International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075, Volume-8 Issue-9, July 2019

Gutman and Degree Monophonic Index of Graphs

V. Kaladevi, G. Kavitha

***Abstract*: *A graph with p points and q edges is denoted by G(p,q). An* edge joining two non-adjacent points of a path P is called a chord of a path P. A path P is called monophonic if it is a chordless path. For any two points *u* and *v* in a connected graph G, the monophonic distance $d_m(u,v)$ from *u* to *v* is defined as the length of a longest *u-v* monophonic path in G. The Gutman monophonic index of a graph G is denoted by *GutMP(G)* and defined by $GutMP(G)=\sum d(u)d(v)d_m(u,v)$ and degree monophonic index of G is denoted by *DMP(G)* and defined by $DMP(G)=\sum[d(u)+d(v)]d_m(u,v)$. The methodology executed in this research paper is to determine the monophonic distance matrix of graphs under consideration. The entries of monophonic distance matrix are calculated by counting the number of edges in the *u-v* monophonic path. In this paper for some standard graphs, *GutMP(G)* and *DMP(G)* are studied which can be applied to derive quantitative structure- property or structure- activity relationships (QSPR / QSAR).**

Index Terms*: *Degree Monophonic Index, Gutman Monophonic Index, Monophonic Index, Monophonic Path.

I. INTRODUCTION

For any two points or vertices *u, v* in a connected graph G, the distance *d(u,v)* is the length of a shortest *u-v* path in G. The length of any longest path between any two vertices is the detour distance and it is denoted by *D(u,v)*. The number of vertices of G, adjacent to a given vertex *v*, is the degree of this vertex and denoted by *d(v)*. The wiener index [11] and the detour index [8,9,12] of a graph are respectively defined by $W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d(u,v)$ and $D(G)=\frac{1}{2}\sum_{u,v\in V(G)} D(u,v)$. A variant of Wiener index and Detour index called Gutman index and Detour Gutman index respectively are also studied. The detour Gutman index and detour degree distance index of pseudo regular graphs are discussed [3,10]. Gutman index is a kind of vertex-valancy weighted sum of the distance between all pair of vertices in a graph. The Gutman index with respect to the distance *d(u,v)* is defined by $Gut(G)=\sum_{u,v\in V} d(u)d(v)d(u,v)$ and $DD(G)=\sum[d(u)+d(v)]d(u,v)$ is called degree distance index of G. An edge joining two non-adjacent points of a path P is called a chord of a path P. A path P is called monophonic if it is a chordless path. For any two points *u* and *v* in a connected graph G, the monophonic distance $d_m(u,v)$ from *u* to *v* is defined as the length of a longest *u-v* monophonic path [1,2,4-7] in G. In this present paper, the Gutman index and degree distance index with respect to monophonic distance are defined and studied for some standard graphs like cycle graph and wheel graph.

II. MATERIALS AND METHODS

In this section the basic definitions and theorems, which are needed for the subsequent sections are collected.

Definition: 2.1 A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G, called edges. The vertex set and the edge set of G are denoted by V(G) or simply V and E(G) or simply E respectively. The number of vertices in G, denoted by *p*, is called the order of G, while the number of edges in G, denoted by *q* is called the size of G. A graph of order *p* and size *q* is called a *(p,q)* – graph. If $u,v\in V$ then e=uv an edge of G. We say that *u* and *v* are adjacent vertices. If two vertices are not joined, then we say that they are non-adjacent. If two distinct edges *e* and *f* are incident with a common vertex *v*, then *e* and *f* are said to be adjacent to each other. If two or more edges join the same pair of (distinct) vertices, then these edges are called parallel edges. If an edge *e* joins a vertex *v* to itself, then *e* is called a loop. A graph G without loops and parallel edges is called a simple graph.

Definition: 2.2 The degree of a vertex *v* in a graph G is the number of edges incident with *v* and is denoted by $\deg_G(v)$ or deg(*v*) or d(v). A vertex of degree 0 in G is called an isolated vertex and a vertex of degree 1 is called a pendant vertex or an end vertex of G. A graph is said to be k-regular if every vertex of G has degree k.

A walk $u=u_0,e_1,u_1,e_2,u_2,...,e_n,u_n=v$ is determined by the sequence $u_0,u_1,u_2,...,u_n$ of its vertices and hence we specify this walk by $w=u_0,u_1,u_2,...,u_n$.

A walk in which all the vertices are distinct is called a path. A walk is said to be open if *u* and *v* are distinct vertices. A walk is closed if $u=v$.

A closed walk $u_0,u_1,u_2,...,u_n$ in which $u_0,u_1,u_2,...,u_{n-1}$ are distinct is called a cycle. A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n.

