

Autoregressive Models and Non-Local Self Similarity in Sparse Representation for Image Deblurring

Y Ravi Sankaraiah, S Varadarajan

Abstract: Local area within a normal natural image can be thought as a stationary process. This can be modelled well using autoregressive models. In this paper, a set of autoregressive models will be learned from a collection of high quality image patches. Out of these models, one will be selected adaptively and will be used to regularize the input image patches. In addition to the autoregressive models, a non-local self-similarity condition was proposed. The autoregressive models will exploit local correlation of individual image, but a natural will have many repetitive structures. These structures, which are basically redundant, are very much useful in image deblurring. The performance of these schemes is verified by applying to image deblurring.

Index Terms: Autoregressive models, deblurring, non-local self-similarity, sparse domain selection.

I. INTRODUCTION

The natural images, generally, can be coded using structural primitives like edges and line fragments [1]. These fragments are similar to simple cell receptive fields [2]. Olshausen *et al.* proposed to use small number of basis functions which are chosen from an over-complete code set to represent natural image [3]. Recently, such kind of sparse coding schemes are extensively being studied to solve inverse problems. This is in proportion to the progress of l_0 -norm and l_1 -norm minimization schemes [4].

Let $x \in \mathcal{R}^n$ is the signal to be encoded, and $\Phi = [\varphi_1, \varphi_2, \dots, \varphi_m] \in \mathcal{R}^{n \times m}$ is the assumed dictionary. The sparse coding of \mathbf{x} over Φ is to find a vector $\alpha = [\alpha_1; \alpha_2; \dots; \alpha_m]$ such that $\mathbf{x} = \Phi\alpha$ [5]. It is note that many coefficients in α are very close to zero. When the sparsity is measured as an l_0 -norm of α , which actually counts the non-zero quantities in α , the problem of sparsity coding becomes $\|\alpha\|_0 \leq T$. Here T is a number governing the amount of sparsity [6]. On the other hand, the vector α can be found as $\hat{\alpha} = \arg \min_{\alpha} \left\{ \|\mathbf{x} - \Phi\alpha\|_2^2 + \lambda \|\alpha\|_0 \right\}$ where λ is an arbitrary constant. The l_0 -norm is non-convex, hence it is many times replaced with l_1 -norm or weighted l_1 -norm, so that the problem becomes convex [7, 8, 9, 4]. The most important part of sparse representation is the selection of dictionary Φ .

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Many efforts are made in choosing the training image dataset and learning a dictionary [10-20, 6]. Let the training image patches are grouped to form the set \mathbf{S} , $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_N] \in \mathcal{R}^{n \times N}$. The objective of dictionary learning is to optimize the dictionary Φ and the representation coefficient matrix $\Lambda = [\alpha_1, \dots, \alpha_N]$ so that $\mathbf{s}_i = \Phi\alpha_i$. This can be formulated using the following:

$$(\hat{\Phi}, \hat{\Lambda}) = \arg \min_{\Phi, \Lambda} \|\mathbf{S} - \Phi\Lambda\|_F^2$$

Still the above minimization issue is non-convex. Different approaches were proposed to optimize the dictionary and coefficient matrix [11-13, 18-21]. Adaptive sparse domain selection (ASDS) scheme was presented in [22]. The scheme learns a series of compact sub-dictionaries and assigns adaptively each local patch a sub-dictionary as the sparse domain.

By learning a set of compact sub-dictionaries from high quality example image patches, the ASDS will perform the restoration. The example image patches are clustered into many clusters. Since each cluster consists of many patches with similar patterns, a compact sub-dictionary can be learned for each cluster. For an image patch to be coded, the best sub-dictionary that is most relevant to the given patch is selected. Since the given patch can be better represented by the adaptively selected sub-dictionary, the whole image can be more accurately reconstructed than using a universal dictionary, which will be validated by the experiments.

With ASDS, a weighted l_1 -norm sparse representation model will be proposed for restoration tasks. Suppose that $\{\Phi_k\}$, $k=1, 2, \dots, K$, is a set of K orthonormal sub-dictionaries. Let \mathbf{x} be an image vector, and $\mathbf{x}_i = \mathbf{R}_i \mathbf{x}$, $i = 1, 2, \dots, N$, be the i^{th} patch (size: root $(n) \times$ root (n)) vector of \mathbf{x} , where \mathbf{R}_i is a matrix extracting patch \mathbf{x}_i from \mathbf{x} . With ASDS, the image restoration problem can be modelled as:

$$\hat{\alpha} = \arg \min_{\alpha} \left\{ \|\mathbf{y} - D\mathbf{H}\Phi\alpha\|_2^2 + \lambda \|\alpha\|_1 \right\}$$

The design of Φ_k can be intuitively formulated by the following objective function:

$$(\hat{\Phi}_k, \hat{\Lambda}_k) = \arg \min_{\Phi_k, \Lambda_k} \left\{ \|\mathbf{S}_k - \Phi_k \Lambda_k\|_F^2 + \lambda \|\Lambda_k\|_1 \right\}$$

where Λ_k is the representation coefficient matrix of \mathbf{S}_k over Φ_k .



