

# Super Resolution Sub-space based ROOT-MUSIC technique for Direction Of Arrival Evaluation in MIMO Radar

N. V. S. V. Vijay Kumar, K. Raja Rajeswari, P. Rajesh Kumar

**Abstract:** *In this paper, subspace based on DOA evaluation with high resolution ROOT-Multiple Signal Classification (MUSIC) method is proposed for MIMO radar. In order to achieve a desired transmitting power distribution, the main component is the fundamental vector was made such that the transmitted power is focuses on power to be transmitted inside the required sectors eliminating the power of off-sector. Using the designed algorithm, population in associate weight-vectors is created which has almost equal size division. These associate vectors utilized in forming multiple transmitting ways, over which an orthogonal waveform is transmitted. Match filtering is done for the collected information and perpendicular transmitted waves. Many of the information vectors similar to the perpendicular waves are generated. Now, carefully taking these waves, virtual information output covariance matrix that enhances the use in super resolution direction of arrival prediction methods. This technique decomposes the eigenvectors in correlation. Signal estimation is performed by taking the maximum values in the signal, corresponding to base in the polynomial. The software output display ROOT MUSIC technique giving best DOA prediction presentation compared to existing techniques that have been used for comparison.*

**Index Terms:** *Beamforming, beam pattern design, MIMO radio detection and ranging, side lobes, Spatial filtering.*

## I. INTRODUCTION

Estimating DOA of multiple targets has uses in most wide fields of applications, like radio detection and ranging, communication and sonar [1]. Variable and non-variable techniques are the direction of arrival prediction methods, of which the non-variable direction of arrival methods are MUSIC, ESPRIT, and Capon beamformer whereas the parametric DOA estimation techniques are the likelihood methods. The parametric techniques suffer from large auxiliary lobes and less resolution levels while the non-parametric techniques give excellent DOA estimation performance with higher computation complexity for different levels. So, it is indeed beneficial to inspect advantages in multiple input and multiple output radio detection and ranging using less computing difficulty. Majority of the DOA estimation methods had expanded in

SIMO, where the presentation in estimation methods had extensively evaluated depending on the eventual belief.

These performances tend in decline when fewer numbers of data previews[10]-[12] or [10]-[13] is available in the direction of arrival prediction. Indeed the high resolution DOA estimation techniques that invert the covariance matrix are valid for a full-rank matrix.

For a past two decades extensive research [7]-[9] has been done on MIMO radio detection and ranging and has been proven that multiple input multiple output radio detection and ranging from collocated antennas enables detection of maximum targets and angular resolution. Most of the DOA estimation algorithms that do not employ proper transmit beam forming suffer from loss of coherent processing gain. Therefore, the signals with weak power are received at the receiver end array which affects the direction of arrival prediction presentation.

When transmit beam forming MIMO radar is not properly designed, it suffers from coherent sending losses which yields less powerful signal at the receiving array, that completely affects the DOA estimation performance. Here we propose a new method where we take into consideration the issue in direction of arrival prediction in various goals in multiple input multiple output radio detection and ranging utilizing the received snapshot data. Thus, this technique was introduced for individual DOA estimation calculation where transmit beam-forming is used to accomplish coherent processing gain. The weight-vectors of the transmitters were perfectly organized to such a degree, to the point that the perfect transmit control scattering configuration concentrates the power within the spatial region while minimizing the power outside the spatial region(s). We refer to this weight-vectors as "principal" weight-vector. These principle weight-vectors deliver a set of weight-vectors with similar dimensions along with equal transfer control like the principle weight-vector. Some of these huge transmitters are picked at the same time to transfer a great deal of symmetrical wave-forms, where the amount of symmetrical wave-forms equals the amount of weight-vectors. The received single snapshot data is matched-filtered with the symmetrical transmit wave-forms for getting practical information equal to the symmetrical wave-forms.