Definition: 2.3 The Wiener index of G denoted by *W(G)* is defined by

$$W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d(u,v)$$

Definition: 2.4 The Detour index of G denoted by D(G) is defined by

$$D(G) = \frac{1}{2}\sum_{u,v\in V(G)} D(u,v)$$

Definition: 2.5 The Monophonic Distance matrix of a graph is a square matrix of order of G whose entries are the monophonic distance $d_m(u,v)$ between every pair of vertices u and v and denoted by MPDM(G)

Definition: 2.6 The Monophonic Polynomial of G denoted by $MPP(G:x)$ and defined by

$$\text{MPP(G:x)} = \frac{1}{2}\sum_{u,v\in V(G)} x^{d_m(u,v)}$$

Definition: 2.7 The Monophonic Index of G is the first order differentiation of $MPP(G:x)$ at x=1. That is,

$$MP(G) = \frac{d}{dx}\,[MPP(G:x)]_{x=1}$$

Definition: 2.8 The Gutman index of G denoted by Gut(G) is defined by

$$Gut(G) = \sum_{u,v\in V} d(u)d(v)d(u,v)$$

Definition: 2.9 The Detour Gutman index of G denoted by DGut(G) is defined by

$$DGut(G) = \sum_{u,v\in V} d(u)d(v)D(u,v)$$

Definition: 2.10 The degree distance index of G denoted by DD(G) is defined by

$$DD(G) = \sum_{u,v\in V} [d(u)+d(v)]d(u,v)$$

Definition: 2.11 The detour degree distance index of G denoted by DDD(G) is defined by

$$DDD(G) = \sum_{u,v\in V} [d(u)+d(v)]D(u,v)$$

III. ALGORITHM

The following stepwise procedure must be followed to get the Gutman monophonic index of a graph G.

Step 1: Draw the graph G under consideration with proper labeling.

Step 2: Write the monophonic distance matrix of G whose entries are $d_m(u,v)$, for all $u,v \in V$.

Step 3: Note down the degree *d(v)* of all vertices $v \in V$. For example, $d(v_i)=k$ in *k*-regular graph.

Step 4: Substitute in the formula *GutMP(G)/DMP(G)* to get the required index.

IV. RESULTS

In this section, the concept of Gutman monophonic index and degree monophonic distance index are introduced and these indices are derived for cycle graph, wheel graph and ladder graph.

Definition: 4.1 The Gutman Monophonic index of G is denoted by *GutMP(G)* and defined by

$$GutMP(G) = \sum_{u,v\in V} d(u)d(v)d_m(u,v)$$

Definition: 4.2 The degree monophonic index of G is denoted by *DMP(G)* and defined by

$$DMP(G) = \sum_{u,v\in V} [d(u)+d(v)]d_m(u,v)$$

Definition: 4.3 The Monophonic Distance matrix of a graph is a square matrix of order of G whose entries are the monophonic distance $d_m(u,v)$ between every pair of vertices u and v and denoted by *MPDM(G)*

Example:

Consider the graph G given in Fig. 1.

G:

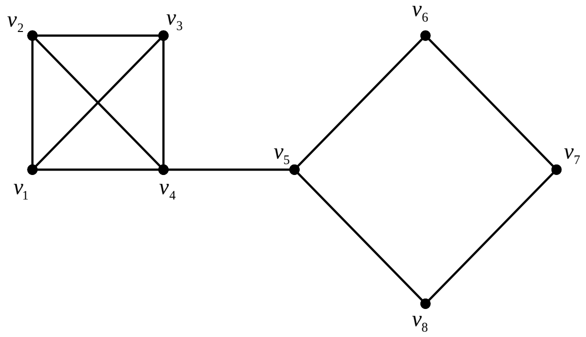

Fig. 1.

The monophonic distance matrix of graph G is shown in Fig. 2.

MPDM(G):

$$\begin{array}{c|cccccccc|}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \\
\hline
v_1 & 0 & 1 & 1 & 1 & 2 & 3 & 4 & 3 \\
v_2 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 3 \\
v_3 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 3 \\
v_4 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 2 \\
v_5 & 2 & 2 & 2 & 1 & 0 & 1 & 2 & 1 \\
v_6 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 \\
v_7 & 4 & 4 & 4 & 3 & 2 & 1 & 0 & 1 \\
v_8 & 3 & 3 & 3 & 2 & 1 & 2 & 1 & 0
\end{array}$$

Fig. 2.

