M^X/G/1 Vacation Queueing System with Two Types of Repair facilities and Server Timeout

V.N.Rama Devi,Y.Saritha, K.Chandan

Abstract We consider a single server vacation queue with two types of repair facilities and server timeout. Here customers are in compound Poisson arrivals with general service time and the lifetime of the server follows an exponential distribution. The server find if the system is empty, then he will wait until the time 'c'. At this time if no one customer arrives into the system, then the server takes vacation otherwise the server commence the service to the arrived customers exhaustively. If the system had broken down immediately, it is sent for repair. Here server failure can be rectified in two case types of repair facilities, case1, as failure happens during customer being served willstays in service facility with a probability of 1-q to complete the remaining service and in case2 it opts for new service also who joins in the head of the queue with probability q. Obtained an expression for the expected system length for different batch size distribution and also numerical results are shown.

Keywords: Expected system length, Server timeout, Two types of repair facilities and. Vacation queue.

I. INTRODUCTION

The concept of Server timeout plays a vital role in a single service system withvacations. Number of problems is modeled by a vacation with queueing system. Queueing theory is used in Inventory control, loading and unloading of ships, machine interference problems, machine service and repair model Vacation queue means the server completely sits idle and were first discussed by Levy and Yechiali [6] introduced and did the utilization of idle time in an M/G/1 queueing system followed by several excellent surveys by B.T. Doshi [1], Later on Jau-chauank, Chian-Hangwu and Zhe George Zhang [3] did on multiple vacation models in queueing theoryand so on. When there is no customers in the system itbecomes empty, the server will wait until time 'c' known as server timeout. Oliver C.Ibe [7], [8] derived an expression to the expected waiting time of a vacation queue in which server couldn't take frequently second vacation upon returning from a first vacation and if he finds system with zero customer or wait indefinitely for a customer to arrive and it also extended for N-policy. E.Ramesh Kumar and Y. Praby Loit [2] obtainedand derived an expression for the mean waiting time from the length of the system with single and n-policy ofvacation queueing system. Y.Saritha, K.Satish KumarandK.Chandan [12] derived the expected system length using different bulk size distributions for M^X/G/1 vacation Queueing system with server timeout. Single Queue subjected to breakdown and repair has been studied by number of statistician. S. Bama M.I.Afthab Begum and P. Fijy Jose[10] analyzed and derived M^X/G/1 queue in which random breakdowns occur with thePoisson process.

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Kailash C. Madan [4] derived an expressionfor mean number of customers in the system for M/G/1 type queue with time homogeneous breakdown and deterministic repair. Kuo-Hsiung Wang, Dong-Yuh-Yang and W.L. Pearn [5] invented the optimal (T, P)-Policy and the optimal (p, N)-Policy M/G/1 queue with SOS, server breakdown and general startup times. S.Pazhani Bala Murugan and K.Santhi [9] studied a non-Markovian queue with Multiple Working vacations and random breakdowns. Tao Li and Liyuan Zhang [11] analyzed M/G/1 retrial G-queue with general retrial times. Y.Saritha, K.Satish Kumar, V.N.RamaDeviandK.Chandan derived the expected system length for M/G/1 Vacation Queueing System with breakdown, repair and Server Timeout [13]. The objective to this paper is to derive the expected system length for M^X/G/1 vacation queueing system with two types of repair facilities and server timeout. Numerical solutions are illustrated for various parameters by applying different batch size distributions to expected system length.

II. MODEL DESCRIPTION

Here customers are assumed to arrivein batches undercompound Poisson process with rate b. let B(Z) be the batch size distribution. The service times of a single server follow general distribution function. The server find if the system is empty, then he will wait until the time 'c' is known as server timeout.. At this time if no one customer arrives into the system, then the server takes vacation otherwise the server commence the service to the arrived customer. At the expiration of the vacation time if no customer arrive the server goes for another vacation, otherwise hecommence the service to the arrived customer exhaustively; Service may be interrupted due to Server breakdowns. Here the server is unable to work unless the machine gets repaired, therefore the server should undergo for repair process. Here breakdown server is facilitated with two case types of repair facilities, case1, as failure happens during customer being served will stays in service facility with a probability of 1-q to complete the remaining service and in case2 it opts for new service also who joins in the head of the queue with probability q. When the repair work is completed the server immediately returns to service system and executes the service for the waiting customer as well as arrived customer in a queue.

