our proposed design gives even better results in terms of frequency of operation, computation time, AT and area when compared with reference designs [2] & [9].

Javeed [3-4] designs has implemented using Virtex-6 FPGA. The throughput is improved around 68% to 75% and AT is decreased around 88% to 90% when compared with our design. Mrabet et al. [5] design provides $4n$ clock cycles to develop the modular inversion, where $n = \log_2^2$ and their design is used implementation hardware in Virtex-5 FPGA.

<table>
<thead>
<tr>
<th>Numbers of Logic Utilization</th>
<th>Proposed Design</th>
<th>Ref. [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slice Registers</td>
<td>1310</td>
<td>1551</td>
</tr>
<tr>
<td>Slice LUTs</td>
<td>4035</td>
<td>5868</td>
</tr>
<tr>
<td>Fully Used LUT-FF Pairs</td>
<td>960</td>
<td>1291</td>
</tr>
<tr>
<td>BUFG/BUFGCTRLs</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bounded IOBs</td>
<td>516</td>
<td>516</td>
</tr>
<tr>
<td>No. of Occupied Slices</td>
<td>1069</td>
<td>1480</td>
</tr>
<tr>
<td>Operating Frequency</td>
<td>168.56</td>
<td>146.38</td>
</tr>
</tbody>
</table>
The development of modular inversion in Virtex-II and Virtex-E are listed out and these designs are obsolete now. So we listed out those design values and estimated parameters but not compared with our designs.

Fig. 3 and Fig. 4 shows that the performance analysis of area-delay product and throughput measures with our design and existing designs. Among these the AT and throughput values of Ref. [1]–[2] and [6] are approximately equal with our design.

The design is also synthesized using Synopsys Design Compiler with Taiwan Semiconductor Manufacturing Company (TSMC) standard logic-cell library (65nm, 1.1V, 25°C). We are calculated power-delay product, area-delay product and throughput with estimated gate-count.

Table III shows various computed parameters used. The “area” is measured in (µm²), which is a total number of 2-input NAND gate equivalent; the “time” is the total computation time, “power” indicates the total power, which is a combination of internal power, switching power and leakage power values, the “energy” is a product of power and delay (Time), the ‘throughput’ is a measure of bit field over the computation time; these factors are calculated for design to finish one operation.

The proposed design methods is proven to be very fast, a frequency of 833.33 MHz with an optimal area of 72.0272 mm² (32.109 K gates) and throughput rate of our design is 626 Mbps for modular inverter over F_{256}. Table III gives the result of three different (Typical case, Best case and Worst case) analysis, obtained from synthesis tool for proposed design over prime field F_{224} and F_{256}.

In our proposed design method requires an average of 1.33MHz clock cycles (CCs) with an extra 10CCs to get proper output, because of the pipelining mechanism, it is sufficient to execute Algorithm 1.

**VI. CONCLUSION**

This paper targeted to ECC applications and presented clearly the hardware architecture implementation for modular inversion over F_{256}. The FSM based binary inversion algorithm results for getting high speed nd low area on FPGA device compared to earlier designs. Our design method has achieved a maximum frequency of 168 MHz and throughput of 127 Mbps with the occupied area of 1069 slices. Synthesis results from Synopsys Design Compiler shows that, a maximum frequency of 833 MHz and throughput of 626 Mbps with consuming area of 32.1 K gates. These results makes our proposed designs more suitable for speed-critical cryptographic applications in affine coordinates. Our design gives best results in terms of area and frequency.