V. GUTMAN MONOPHONIC INDEX OF CYCLE AND WHEEL GRAPH

Theorem: 5.1 If G=C_p is the cycle graph with *p* vertices, then the Gutman monophonic index of C_p is $GutMP(C_p) = \frac{p}{2}[3p^2 - 12p + 16]$ if *p* is even and $p \geq 4$ and if *p* is odd and $p \geq 3$. $GutMP(C_p) = \frac{p}{2}[3p^2 - 12p + 17]$

Proof: Let $V(C_p) = \{v_1, v_2, \ldots, v_p\}$ *and* $E(C_p) = \{v_i v_{i+1} : 1 \leq i \leq p-1\} \cup \{v_p v_1\}$ be the vertex set and edge set of G respectively. Since C_p is a 2-regular graph $d(v_i) = 2$ *for all* $i = 1$ *to* p. The distance $d_m(u,v)$ can be obtained by forming the monophonic distance matrix *MPDM*(C_p). The proof of the theorem is considered in two different

Retrieval Number: I8911078919/19©BEIESP
DOI:10.35940/ijitee.I8911.078919

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

cases when p is even and p is odd.

Case i: when p is even

If p=4, then

$$GutMP(C_4)=\sum_{i=1}^{4}\sum_{j=i}^{4} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=4(2)2+4(2)2$$
$$=16+16=32$$

When p=6, the cycle graph C_6 and $MPDM(C_6)$ are respectively given in Fig. 3 and Fig. 4.

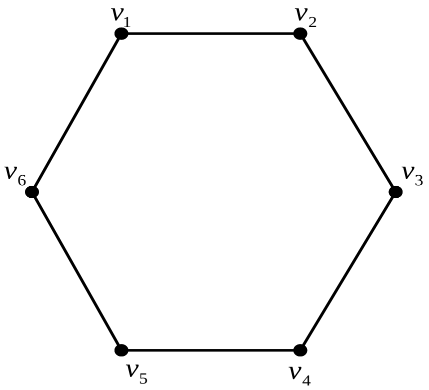

Fig. 3. Cycle C_6

$$\begin{array}{c|cccccc|}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
\hline
v_1 & 0 & 1 & 4 & 3 & 4 & 1 \\
v_2 & 1 & 0 & 1 & 4 & 3 & 4 \\
v_3 & 4 & 1 & 0 & 1 & 4 & 3 \\
v_4 & 3 & 4 & 1 & 0 & 1 & 4 \\
v_5 & 4 & 3 & 4 & 1 & 0 & 1 \\
v_6 & 1 & 4 & 3 & 4 & 1 & 0
\end{array}$$

Fig. 4. MPDM(C_6)

$GutMP(C_6)$ is given by

$$GutMP(C_6)=\sum_{i=1}^{6}\sum_{j=i}^{6} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 6(2)(2)1+3(2)(2)3+6(2)(2)4$$
$$= 156$$

When p=8, the cycle graph C_8 and $MPDM(C_8)$ are respectively given in Fig. 5 and Fig. 6.

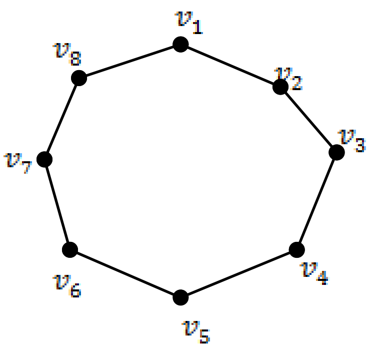

Fig. 5. Cycle C_8

$$\begin{array}{c|cccccccc|}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \\
\hline
v_1 & 0 & 1 & 6 & 5 & 4 & 5 & 6 & 1 \\
v_2 & 1 & 0 & 1 & 6 & 5 & 4 & 5 & 6 \\
v_3 & 6 & 1 & 0 & 1 & 6 & 5 & 4 & 5 \\
v_4 & 5 & 6 & 1 & 0 & 1 & 6 & 5 & 4 \\
v_5 & 4 & 5 & 6 & 1 & 0 & 1 & 6 & 5 \\
v_6 & 5 & 4 & 5 & 6 & 1 & 0 & 1 & 6 \\
v_7 & 6 & 5 & 4 & 5 & 6 & 1 & 0 & 1 \\
v_8 & 1 & 6 & 5 & 4 & 5 & 6 & 1 & 0
\end{array}$$

Fig. 6. MPDM(C_8)

From $MPDM(C_8)$, $GutMP(C_8)$ is given by

$$GutMP(C_8)=\sum_{i=1}^{8}\sum_{j=i}^{8} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=8(2)(2)1+8(2)(2)6+8(2)(2)5$$
$$+(4)(2)(2)4$$
$$=448$$

Similarly, one can write Gutman Monophonic indices by forming the monophonic distance matrix of the corresponding graphs. Hence

$$GutMP(C_{10})=\sum_{i=1}^{10}\sum_{j=i}^{10} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=10(2)(2)1+10(2)(2)8+10(2)(2)7$$
$$+10(2)(2)6+5(2)(2)5$$
$$=780$$

$$GutMP(C_{12})=\sum_{i=1}^{12}\sum_{j=i}^{12} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=12(2)(2)1+12(2)(2)7+12(2)(2)8$$
$$+12(2)(2)9+12(2)(2)10+6(2)(2)6$$
$$=1824$$

and so on. Hence in general if p is even and $p\geq 4$

$$GutMP(C_p)=\frac{p}{2}\left[3p^2-12p+16\right]$$

Case ii: When p is odd

If p = 3, then

$$GutMP(C_3)=\sum_{i=1}^{3}\sum_{j=i}^{3}d(v_i)d(v_j)d_m(v_i.v_j)$$

$$=3(2)2$$

$$=12$$

When p=5, the cycle graph C_5 and $MPDM(C_5)$ are respectively given in Fig. 7 and Fig. 8.