III. ANALYSINGTHE MODEL

The customer arrives to the system and occurs according to a compound Poisson process with arrival rate'b', and service times are exponential with mean rate µ.

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The time, X, is to serve a customer and has a general distribution with cumulative distribution function CDF $F_X(x)$, with mean E(x) and its second moment E(X^2). The Laplace stieltjes transform function of X is $B_X(s)$, which is defined by

$$\underset{X}{B}(s) = E(e^{-SX}) = \int_0^\infty e^{-Sx} \underset{X}{f}(X)dx \quad (1)$$

The duration of a vacation is symbolized as V and have to follow general distribution with CDF $F_V(v)$ and Laplace stieltjes transform function of V is $B_V(s)$. The average of V is E (V) and its second moment E (V^2).

$$\underset{V}{B}(S) = \frac{\gamma}{S+\gamma} \quad (2)$$

Here X and V are mutually independent.
Similarly if the system has breakdown with rate **α**, then system go for two types of repair facilities based on customer 's choice. After repair , the server start the service to the customer. Let $G_{L\ (M/G/1)}$ (z) is PGF the number in system is given by [Ref 10]

$$g_{L(M^X/G/1)}(Z) = \frac{(1-\rho)h_\alpha k_1 S^* k_2(z-1)}{h_\alpha k_1(z-S^*k_2) - z\alpha q R^* k_1(1-S^*k_2)} \quad (3)$$

Where $k_1 = \lambda - \lambda x(z)$ and $k_2 = \alpha + \lambda - \lambda x(z)$

Let A indicates number of customers in the system at the beginner of busy period. The PMF of A is P_A (a) =P [A=a], whose z-transformation is G_A (z) is given by

$$\underset{A}{g}(Z) = E(Z^A) = \sum_{a=1}^{\infty} Z^a p_A(a) \quad (4)$$

The mean of A is E (A) and its second moment is E (A^2). Let the random variable B denotes the number of customers left the system by an arbitrary departing customer. The PMF of B is P_B (b), whose z-transform is given by G_B (z).

$$g_B(Z) = \frac{1-g_A Z}{(1-Z)E(A)} \quad (5)$$

Its meanis E (B) and its second moment is E (B^2).

Assume that L be the number of customers in the system.The Z-transform of the pmf of L i.e.; $P_L(l)$ is given by g_L (z)

$$\underset{L}{g}(Z) = \underset{B}{g}(Z) + g_{L(M^X/G/1)}(Z) \quad (6)$$

Its mean is E (L) and its second moment is E (L^2).

Special Batch Size Distributions:
As the batch size'b' is a random variable, it has a probability distribution. In particular Deterministic, Geometric and Positive Poisson are considered.

1.) If we take Deterministic, then the generating function is

$$B(z) = z^b \quad (7)$$

This gives mean $B'(z) = z^b$ and second moment $\bar{\bar{B}} = B''(z) = b^2 - b$, where b is the average batch size.

2.) If we take Positive Poisson, then the generating function becomes

$$B(z) = \frac{ne^{-\alpha}(e^{\alpha z}-1)}{\alpha}, \text{ where } n = \frac{\alpha}{1-e^{-\alpha}} \quad (8)$$

This gives mean $\bar{B} = B'(Z) = n$ and second moment $\bar{\bar{B}} = B''Z=\alpha 21-e-\alpha$, where n is the average batch size.

3.) If we take Geometric, then the generating function becomes

$$B(z) = p[z^{-1} - (1-p)]^{-1} \quad (9)$$

This gives mean $\bar{B} = B'(Z) = \frac{1}{\rho}$ and second moment $\bar{\bar{B}} = B''(Z) = \frac{2(1-\rho)}{\rho^2}$, where $\frac{1}{\rho}$ is the average batch size.