II. AUTOREGRESSIVE MODELS

As described in previous section, the training data set was divided into \mathbf{K} sub-datasets \mathbf{S}_K . One autoregressive model can be trained for each \mathbf{S}_K by utilizing all the patches in it. Assume that the support of the autoregressive model is a square window, and the projected autoregressive model will estimate the middle pixel in the square with the knowledge of its neighbor pixels which are the boundary pixels of the square window. Determination of size window is very difficult. A high order of the window will result in data over-fitting. In this paper, a window size of 3x3 is used. So, the total number of pixels becomes 9, and the number of neighbors of middle pixel is 8, which is the order of the autoregressive model. The vector of autoregressive model \mathbf{ar}_K of one of the sub-dataset \mathbf{S}_K can be obtained using the solution of the least square problem given below.

$$\mathbf{ar}_K = \arg \min_a \sum_{s_i \in \mathbf{S}_K} (s_i - \mathbf{a}^T \mathbf{q}_i)^2$$

' s_i ' is the middle pixel of the image patch S_i and \mathbf{q}_i is the vector with all its neighbors. When this training process is applied to all the sub-dataset, a set of autoregressive models $\{\mathbf{ar}_1, \mathbf{ar}_2, \dots, \mathbf{ar}_K\}$. This set of autoregressive models will be used for regularization. In the previous section, selection of sub-dictionaries was presented. Similar to that, the selection of autoregressive models will be done. Using the estimate \hat{x} , the high pass filtered version \hat{x}_i^h can be calculated. Now, assume $k_i = \arg \min_k \|\Phi_c \hat{x}_i - \Phi_c \mu_k\|_2$, and the autoregressive model \mathbf{ar}_{k_i} to be assigned to the image patch \mathbf{x}_i . The vector holding the holding the neighbors of x_i is denoted by χ_i . Now, the prediction error of x_i using \mathbf{ar}_{k_i} and χ_i should be minimum. In other words the term $\|\mathbf{x}_i - \mathbf{ar}_{k_i}^T \mathbf{X} \mathbf{Y}_i\|_2^2$ need to be optimized to be smallest. Incorporating this condition on the sparse representation, the objective function will look like the following.

$$\hat{\alpha} = \arg \min_{\alpha} \left\{ \|y - DH\Phi\alpha\|_2^2 + \sum_{i=1}^N \sum_{j=1}^N \lambda_{i,j} |\alpha_{i,j}| + \gamma \sum_{x_i \in \mathbf{X}} \|x_i - \mathbf{ar}_{k_i}^T \chi_i\| \right\}$$

Here, γ balances the effect of autoregressive regulating term.

III. NON-LOCAL SIMILARITY

Adaptive regularization based on autoregressive model feats the local features in each and every image patch. In addition to the local features, many repetitive patterns are present on any natural image. These non-local redundancy is very much helpful in further improvement of quality of reconstructed images. For each local patch \mathbf{x}_i , similar patches in the image need to be identified. A patch \mathbf{X}_i^l is said to be similar patch if $e_i^l = \|\hat{\mathbf{X}}_i - \hat{\mathbf{X}}_i^l\|_2^2 \leq t$. Here t is a threshold. Assume x_i and x_i^l to be middle pixels of the patches \mathbf{X}_i and \mathbf{X}_i^l respectively.