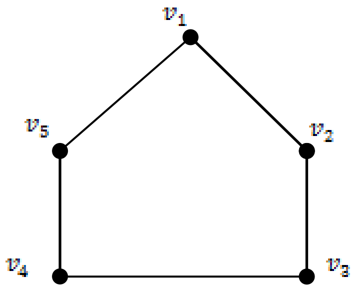

Fig. 7. Cycle C_5

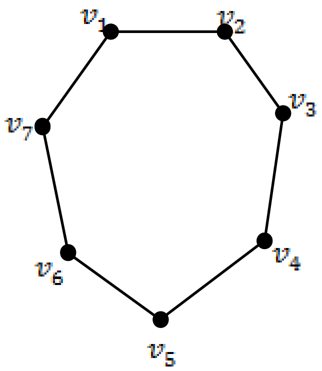

Fig. 8. MPDM (C_5)

$GutMP(C_5)$ is given by

$$GutMP(C_5)=\sum_{i=1}^{5}\sum_{j=i}^{5}d(v_i)d(v_j)d_m(v_i.v_j)$$

$$=5(2)(2)1+5(2)(2)3$$

$$=80$$

When p=7, the cycle graph C_7 and $MPDM(C_7)$ are respectively given in Fig. 9 and Fig. 10.

$$\begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} & \begin{bmatrix} 0 & 1 & 3 & 3 & 1 \\ 1 & 0 & 1 & 3 & 3 \\ 3 & 1 & 0 & 1 & 3 \\ 3 & 3 & 1 & 0 & 1 \\ 1 & 3 & 3 & 1 & 0 \end{bmatrix} \end{array}$$

Fig. 9. Cycle C_7

$$\begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{matrix} & \begin{bmatrix} 0 & 1 & 5 & 4 & 4 & 5 & 1 \\ 1 & 0 & 1 & 5 & 4 & 4 & 5 \\ 5 & 1 & 0 & 1 & 5 & 4 & 4 \\ 4 & 5 & 1 & 0 & 1 & 5 & 4 \\ 4 & 4 & 5 & 1 & 0 & 1 & 5 \\ 5 & 4 & 4 & 5 & 1 & 0 & 1 \\ 1 & 5 & 4 & 4 & 5 & 1 & 0 \end{bmatrix} \end{array}$$

Fig. 10. MPDM(C_7)

From MPDM(C_7), GutMP(C_7) is given by

$$GutMP(C_7)=\sum_{i=1}^{7}\sum_{j=i}^{7}d(v_i)d(v_j)d_m(v_i.v_j)$$

$$=7(2)(2)+7(2)(2)+7(2)(2)5$$

$$=280$$

Similarly, one can write Gutman Monophonic indices by forming the monophonic distance matrix of the corresponding graphs. Hence

$$GutMP(C_9)=\sum_{i=1}^{9}\sum_{j=i}^{9}d(v_i)d(v_j)d_m(v_i.v_j)$$

$$=9(2)(2)1+9(2)(2)5+9(2)(2)6+9(2)(2)7$$

$$=684$$

$$GutMP(C_{11})=\sum_{i=1}^{11}\sum_{j=i}^{11}d(v_i)d(v_j)d_m(v_i.v_j)$$

$$=11(2)(2)1+11(2)(2)6+11(2)(2)7$$

$$+11(2)(2)8+11(2)(2)9$$

$$=1364$$

$$GutMP(C_{13})=\sum_{i=1}^{13}\sum_{j=i}^{13}d(v_i)d(v_j)d_m(v_i.v_j)$$

$$=13(2)(2)1+13(2)(2)7+13(2)(2)8$$

$$+13(2)(2)9+13(2)(2)10+13(2)(2)11$$

$$=2392$$

Hence in general if $p\geq 3$ and p is odd

$$GutMP(C_p)=\frac{p}{2}\left[3p^2-12p+17\right]$$

Note: For the cycle graph C_p, degree monophonic index and the Gutman monophonic index are equal. That is

$$DMP(C_P)=GutMP(C_p)$$

Theorem: 5.2 For the wheel graph W_p with p vertices

$$GutMP(W_p) = \frac{3}{8}\left[9p^3 - 46p^2 + 8p + 344\right]$$ if p is even and $p \geq 4$, $GutMP(W_p) = \frac{3p}{8}\left[9p^2 - 55p + 131\right] - \frac{255}{8}$ if p is odd and $p \geq 5$