**Revised Manuscript Received on July 05, 2019.**

**N. V. S. V. Vijay Kumar**, Electronics & Communication Engineering, GITAM (Deemed to be University), Visakhapatnam, India.

**K. Raja Rajeswari**, Electronics & Communication Engineering, G.V.P College of Engineering for Women, Visakhapatnam, India.

**P. Rajesh Kumar**, Electronics & Communication Engineering, Andhra University College of Engineering, Visakhapatnam, India.



The above process involves constructing a covariance matrix with full-rank where the amount of symmetrical wave-forms is carefully selected which allows the use of huge-objective methods for DOA calculation. Proliferation results show the efficiency of DOA estimation system.

## II. SYSTEM MODEL

Consider a multiple input multiple output radar having  $M$  transmit antennas along with  $N$  receiving antennas. All these antennas were sorted in the form of uniform linear array(ULA) which are closely located from each other. Let  $\mathbf{v}$  be the  $M \times 1$  principal transmit beamforming vector that concentrates on the transmit power inside the spatial sector  $\Theta = [\phi_{\min}, \phi_{\max}]$  simultaneously reducing the power emitted outside the spatial sector. The transmit beamforming vector  $\mathbf{v}$  can be studied by determining the improvement equation

$$\begin{aligned} \min_w \max_i & \left| \mathbf{v}^H s(\phi_i) - e^{j\Phi(\phi_i)} \right|, \phi_i \in \Theta, i = 1, \dots, l \\ \text{subject to} & \left| \mathbf{v}^H s(\phi_k) \right| \leq \delta, \phi_k \in k = 1, \dots, K \end{aligned} \quad (1)$$

here  $s(\phi)$  denotes transmit steering vector,  $\Phi(\phi)$  is the arbitrary phase profile,  $\phi_i$  and  $\phi_k$  are the angles selected homogenously or non-homogenously to inexact  $\Theta$  and  $\tilde{\Theta}$ ,  $(\cdot)^H$  denotes the Hermitian operator and  $\delta$  was an persons-specific non-negative integer which controls the side-lobe intensity. Interior point methods [17] can be used for solving the optimization problem (1).

Let  $v_q, q = 1, \dots, Q$ , is the bunch of associated heap transmitters each having magnitude of  $M \times 1$ , that are created from main weight-vector  $\mathbf{v}$  [15]. Every single associated heap transmitter are assumed to be normalized to get unit norm which is having similar comparable communicate control scattering structure as the fundamental weight-vector, i.e.,

$$\left| \mathbf{v}^H s(\phi) \right|^2 = \left| v_q^H s(\phi) \right|^2, \phi \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \quad (2)$$

where  $\mathbf{z}$  is the  $Q \times 1$  presumed set of unrelated undulations,  $\mathbf{z}$  is rapid instance, &  $(\cdot)^T$  denotes transpose.

The associate heavy resultant form communicates for transmitting the majority of the symmetrical outputs. Along these lines, the message signal delineation of transfer hail sequence  $f(t)$  may be conferred as a mix of symmetrical signal, i.e.,

$$f(t) = \eta V w(t) \quad (3)$$

where  $V = [v_1^*, \dots, v_Q^*]$  is the  $M \times Q$  transmit beam forming matrix,  $\eta = \sqrt{M/Q}$  is the normalization factor which ensures entire transmitted control is constant  $P_t = M$ , &  $(\cdot)^*$  refers complex conjugate. The vector  $f(t)$  denotes input data which is utilized at transmitter of MIMO waveguide environment. Nevertheless,

from a destination end point of view, the wave design prototype (3) performs pulse compression which uses the semaphore matrix  $w(t)$  that doesn't necessarily be the unchangeable absolute value. That gives extra grades of Free State while generating the orthogonal outputs. The  $N \times 1$  wave matrix which is produced at the output side which is in the direction  $\phi$  can be modeled as

$$\begin{aligned} x(t) &= \eta a_\phi a(\phi) s^T(\phi) f(t) + n(t) \\ &= \eta a_\phi a(\phi) s^T(\phi) V w(t) + n(t) \end{aligned} \quad (4)$$

where  $a_\phi$  is the guided mirror coefficient that defines the Swirling II target\_model,  $a(\phi)$  denotes  $N \times 1$  output matrix,  $I_N$  denotes the  $N \times N$  identity matrix and  $n(t)$  denotes the  $N \times 1$  AWGN-Vector with zero sum & covariance  $\sigma_z^2 I_N$ .