Now use the weighted average of x_i^l , $\sum_{l=1}^L b_i^l x_i^l$ can be used to estimate x_i . Here the weight given to x_i^l is fixed using the relation $b_i^l = \frac{1}{c_i} \exp(-e_i^l/h)$. Here h is governing factor of weight and c_i is the normalizing factor given by $c_i = \sum_{l=1}^L \exp(-e_i^l/h)$. The estimate error $\left\| x_i - \sum_{l=1}^L b_i^l x_i^l \right\|_2^2$ can be rewritten as $\|x_i - \mathbf{b}_i^T \beta_i\|_2^2$, where \mathbf{b}_i is the column vector containing all the weights b_i^l and β_i is the column vector that holds all x_i^l . After incorporating the non-local similarity regularization into sparse representation, we get the following.

$$\hat{\alpha} = \arg \min_{\alpha} \left\{ \|y - DH\Phi\alpha\|_2^2 + \sum_{i=1}^N \sum_{j=1}^N \lambda_{i,j} |\alpha_{i,j}| + \eta \sum \|x_i - \mathbf{b}_i^T \beta_i\|_2^2 \right\}$$

IV. RESULTS AND DISCUSSIONS

In this work, to learn dictionaries and thereby sub-dictionaries, two sets of high quality test images were considered. From each set on dictionary and corresponding sub dictionaries are formed. The selection of test images for learning the dictionary is so critical. The test images should have all the possible patterns to represent the features that are there in input images. The effect of the selection of these high quality images is presented in this section. Two kinds of blurs are considered, uniform and Gaussian blurs. Uniform blur is created by convolving the original image with a kernel of specific order. In the experiments two sizes are considered, 3x3 and 9x9. When the dimension of the kernel is more, the effect of the blur will also be more. So, the effect of kernel of 9x9 will be more than that with a kernel of 3x3. Gaussian blur is also applied in two varieties, one with standard deviation of 1 and another with 3. The effect of Gaussian blur with the standard deviation of 3 is predominant than the other. The procedure is an iterative process, and it is iterated for 1000 times. For each 40 iterations, the peak signal to noise ratios is noted. Fig.1.

Table 1 gives the PSNR values obtained iteration wise and structural similarity (SSIM) between the input images and deblurred image obtained after 1000 iterations in uniform blur case with kernel size of 3x3. Tables 2, 3 and 4 presents the PSNR and SSIM values in uniform blur case with kernel size of 9x9. Gaussian blur case with standard deviation of 3 respectively.