Proof:

Let $V(W_p) = \{v_0, v_1, v_2, ..., v_{p-1}\}$ and $E(G) = \{v_0 v_i ; 1 \leq i \leq p-1\} \cup \{v_i v_{i+1} : 1 \leq i \leq p-2\} \cup \{v_{p-1} v_1\}$ be the vertex set and edge set of W_p respectively. In wheel graph W_p $d(v_0)=p$, $d(v_i)=3$ for all $i = 1$ *to* p. For the wheel graph W_p, $p = 4, 6, 8, \ldots$ the Gutman monophonic indices are given below:

$$GutMP(W_4) = \sum_{i=1}^{3} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{3}\sum_{j=i}^{3} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 6(3)(3)1$$
$$= 54$$

$$GutMP(W_6) = \sum_{i=1}^{5} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{5}\sum_{j=i}^{5} d(v_i)d(v_j)d_m(v_i, v_j)$$

$$GutMP(W_8) = \sum_{i=1}^{7} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{7}\sum_{j=i}^{7} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 7(7)(3)1 + 7(3)(3)1 + 7(3)(3)4 + 7(3)(3)5$$
$$= 777$$

$$GutMP(W_{10}) = \sum_{i=1}^{9} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{9}\sum_{j=i}^{9} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 9(9)(3)1 + 9(3)(3)1 + 9(3)(3)5 + 9(3)(3)6 + 9(3)(3)7$$
$$= 1782$$

$$GutMP(W_{12}) = \sum_{i=1}^{11} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{11}\sum_{j=i}^{11} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 11(11)(3)(1) + 11(3)(3)1 + 6(3)(3)6 + 7(3)(3)7 + 7(3)(3)8 + 7(3)(3)9$$
$$= 3432$$

Hence if p is even, $p \geq 4$

$$GutMP(W_p) = \frac{3}{8}\left[9p^3 - 46p^2 + 8p + 344\right]$$

For $p = 5, 7, 9, \ldots$ Gutman monophonic indices of W_p are shown below:

$$GutMP(W_5) = \sum_{i=1}^{4} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{4}\sum_{j=i}^{4} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 4(4)(3)1 + 4(3)(3)1 + 2(3)(3)2$$
$$= 120$$

$$GutMP(W_7) = \sum_{i=1}^{6} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{6}\sum_{j=i}^{6} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 6(6)(3)1 + 6(3)(3)1 + 3(3)(3)3 + 6(3)(3)4$$
$$= 459$$

$$GutMP(W_9) = \sum_{i=1}^{8} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{8}\sum_{j=i}^{8} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 1200$$

$$GutMP(W_{11}) = \sum_{i=1}^{10} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{10}\sum_{j=i}^{10} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 2505$$

$$GutMP(W_{13}) = \sum_{i=1}^{12} d(v_0)d(v_i)d_m(v_0, v_i) + \sum_{i=1}^{12}\sum_{j=i}^{12} d(v_i)d(v_j)d_m(v_i, v_j)$$
$$= 4536$$

Hence if p is odd and $p \geq 5$

$$GutMP(W_p) = \frac{1}{8}\left[27p^3 - 165p^2 + 393p - 255\right]$$

Corollary 5.3:

$DMP(W_p) = \frac{1}{4}\left[9p^3 - 49p^2 + 31p + 249\right]$ if p is odd and $p \geq 5$

$DMP(W_p)=\frac{1}{4}\left[9p^3-59p^2+154p-104\right]$ if p is even and $p\geq 4$

Proof:

$$DMP(W_p)=\sum_{i=1}^{p}\sum_{j=i}^{p}d(v_i)d(v_j)d_m(v_i,v_j)$$

$$DMP(W_4)=6(3+3)1$$
$$=36$$

$$DMP(W_5)=4(4+3)1+4(3+3)1$$
$$+2(3+3)2$$
$$=76$$

$$DMP(W_6)=5(5+3)1+5(3+3)1$$
$$+5(3+3)(3)$$
$$=165$$

$$DMP(W_7)=6(6+3)1+6(3+3)1+3(3+3)3$$
$$+6(3+3)4$$
$$=288$$

$$DMP(W_8)=490$$
$$DMP(W_9)=760$$
$$DMP(W_{10})=1234$$
$$DMP(W_{11})=1600$$
$$DMP(W_{12})=220$$

If p is odd and $p\geq 5$,

$$DMP(W_p)=\frac{1}{4}\left[9p^3-49p^2+31p+249\right]$$

Degree monophonic index of W_p, when p is even and $p\geq 4$

$$DMP(W_4)=6(3+3)1$$
$$=36$$

$$DMP(W_6)=5(5+3)1+5(3+3)1+5(3+3)3$$
$$=160$$

$$DMP(W_8)=7(7+3)1+7(3+3)1+7(3+3)4$$
$$+7(3+3)5$$
$$=490$$

$$DMP(W_{10})=9(9+3)1+9(3+3)1+9(3+3)5$$
$$+9(3+3)6+9(3+3)7$$
$$=1134$$

$$DMP(W_{12})=11(11+3)1+11(3+3)1$$
$$+11(3+3)6$$
$$=2200$$

$$DMP(W_{14})=13(13+3)1$$
$$+(13)(6)[1+7+8+9+10+11]$$
$$+11(3+3)7+11(3+3)8+11(3+3)9$$
$$=3796$$

Hence in general for $p\geq 4$, p is even

$$DMP(W_p)=\frac{1}{4}\left[9p^3-59p^2+154p-104\right]$$

VI. GUTMAN AND DEGREE MONOPHONIC INDEX OF P_2XP_n

Theorem: 6.1 Let $G=P_2XP_n$ be the Cartesian product of P_2 and P_n. Let $G=P_2XP_n$ be the ladder graph. Then $GutMP(G)=\sum_{u,v\in G}d(u)d(v)d_m(u,v)$ where *d(u), d(v)* are the degrees of *u* and *v* respectively and $_{dm}(u,v)$ be the monophonic distance between *u* and *v*.

Proof:

Let $V(P_2)=\{u_1,u_2\}$ and $V(P_n)=\{v_1,v_2,\ldots\ldots,v_n\}$ be the vertex set of P_2 and P_n respectively.

Then $V(P_2\times P_n)=\{(u_1,v_i),(u_2,v_i):1\leq i\leq n\}$.

For simplicity let us denote $(u_1,v_i)=w_i$; $1\leq i\leq n$ and $(u_2,v_i)=w_i$; $n+1\leq i\leq 2n$. The edge set

$$E(P_2\times P_n)=\{w_iw_{i+1}:1\leq i\leq n-1\}$$
$$\cup\{w_iw_{i+1}:n+1\leq i\leq 2n-1\}\cup\{w_iw_{n+i}:1\leq i\leq n\}$$
$$d(w_1)=2=d(w_n)=d(w_{n+1})=d(w_{2n})$$

For all other vertices $w_i, d(w_i)=3$.

Let $n=3$. The graph $P_2\times P_3$ and *MPDM* $(P_2\times P_3)$ are given in Fig. 11 and Fig. 12 respectively.

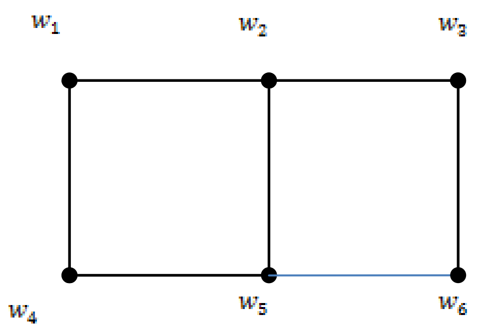

Fig. 11. $P_2\times P_3$

$$\begin{array}{c|cccccc|}
 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \\
\hline
w_1 & 0 & 1 & 4 & 1 & 2 & 3 \\
w_2 & 1 & 0 & 1 & 2 & 1 & 2 \\
w_3 & 4 & 1 & 0 & 3 & 2 & 1 \\
w_4 & 1 & 2 & 3 & 0 & 1 & 4 \\
w_5 & 2 & 1 & 2 & 1 & 0 & 1 \\
w_6 & 3 & 2 & 1 & 4 & 1 & 0
\end{array}$$

Fig. 12. MPDM$(P_2 \times P_3)$

For n = 3, 4, 5, … the $GutMP(G)$ are derived and shown below:

$$\begin{aligned}
GutMP(P_2XP_3) &= \sum_{i=1}^{6}\sum_{j=i}^{6} d(w_i)d(w_j)d_m(w_i.w_j) \\
&= [4(2.3)+2(2.2)+(3.3)]1 \\
&\quad + 4(2.3)2 + 2(2.2)3 + 2(2.2)4 \\
&= 145
\end{aligned}$$

When $n = 4$, the graph $P_2 \times P_4$ and MPDM $(P_2 \times P_4)$ are given in Fig. 13 and Fig. 14 respectively.