Let W_q denotes the waiting time in the system, to determine expected system length. Thus applying little's law we obtain the expected system length of the customer is [Ref 12]

$$E(W_q) = \frac{\mathbf{1}}{\lambda}\frac{\boldsymbol{dG_L(Z)}}{\boldsymbol{dZ}}\bigg|_{\boldsymbol{Z=1}} - E(X) \quad (10)$$

$$\lambda E(W_q) = \frac{dG_L(Z)}{dZ}\bigg|_{Z=1} - E(X)$$

$$\lambda\left(E(W_q) + E(X)\right) = \frac{dG_L(Z)}{dZ}\bigg|_{Z=1} = E(L) \quad (10(a))$$

By using g_A(z), we can derive E (L). To get the Expressions for g_A (z) the Laplace –Stieltjes transform of Expected system length. [Ref 12]

$$g_A(Z) = ZP_2 + \frac{M_V(\lambda - \lambda Z) - M_V(\lambda)}{\mathbf{1} - M_V(\lambda)} P_3 \quad (11)$$

Where $P_2 = \frac{M_V(\lambda)\{\mathbf{1}-\boldsymbol{e}^{-\lambda c}\}}{(\mathbf{1}-\boldsymbol{e}^{-\lambda c})M_V(\lambda)}$ and $P_3 = \frac{\{\mathbf{1}-M_V(\lambda)\}}{(\mathbf{1}-\boldsymbol{e}^{-\lambda c})M_V(\lambda)}$

From these, we get

$$g'_A(1) = E(A) = P_2 + \frac{\lambda D'(1)E(V)}{1 - M_V(\lambda)} P_3$$

$= \frac{M_V(\lambda)(1-e^{-\lambda c})+\lambda D''(1)E(V)}{1-e^{-\lambda c}M_V(\lambda)}$ (12)

and

$$g''_A(1) = E(A^2) = \frac{\lambda D''(1)E(V) + \lambda^2 D'(1)^2 E(V^2)}{1 - M_V(\lambda)} P_3 = \frac{\lambda D''(1)E(V) + \lambda^2 D'(1)^2 E(V^2)}{1 - e^{-\lambda c} M_V(\lambda)}$$

(13)

By assuming the equation (2), we can define equation (12) and (13) by using $B_V(S) = \frac{\gamma}{S+\gamma} = B_V(\lambda) = \frac{\gamma}{\lambda+\gamma}$ and obtain the following equations as below $E(A) = \frac{\gamma^2(1-e^{-\lambda c})-\lambda(\lambda+\gamma)}{((\lambda+\gamma)-e^{-\lambda c}\gamma)\gamma}$ (14)

$E(A^2) = \frac{2\lambda^2(\lambda+\gamma)}{((\lambda+\gamma)-e^{-\lambda c}\gamma)\gamma^2}$ (15)

By substituting equation (3), (5) in equation (6), we get

$$g_L(Z) = \frac{1 - g_A Z}{(1-Z)E(A)} + \frac{(1-\rho)h_\alpha k_1 S^* k_2 (z-1)}{h_\alpha k_1 (z - S^* k_2) - z\alpha q R^* k_1 (1 - S^* k_2)}$$

(16)

$g_L(Z) = \frac{1-g_A Z}{(1-Z)E(A)} + \frac{(1-\rho)h_\alpha k_1 S^* k_2 (1-z)}{z\alpha q R^* k_1(1-S^*k_2) - h_\alpha k_1(z-S^*k_2)}$(17)

$g_L(Z) = k * \frac{(1-g_A(Z))\, h_\alpha k_1 S^* k_2}{z\alpha q R^* k_1(1-S^*k_2) - h_\alpha k_1(z-S^*k_2)}$ (18)

Consider
$k = \frac{1-\rho}{E(A)}$ (19) By differentiating the above equation (18) w.r.to z, we get

$g'_L(z)\{z\alpha q R^* k_1(1 - S^* k_2) - h_\alpha k_1(z - S^* k_2)\} + g_L(Z)\{\alpha q R^* k_1(1 - S^* k_2)(1 - h_\alpha)\} =$ $K*\{(-g'A(Z))\ h\alpha\ k1S*k2+1-gA(Z))\ h\alpha\ k1S*k2\}$
(20)

Now substituting K, K_1, K_2, z=1 also equation (14) and (15) in equation (20), we get

$$g_L(1) = \frac{\{(E(A))\, \alpha q R^* h_\alpha k_1 S^* k_2}{\alpha q R^*\, h_\alpha S^* k_2 E(A)}$$

$g_L(1) = 1$ (21)