## III. HIGH-RESOLUTION DOA ESTIMATION USING ROOT-MUSIC ALGORITHM

The technique is a model based parameter estimation technique where the steering vector is modeled such that the parameter  $\phi$  is calculated depending upon the data obtained at output and model. Considering the signals to be uncorrelated, the correlation matrix  $M$  can be estimated depending on the output data, where the estimate is average of the signal subspace. The signal correlation matrix is an  $M \times M$  order matrix that contains  $K$  number of incoming signals which can be written as

$$Q R Q^H + \sigma_z^2 I_N \quad (5)$$

where  $Q$  is the signal steering vector matrix,  $R$  is the signal power matrix and  $\sigma_z^2$  variance of the noise vector matrix. The above equation confirms the orthogonality between the eigenvectors that make the noise, i.e.,  $\mathbf{U}_n$  and the eigenvectors that make the signal vector. From the above set of equations, using the property of orthogonality of vectors, we can derive the angle of the incident signals from the signal vectors. We consider this product to be a polynomial in  $z$ . Therefore, the signal directions are given by the roots of the polynomial in  $z$ .

To evaluate the roots we construct a polynomial such that

$$P(z) = m_i^H Q R Q^H m_i \quad (6)$$

The roots of the above polynomial give the angle of the incoming signals. Practically, these roots are positioned on unit circle but, due to the presence of noise, we consider those which are nearer to that unit circle are the one that determine the orientation of input signal. Out of  $M-1$  roots that lie within the unit circle, select  $N$  roots which are nearer to that unit circle,  $(z_j, j = 1, \dots, N)$ .

Orientation of accomplishing angle for each of the roots is found using



$$\phi_j = \sin^{-1} \left[ \frac{\lambda}{2\pi d} \arg(z_j) \right] \quad (7)$$

Expecting  $M$  targets correspond to be particular range, matched-filtering the information (4) to all of orthogonal delivered wave-forms gives the virtual data snapshots

$$z_q = \tilde{\eta} \sum_{a=1}^L \alpha_l (V_q^H S(\phi_l)) a(\phi_l) + \eta_q, q = 1, \dots, Q \quad (8)$$

where,  $\alpha_l$  is the reflection coefficient related to the  $l^{\text{th}}$  target,  $\eta_q$  is noise vector at the yield of the matched filter which has the same statistics as that of  $\mathbf{n}(t)$ , and  $\tau_p$  is the radar pulse duration. After performing eigen-decomposition of (6), that can produce the  $N \times L$  matrix  $Y_s$  &  $N \times (N-L)$  matrix  $Y_n$ , that relate to the signal subspace & noise subspace, respectively. Root music spectrum can be calculated as:

$$S(\theta) = \frac{a^H(\phi) a(\phi)}{a^H(\phi) Y_n Y_n^H a(\phi)} \quad (9)$$

The Root-Music algorithm deals with the phase of the roots, so even at very low SNR values Root Music provides extraordinary performance results compared to Music and the traditional methods.

#### IV. SIMULATION RESULTS

The results have been simulated by choosing a MIMO radar which transmit ULA of  $M=14$  antennas & receive sequence of  $N=14$  antennas placed 1/2 a distance between two antennas arrays. The noise is expected as additive white Gaussian noise with zero mean. We assume targets are placed in the designed free space region. Attenuation for the side lobes is maintained at 20dB. The principal weight-vector is obtained by performing the calculations on the optimization problem (1), using which we can create 4095 associated weight-vectors. Some array of  $Q=12$  weight-vectors are stochastically chosen among the total strength of 4096 weight-vectors. These weight-vectors are helpful for the advanced single-snapshot DOA estimation technique. The values taken for existing methods are  $K=3$  principal weight-vectors for spheroidal sequences, the principal weight-vector is utilized at source part for conventional beamformer.