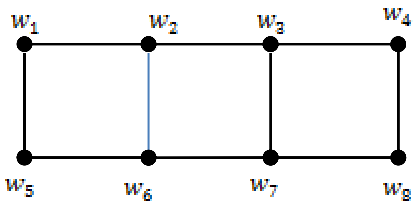

Fig. 13. $P_2 \times P_4$

$$\begin{array}{c|cccccccc|}
 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\
\hline
w_1 & 0 & 1 & 4 & 5 & 1 & 2 & 3 & 4 \\
w_2 & 1 & 0 & 1 & 4 & 2 & 1 & 2 & 3 \\
w_3 & 4 & 1 & 0 & 1 & 3 & 2 & 1 & 2 \\
w_4 & 5 & 4 & 1 & 0 & 4 & 3 & 2 & 1 \\
w_5 & 1 & 2 & 3 & 4 & 0 & 1 & 4 & 3 \\
w_6 & 2 & 1 & 2 & 3 & 1 & 0 & 1 & 4 \\
w_7 & 3 & 2 & 1 & 2 & 4 & 1 & 0 & 1 \\
w_8 & 4 & 3 & 2 & 1 & 3 & 4 & 1 & 0
\end{array}$$

Fig. 14. MPDM$(P_2 \times P_4)$

$$\begin{aligned}
GutMP(P_2XP_4) &= [4(2.3)+2(2.2)+4(3.3)]1 \\
&\quad + [4(2.3)+2(3.3)]2 \\
&\quad + 4(2.3)3 + [2(2.2) \\
&\quad + 4(2.3)]4 + 2(2.2)5
\end{aligned}$$

$$\begin{aligned}
GutMP(P_2XP_5) &= [4(2.3)+2(2.2)+7(3.3)]1 \\
&\quad + [4(2.3)+4(3.3)]2 \\
&\quad + 3[4(2.3)+2(3.3)] \\
&\quad + [8(2.3)+2(3.3)]4 \\
&\quad + [4(2.3)+2(2.2)]5 \\
&\quad + 2(2.2)6
\end{aligned}$$

$$\begin{aligned}
GutMP(P_2XP_6) &= [4(2.3)+2(2.2)+10(3.3)]1 \\
&\quad + [4(2.3)+6(3.3)]2 \\
&\quad + [4(2.3)+4(3.3)]3 \\
&\quad + [8(2.3)+6(3.3)]4 \\
&\quad + [8(2.3)+2(3.3)]5 \\
&\quad + [4(2.3)+2(2.2)]6 \\
&\quad + 2(2.2)7
\end{aligned}$$

$$\begin{aligned}
GutMP(P_2XP_7) &= [4(2.3)+2(2.2)+13(3.3)]1 \\
&\quad + [4(2.3)+8(3.3)]2 \\
&\quad + [4(2.3)+6(3.3)]3 \\
&\quad + [8(2.3)+10(3.3)]4 \\
&\quad + [8(2.3)+6(3.3)]5 \\
&\quad + [8(2.3)+2(3.3)]6 \\
&\quad + [4(2.3)+2(2.2)]7 \\
&\quad + 2(2.2)8
\end{aligned}$$

$$\begin{aligned}
GutMP(P_2XP_8) &= [4(2.3)+2(2.2)+16(3.3)]1 \\
&\quad + [4(2.3)+10(3.3)]2 \\
&\quad + [4(2.3)+8(3.3)]3 \\
&\quad + [8(2.3)+14(3.3)]4 \\
&\quad + [8(2.3)+10(3.3)]5 \\
&\quad + [8(2.3)+2(3.3)]6 \\
&\quad + [8(2.3)+2(3.3)]7 \\
&\quad + [4(2.3)+2(2.2)]8 \\
&\quad + 2(2.2)9
\end{aligned}$$

Hence in general for any $n \geq 5$,

Retrieval Number: I8911078919/19©BEIESP
DOI:10.35940/ijitee.I8911.078919

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$
\begin{aligned}
GutMP(P_2XP_n) = {} & [4(2.3)+2(2.2)+(3n-8)(3.3)]1 \\
& +[4(2.3)+(2n-6)(3.3)]2 \\
& +[4(2.3)+(2n-8)3.3]3 \\
& +[8(2.3)+(4n-18)(3.3)]4 \\
& +[8(2.3)+(4n-22)(3.3)]5 \\
& +[8(2.3)+(4n-26)(3.3)]6 \\
& +[8(2.3)+(4n-30)(3.3)]7+\ldots \\
& +[8(2.3)+2(3.3)](n-1) \\
& +[4(2.3)+2(2.2)]n \\
& +2(2.2)(n+1)
\end{aligned}
$$

Corollary 3.2:

$$
\begin{aligned}
DMP(P_2XP_n) = {} & [4(2+3)+2(2+2)+(3n-8)(3+3)]1 \\
& +[4(2+3)+(2n-6)(3+3)]2 \\
& +[4(2+3)+(2n-8)(3+3)]3 \\
& +[8(2+3)+(4n-18)(3+3)]4 \\
& +[8(2+3)+(4n-22)(3+3)]5 \\
& +[8(2+3)+(4n-26)(3+3)]6 \\
& +[8(2+3)+(4n-30)(3+3)]7+.. \\
& +[8(2+3)+2(3+3)](n-1) \\
& +[4(2+3)+2(2+2)]n \\
& +2(2+2)(n+1)
\end{aligned}
$$