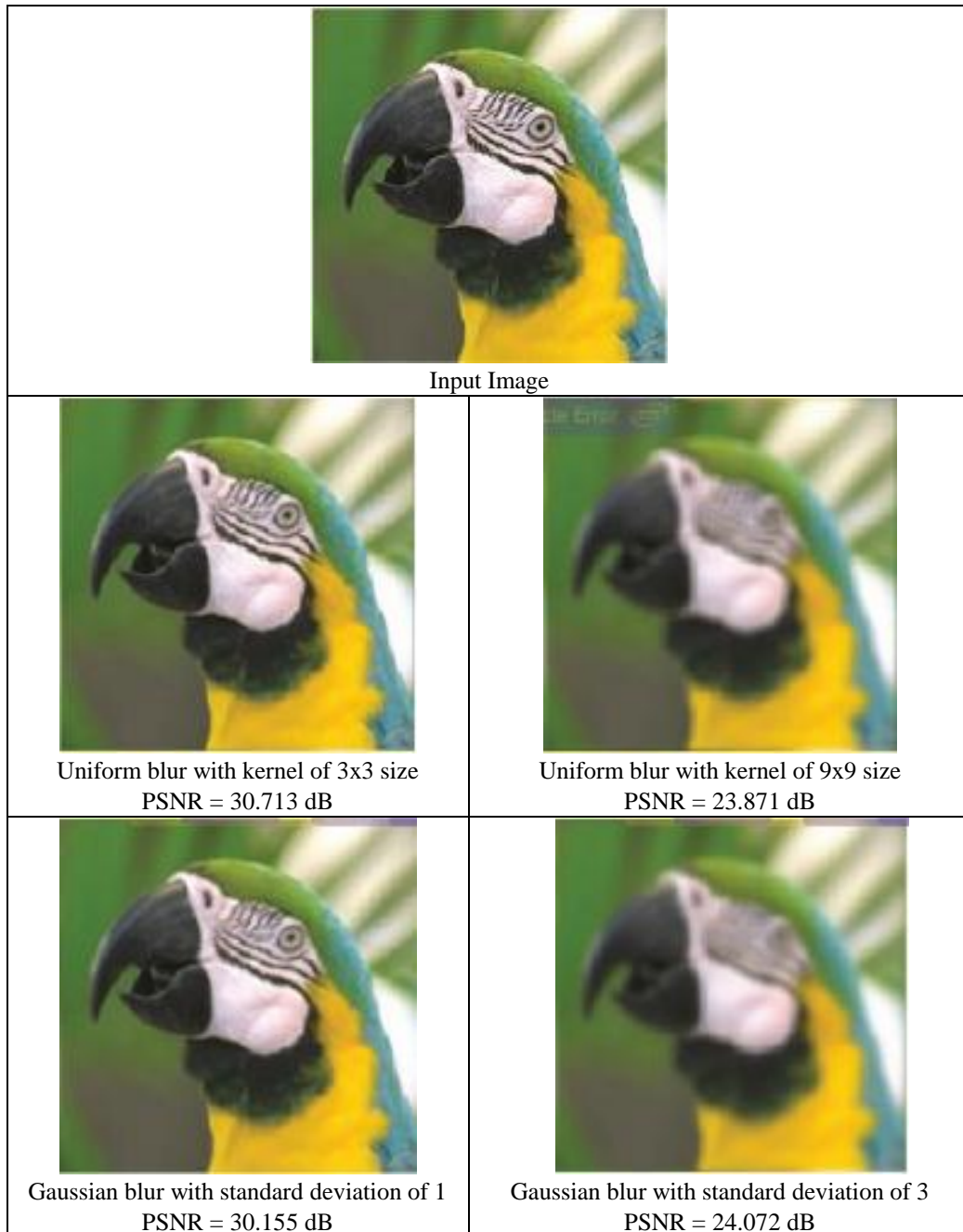


Fig.1. Input image and blurred images

Table 1. Simulation results on Uniform blur with 3x3 kernel

It	Dictionary – 1			Dictionary – 2		
	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity
0	30.714	30.714	30.714	30.714	30.714	30.714
80	35.375	35.759	36.587	35.381	35.758	36.591
160	35.117	35.690	36.663	35.132	35.697	36.672
240	35.043	35.685	36.679	35.061	35.692	36.681
320	34.999	35.748	36.631	35.021	35.743	36.621
400	34.979	35.801	36.596	34.999	35.790	36.584
480	34.966	35.808	36.595	34.984	35.794	36.584
560	34.956	35.808	36.594	34.970	35.794	36.585
640	34.950	35.797	36.571	34.962	35.789	36.567
720	34.945	35.791	36.563	34.957	35.785	36.561
800	37.089	38.053	38.606	37.096	38.024	38.603
880	37.232	38.106	38.625	37.231	38.076	38.626
960	38.036	38.568	38.787	38.037	38.540	38.778
1000	38.030	38.570	38.790	38.040	38.540	38.780
SSIM	0.965	0.967	0.968	0.965	0.967	0.968

Table 2. Simulation results on Uniform blur with kernel of 9x9

It	Dictionary - 1			Dictionary – 2		
	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity
0	23.871	23.871	23.871	23.871	23.871	23.871
80	28.625	28.611	28.620	28.619	28.604	28.614
160	29.316	29.282	29.157	29.310	29.272	29.151
240	29.839	29.767	29.444	29.835	29.757	29.438
320	30.152	30.068	29.822	30.144	30.056	29.816
400	30.361	30.304	30.330	30.353	30.293	30.317
480	30.505	30.459	30.563	30.501	30.454	30.554
560	30.607	30.562	30.685	30.605	30.565	30.688
640	30.684	30.639	30.846	30.679	30.644	30.857
720	30.739	30.695	30.985	30.733	30.701	30.994
800	31.124	31.019	31.160	31.111	31.019	31.166
880	31.260	31.144	31.241	31.236	31.137	31.247
960	31.242	31.126	31.187	31.204	31.103	31.189
1000	31.270	31.150	31.200	31.230	31.130	31.200
SSIM	0.900	0.899	0.898	0.900	0.899	0.898

Table 3. Simulation results on Gaussian blur with standard deviation of 1

It	Dictionary – 1			Dictionary – 2		
	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity
0	30.155	30.155	30.155	30.155	30.155	30.155
80	36.061	35.852	34.167	36.045	35.856	34.186
160	36.049	35.820	32.908	36.030	35.827	32.936
240	36.073	35.831	32.174	36.056	35.833	32.206
320	36.033	35.660	31.680	36.020	35.664	31.712
400	35.968	35.387	31.333	35.967	35.402	31.364
480	35.950	35.320	31.079	35.955	35.339	31.111
560	35.943	35.297	30.888	35.950	35.318	30.922
640	35.935	35.177	30.743	35.943	35.202	30.778
720	35.931	35.088	30.629	35.938	35.119	30.666
800	37.519	36.353	30.511	37.474	36.391	30.562
880	37.593	36.585	30.372	37.534	36.613	30.436
960	37.886	37.368	30.260	37.827	37.347	30.342
1000	37.890	37.390	30.200	37.830	37.370	30.290
SSIM	0.965	0.953	0.721	0.965	0.953	0.726