Again differentiating the above equation (20) w.r.to z, we get

$$g''_L(z)\{z\alpha q R^* k_1(1 - S^* k_2) - h_\alpha k_1(z - S^* k_2)\} + 2g'_L(z)\{\alpha q R^* k_1(1 - S^* k_2)(1 - h_\alpha)\} + \boldsymbol{g_L}(1) = K * \{(-g''_A(Z))\, h_\alpha k_1 S^* k_2 + (-g'_A(Z))\, h_\alpha k_1 S^* k_2\}$$

(22)

Again substituting K, K_1, K_2, z=1 also equation (14) and (15) in equation (22), we get

$$g'_L(1) = E(L) = \frac{(1-\rho)\mu[r\{\lambda - (\lambda + r)e^{-\lambda c}\}]}{\alpha\{\alpha^2(\mu-1)\}[r^2(1-e^{-\lambda c}) + \lambda(\lambda+r)]} + \frac{(1-\rho)\mu[\mu+\alpha)[r\{\lambda - (\lambda + r)e^{-\lambda c}\}]}{\{\alpha^2(\mu-1)\}[r^2(1-e^{-\lambda c}) + \lambda(\lambda+r)]} + \frac{(1-\rho)\alpha q[\mu+\alpha)(2\mu+\alpha)}{\{\alpha^2(\mu-1)\}^2} + \frac{(1-\rho)\alpha q[\mu+\alpha)(2\mu^2+\alpha q)}{\{\alpha^2(\mu-1)\}^2} + \frac{(1-\rho)\mu[\mu+\alpha)(2\mu+\alpha)}{\{\alpha^2(\mu-1)\}^2} + \frac{(1-\rho)\mu\alpha[\mu+\alpha)(2\mu+\alpha)}{\{\alpha^2(\mu-1)\}^2} \quad (23)$$

Thus by above expression, we obtain expected system length.

Particular case Here (in equation (23)) the resulting expression is the special case for the M/G/1 model.

IV. NUMERICAL RESULTS

Thus by using equation (23) and varying different parameters, we get some numerical illustration in Table 1 is given below:

Table1: Effect of Different Variables (b, λ, μ, c, α, and γ) on expected system length for fixed values of b=2, λ=2, μ=50, c=1, α=0.2 and γ =0.25.

S. No	Parameters	Deterministic distribution of E(L)	Positive Poisson distributionof E(L)	Geometric distribution of E(L)
1	Varying batch size b			
	2	8.551	9.86	10.05
	3	11.701	10.85	12.767
	4	16.267	11.29	17.208
	5	21.466	11.844	22.48156
	6	27.542	24.85	8.7996
2	Varying arrival rate λ			
	2	8.5515	9.8621	10.05123
	6	8.8980	10.793	10.32097
	10	10.441	11.749	15.24963
	14	11.464	13.273	18.60797
	18	13.061	15.660	25.66514
3	Varying service rate μ			
	50	8.5515	9.8621	10.05123
	60	8.5345	9.8365	9.017707
	70	8.5205	9.8155	9.99023
	80	8.5088	9.7979	9.96731
	90	8.4989	9.7830	9.94790
4	Varying break down rate α			
	0.2	8.5515	9.8621	10.05123
	0.4	8.5575	9.8683	10.05738

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	0.6	8.5589	9.8691	10.0582
	0.8	8.5585	9.8692	10.0583
	1	8.5585	9.8692	10.0583
5	Varying repair rate γ			
	.25	8.5515	9.8621	10.05123
	.75	4.3933	5.6704	6.04786
	1.5	2.2247	3.4776	4.66395
	2	1 .9713	2.2177	1.3564
	2.5	0.8404	1.0865	1.19327
6	Varying server timeout c			
	1	8.5515	9.8621	10.0512
	2	5.3521	6.5706	7.5431
	3	3.2475	4.6325	5.5963
	4	1.4329	3.2212	2.1456
	5	0.7456	1.0214	1.1963

From the above result
As b, λ, and α were increasing then expected system length E (L) is also increasing.
As µ, γ and c are increasing then expected system length E (L) is decreasing.

V. CONCLUSION

The model is derived with an expression of expected system length for M^X/G/1 vacation queueing system with two types of repair facilities and server timeout. Numerical results are given for different varying parametersfor the given batch size distribution that shows the impact of the timeout period on the system length.

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