VII. CONCLUSION

The Gutman index and degree distance index are studied for various graphs by several authors with respect to the shortest distance and the longest distance. In this paper we have introduced the indices based on monophonic distance and computed Gutman Monophonic index and Degree Monophonic index for cycle graph, wheel graph and ladder graph by monophonic distance matrix method. The study of these indices for molecular graphs, which are helpful for drug design are under investigation. The indices, which are presented in this paper may be used to derive quantitative structure- property or structure- activity relationships (QSPR / QSAR).

REFERENCES

1. P. Arul Paul Sudhahar and M. Little Flower, "Total Edge Detour Monophonic Number of a Graph", DOI: 10.29055/jcms/1004, 2019.
2. S. Arumugam, "The Upper Connected Vertex Detour Monophonic Number of a Graph", Indian Journal of Pure and Applied Mathematics, vol. 49(2), 2018, pp. 365-379.
3. S. Kavitha and V. Kaladevi, "Gutman and Detour Gutman index of Pseudo-regular graphs", Journal of applied mathematics, vol. 17, 2017, pp. 1-8.
4. P. Titus and K. Ganesamoorthy, "Upper detour monophonic number of a graph", Electronic Notes in Discrete Mathematics, vol. 53, 2016, pp. 331-342.
5. P. Titus and K. Ganesamoorthy, "On the detour monophonic number of a graph", Ars Combinatoria, vol. 129, 2016, pp. 33-42.
6. A. P. Santhakumaran and P. Titus, P, "A note on "Monophonic distance in graphs", Discrete Mathematics, Algorithms and Applications, vol. 4(2):12500189, 2012.
7. A. P. Santhakumaran and P. Titus, "Monophonic distance in graphs", Discrete Mathematics, Algorithms and Applications, vol. 3(2), 2011, pp. 159-169.
8. K. M. Kathiresan, C. Parameswaran, G. Marimuthu and S. Arockiaraj, "Detour Wiener Indices of Graphs", Bulletin of the ICA, vol. 62, 2011, pp. 33-47.
9. V. Kaladevi and P. Backialakshmi, "Detour Distance Polynomial of Double Star Graph and Cartesian Product of $P_2 \times C_n$", Antarctica Journal of Mathematics, vol. 8(5), 2011, pp. 399-406.
10. G. Chantrand and P. Zhang, "Distance in Graphs, Taking the LongView", AKCE. J. Graphs Combin., vol. 1, 2004, pp. 1-13.
11. B. Furtula, Ivan Gutman, Zeljko Tomovic, Aleksander Vesel and Igor Pesek, "Wiener-type Topological Indices of Phenyleness", Indian J. Chem., vol. 41 A, 2002, pp. 1767-1772.
12. L. Lukovits, "The Detour Index", Croat. Chem. Acta., vol. 69, 1996, pp. 873-882.

AUTHORS PROFILE

V.Kaladevi, received her Ph.D from Bharathidasan University, Trichirapalli, India. She received her Master of Science in Mathematics from Nirmala College, affiliated to University of Madras, Chennai, India and Master of Philosophy in Mathematics from Madurai Kamaraj University, Madurai, India. She obtained her Bachelor of Science in Mathematics from Standard Fireworks Rajarathinam College for Women, Sivakasi, affiliated to Madurai Kamaraj University, Madurai, India. Her research interests are Graph Theory and Number Theory. She has published 50 research papers in International journals of repute and 4 students are awarded Ph.D. degree. Currently, she is affiliated with Department of Mathematics, as an Associate Professor in Hindustan Institute of Technology & Science(Deemed to be University), Chennai, India.

G.Kavitha, received her Ph.D from the Department of Mathematics, Hindustan Institute of Technology & Science, Chennai, India. She received her Master of Science in Mathematics from Queen Mary's College, affiliated to University of Madras, Chennai, India and Master of Philosophy in Mathematics from Ramanujan Institute for Advanced Studies in Mathematics, University of Madras, Chennai, India. She obtained her Bachelor of Science in Mathematics from Guru Nanak College, Velachery, affiliated to University of Madras, Chennai, India. Her research interests are Stochastic Optimization, Stochastic Graphs, Time series Analysis, Artificial Intelligence, Block chain Technology, Soft Computing Techniques, Mathematical Finance, Finance and Modeling, Data Analysis, Forecast Analysis, Statistical modeling. She has published 3 text books and 7 research papers in International journals of repute. Currently, she is affiliated with Department of Mathematics, as an Assistant Professor (SG) in Hindustan Institute of Technology & Science(Deemed to be University), Chennai, India.