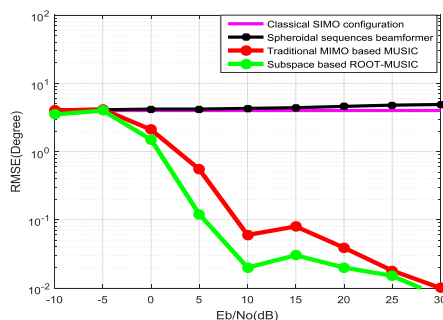


Fig. 1. Comparing ROOT-MUSIC algorithm with existed non-parametric methods among RMSE and SNR.

The simulated results depicted in Fig. 1 show the root mean square error (RMSE) with the SNR for all the entire methods. Simulation outputs averaged on 500 iterations are considered in each method. It is observed that the two, traditional & ellipsoidal beamformers show very low potentials at low & high SNR outputs due to the low resolution. The graph depicts the multiple input multiple output built MUSIC method has better root mean square error presentation in large signal to noise ratio parameters. Our subspace based ROOT-MUSIC technique had best representation when related to the remaining techniques. Fig. 2 depicts probability in source resolution vs signal to noise ratio. Target resolution is achieved if a minimum of two peaks were seen and also if state of it is contented [1]

$$\left| \tilde{\phi}_i - \phi_i \right| \leq \frac{|\phi_2 - \phi_1|}{2}, l = 1, 2$$

(10)

Because of less resolution capabilities in conventional beamformer and the ellipsoidal shaped sequence built techniques, it can be observed from Fig.2 that these two methods do not achieve optimum probability in target resolution presentation at large signal to noise ratio parameters. Proposed advanced single snapshot MUSIC and conventional multiple input and multiple output MUSIC get maximum target resolution in signal to noise ratio parameters greater than 4 dB and 15 dB. For relatively less signal to noise ratio parameters, the following advanced single-snapshot MUSIC technique gives good resolution abilities when

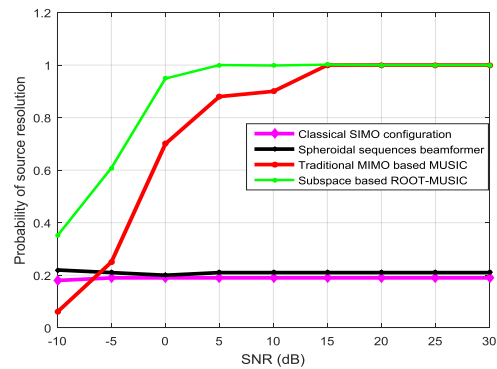


Fig. 2. Probability of source resolution vs SNR for ROOT-MUSIC algorithm

contrast to the conventional multiple input multiple output MUSIC. The ROOT-MUSIC algorithm resolution is very high at the low SNR compared to existing techniques for example at SNR= -10dB the resolution is around 0.37 and for remaining techniques the SNR is below 0.1. For High SNR the resolution performance is far better than remaining non-parametric techniques. Result separation utilized for this instance is  $\Delta\phi = \phi_2 - \phi_1 = 12^\circ$  which was lesser compared to Rayleigh resolution, resulting in failure to resolve targets by standard and ellipsoidal shaped methods beamformers at all the signal to noise ratio parameters.