### Table II. Performance Analysis for Modular Inversion over F_{256}

<table>
<thead>
<tr>
<th>Ref. Design</th>
<th>FPGA Device</th>
<th>Area (slices)</th>
<th>Frequency (MHz)</th>
<th>Time (µs)</th>
<th>Area×Time (AT)</th>
<th>Throughput (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed work</td>
<td>Virtex-7</td>
<td>1069</td>
<td>168.56</td>
<td>2.013</td>
<td>1</td>
<td>127.17</td>
</tr>
<tr>
<td>Hosain [1]</td>
<td>Virtex-7</td>
<td>1480</td>
<td>146.38</td>
<td>2.329</td>
<td>1.59</td>
<td>109.91</td>
</tr>
<tr>
<td>Hosain [2]</td>
<td>Kintex-7</td>
<td>1480</td>
<td>142.38</td>
<td>2.33</td>
<td>1.59</td>
<td>109.87</td>
</tr>
<tr>
<td>Javeed [3]</td>
<td>Virtex-6</td>
<td>1190</td>
<td>151.00</td>
<td>3.39</td>
<td>8.71</td>
<td>75.51</td>
</tr>
<tr>
<td>Javeed [4]</td>
<td>Virtex-6</td>
<td>1340</td>
<td>146.00</td>
<td>3.52</td>
<td>10.18</td>
<td>72.72</td>
</tr>
<tr>
<td>Marabet [5]</td>
<td>Virtex-5</td>
<td>592</td>
<td>129.00</td>
<td>7.937</td>
<td>10.15</td>
<td>32.25</td>
</tr>
<tr>
<td>Kudithi [6]</td>
<td>Virtex-7</td>
<td>1577</td>
<td>138.3</td>
<td>2.45</td>
<td>1.74</td>
<td>104.23</td>
</tr>
<tr>
<td>Lee [8]</td>
<td>Virtex-II</td>
<td>9213</td>
<td>37.00</td>
<td>4.98</td>
<td>21.25</td>
<td>51.40</td>
</tr>
<tr>
<td>Vliegen [9]</td>
<td>Virtex-II</td>
<td>2085</td>
<td>68.17</td>
<td>1160</td>
<td>1120.07</td>
<td>0.22</td>
</tr>
<tr>
<td>Ghosh [10]</td>
<td>Virtex-II</td>
<td>9146</td>
<td>34.00</td>
<td>14.60</td>
<td>61.81</td>
<td>17.53</td>
</tr>
<tr>
<td>Daly [12]</td>
<td>Virtex-II</td>
<td>5477</td>
<td>50.00</td>
<td>6.40</td>
<td>16.23</td>
<td>40</td>
</tr>
<tr>
<td>Dormale [22]</td>
<td>Virtex-E</td>
<td>1390</td>
<td>41.00</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

|† Normalized value, AT = 1/0.00215932655 = 463.10735, where A: area (slices) and T: time (s).
of operation making it best suitable for ECC applications.

Fig.3 Comparison of our design AT with existing designs.

Fig.4 Comparison of our design throughput with existing designs.

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REFERENCES


AUTHORS PROFILE

Thirumalesu Kudithi received the B.Tech degree in Electrical & Electronics Engineering from Jawaharlal Nehru Technological University (JNTU), Hyderabad, India, in 2003 and M.Tech degree in VLSI Design and Embedded Systems from Visvesvaraya Technological University (VTU), Belgaum, India in 2006. After he served eight years as an Assistant Professor in the Department of Electronics & Communication Engineering at Sreenivasa Institute of Technology and Management Studies (SITAMS), Chittoor, India. Currently he is pursuing Ph.D degree in the school of Electronics Engineering at Vellore Institute of Technology (VIT), Vellore, India. His research interests include efficient arithmetic for cryptographic algorithms, stochastic computing and embedded systems. He is a member of Indian Society for Technical Education (ISTE). He published several technical papers in journals/conferences.

Sakhthivel R received Bachelor degree in Electrical Engineering from Madras University, Madras, India in 2000 and his M.E degree in Applied Electronics from Anna University, Madras, India in 2004. He has received his Doctorate in the area of Low Power High speed architecture development for signal processing and cryptography from VIT University, Vellore, India, in 2014. He is currently working as an Associate Professor in the School of Electronics Engineering at Vellore Institute of Technology (VIT), Vellore. Currently he works on high-speed and low-power hardware architectures, Neuromorphic computing, efficient VLSI architectures for AI computing. He is a member of IEEE, Indian Society for Technical Education (ISTE), System Society of India (SSI) and VLSI Society of India (VSI). He has authored two books and published around 35 peer reviewed papers in journals/conferences.

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