Table 4. Simulation results on Gaussian blur with standard deviation of 3

It	Dictionary – 1			Dictionary – 2		
	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity	ASDS	ASDS Autoregressive	ASDS Autoregressive with non-local similarity
0	24.072	24.072	24.072	24.072	24.072	24.072
80	26.840	26.824	26.834	26.838	26.821	26.834
160	27.149	27.087	27.122	27.145	27.084	27.123
240	27.351	27.233	27.305	27.341	27.234	27.310
320	27.478	27.369	27.432	27.465	27.370	27.435
400	27.581	27.516	27.529	27.566	27.517	27.532
480	27.652	27.610	27.607	27.639	27.604	27.610
560	27.702	27.676	27.671	27.691	27.667	27.674
640	27.742	27.742	27.723	27.733	27.733	27.725
720	27.773	27.802	27.768	27.766	27.792	27.765
800	27.795	27.873	27.880	27.781	27.853	27.871
880	27.825	27.913	27.930	27.801	27.882	27.917
960	27.835	27.940	27.966	27.800	27.898	27.949
1000	27.840	27.950	27.980	27.810	27.910	27.960
SSIM	0.863	0.868	0.868	0.862	0.867	0.867

The effect of autoregressive model and extracting non-local similarity is apparent when the blurring is predominant. Hence, the effect can well be observed in Gaussian blur with standard deviation of 3 and uniform blur

with kernel 9x9. This effect is shown in Fig. 2.



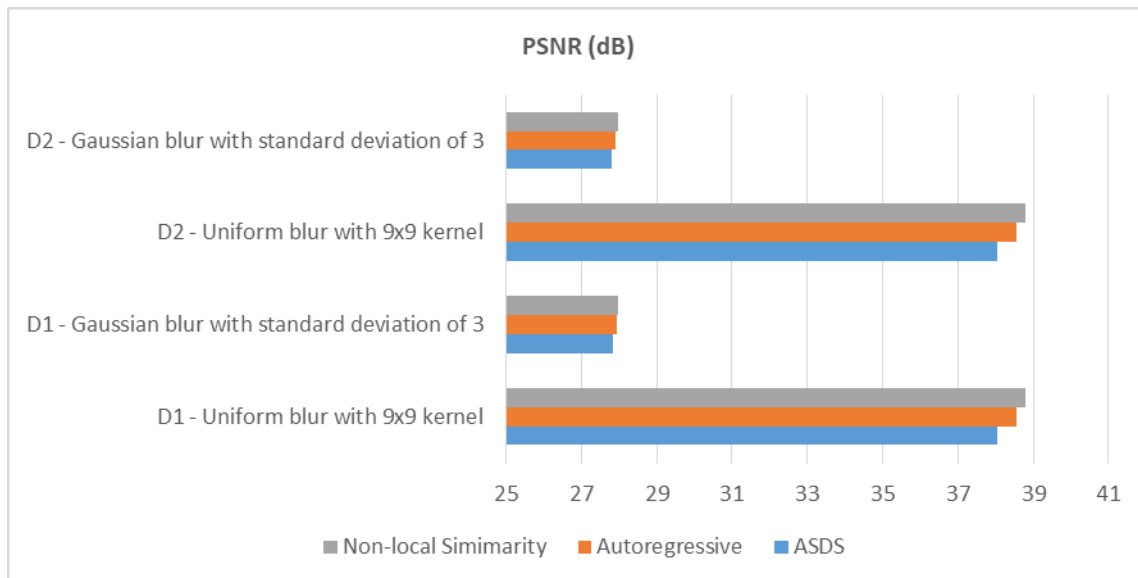


Fig. 2 PSNR values obtained with different techniques when blurring is more

V. CONCLUSIONS

In this paper, sparse representation is proposed with few improvement terms and applied on image deblurring. Uniform and Gaussian blurs are simulated and deblurring was done using sparse representation based schemes. Adaptive selection of sub-dictionaries was presented. In addition to the proposal of the said sparse representation, to characterize local image structures, autoregressive models are proposed. These autoregressive models pre-learned from training dataset. Out of the all autoregressive models learned, one or few models will be adaptively chosen to regularize the solution space. Along with the autoregressive models, a non-local self-similarity condition was also proposed. A natural image will have several repetitive image structures. These conditions help in preserving the sharp edges. The simulation results proved the superiority of the two improvements proposed, specifically when the blurring quantity is more.

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