## V. CONCLUSION

The issues in Direction of Arrival prediction in various targets in multiple input multiple output radar utilizing subspace based method had been addressed. Transmit coherent gain had been gotten with proper calculation of the normalized principal weight-vector in addition to limiting the power emanated with out-of-the spatial sector. The designed weight-vector had been utilized in order to create a set of weight-vectors with equivalent transmitting control like principal weight-vector. The associate weight-vectors empower passing numerous orthogonal waveforms giving various virtual data snapshots information equivalent to the quantity of the orthogonal waveforms, in this manner allowing utilization of large-goals for Direction of arrival prediction methods. Now the proposed strategy appeared to over perform the traditional multiple input multiple output radar, the traditional beamformer, and the circular shaped sequences based beamformer. The ROOT-MUSIC algorithm gives best performance for high resolution subspace at low and high SNR. The DOA estimation and eigen value decomposition is accurately developed in the ROOT MUSIC algorithm such that the resolution at different SNR's are high and MSE also reduced.

## REFERENCES

1. A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars," *IEEE Trans. Signal Processing*, vol. 58, no. 6, pp. 3137–3151, June 2010
2. A. Hassanien and S. A. Vorobyov, "Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas," *IEEE Trans. Signal Processing*, vol. 59, no. 6, pp. 2669–2682, June 2011.
3. A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Capon-based single snapshot DOA estimation in monostatic MIMO Radar," in *Proc Symposium SPIE Sensing Technology + Applications*, Baltimore, MD, Apr. 2015.
4. A. Khabbazbasmenj, A. Hassanien, S. Vorobyov, and M. Morrency, "Efficient transmit beamspace design for search-free based DOA estimation in MIMO radar," *IEEE Trans. Signal Processing*, vol. 62, no. 6, pp. 1490–1500, Mar. 2014.
5. A. Khabbazbasmenj, A. Hassanien, and S. A. Vorobyov, "How many beamforming vectors generate the same beampattern?," *IEEE Signal Processing Lett.*, submitted, arXiv preprint, arXiv:1402.1682, 2014.
6. D. Nion and N. D. Sidiropoulos, "Tensor algebra and multidimensional harmonic retrieval in signal processing for MIMO radar," *IEEE Trans Signal Processing*, vol. 58, no. 11, pp. 5693–5705, Nov. 2010.
7. G. Hua and S. S. Abeysekera, "MIMO radar transmit beampattern design with ripple and transition band control," *IEEE Trans. Signal Processing*, vol. 61, no. 11, pp. 2963–2974, Jun. 2013.
8. H. Krim and M. Viberg, "Two decades of array signal processing research: the parametric approach," *IEEE Signal Processing Mag.*, vol. 13, no. 4, pp. 67–94, Aug. 1996.
9. H. L. Van Trees, *Optimum Array Processing*. Wiley, NY, 2002.
10. J. Li and P. Stoica, *MIMO Radar Signal Processing*. New Jersey: Wiley 2009.
11. J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Processing Mag.*, vol. 24, pp. 106–114, Sept. 2007.
12. L. Xu, J. Li, and P. Stoica, "Target detection and parameter estimation for MIMO radar systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 3, pp. 927–939, July 2008
13. P. H'acker and B. Yang, "Single snapshot DOA estimation," *Adv. Radio Sci.* 8, no. 16, pp. 251–256, 2010.
14. P. Heidenreich and A. M. Zoubir, "Computationally simple DOA estimation of two resolved targets with a single snapshot," in *Proc. 37<sup>th</sup> IEEE Int. Conf. Acoustics, Speech and Signal Processing (ICASSP'12)*, Kyoto, Japan, Mach 2012, pp. 2553–2556.
15. S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2009.
16. S. Fortunati, R. Grasso, F. Gini, and M. S. Greco, "Single snapshot DOA estimation using compressed sensing," in *Proc. 39<sup>th</sup> IEEE Int. Conf.*

*Acoustics, Speech and Signal Processing (ICASSP'14)*, Florence, Italy, May 2014, pp. 2297–2301.

17. Y. D. Zhang, M. G. Amin, and B. Himed, "Joint DOD/DOA estimation in MIMO radar exploiting time-frequency signal representations," *EURASIP Journal on Advances in Signal Processing*, vol. 2012, no. 1, July 2012.
- 18.