

# Capacity Analysis and Linear Detection in Massive MIMO for 5G Wireless Systems

Haresh L. Judal, Kishor G. Maradia

**Abstract:** Multiple Input Multiple Output (MIMO) is an attractive air interface solution which is used in the 4<sup>th</sup> generation wireless networks to achieve higher data rate. With a very large antenna array in Massive MIMO the capacity will increase drastically. In this paper channel capacity comparison for MIMO using known Channel State Information (CSI) and unknown CSI has been carried out for a higher number of antennas at transmitter and receiver side. It has shown that at lower SNR known CSI will give better performance compared to unknown CSI. At higher SNR known CSI and unknown CSI will provide similar results. Capacity comparison has been evaluated with help of MATLAB for known CSI and unknown CSI from a small number of antennas to hundred of antennas. Also, the performance evaluated with MATLAB simulation of linear detectors zero-forcing (ZF) and maximum ratio combining (MRC) method for large number of antennas at Base station (BS) which are serving a small number of single antenna users. Performance is evaluated in terms of Symbol Error Rate (SER) for ZF and MRC, and results show that ZF will outperform MRC. It has also been analyzed that increasing the antennas at BS for a small number of users will also help to reduce SER.

**Keywords:** MIMO, ZF, MRC, BS, SER

## I. INTRODUCTION

The Multiple-Input Multiple-Output (MIMO) has the potential to improve the capacity of wireless systems. MIMO communication system employs multiple antennas at the transmitter and the receiver side. MIMO systems offer additional degrees of diversity which can be used to combat multipath fading in a wireless channel. Massive MIMO system is currently considered the most compelling technology for the 5G wireless networks, [1][9][10][12]. The Base station (BS) contains a large number of antenna elements in the order of hundreds or more which is simultaneously serving tens of mobile terminals (MTs) in Massive MIMO. Uncorrelated noise and multiuser interference effect can be made small due to coherently combining signals at BS antennas [2], [3], [11]. Hence network capacities can be unprecedentedly achieved. If the number of BS antennas grows larger than the random channel vectors between the BS and the users, become pairwise orthogonal [4]. With simple matched filter processing at the BS uncorrelated noise and intracell interference disappear completely if the infinite numbers of antennas can be taken at the BS. Massive-MIMO technology is a promising solution to meet a demand for higher data capacity in 5G Wireless Communication Networks. The increase in antennas at transmission and reception open a door for research in the theory of Communications.

Revised Manuscript Received on July 09, 2019

Haresh L. Judal, Ph.D. Scholar, E & C Engineering Department, Gujarat Technological University Ahmedabd, Gujarat, India.

Kishor G. Maradia, Professor & Head, E& C Engineering Department, Government Engineering College, Gandhinagar Sector 28, Gujarat, India.

In this paper, simulation is done based on the expression for the capacity of the MIMO channel for known Channel State Information (CSI) and unknown CSI. It has also shown that using a higher number of antennas at transmitter and receiver provide a capacity gain in terms of bits/s/Hz. In this paper, the comparison has been made between maximum ratio combining (MRC) and zero-forcing (ZF) linear detectors in Massive MIMO environment.

In this paper Section II explains the MIMO architecture and the theory behind it. In Section III explanation is given for the capacity of MIMO for unknown CSI and known CSI. Massive MIMO is explained in Section IV. Section V explains linear detectors used in Massive MIMO. The results from simulation are presented in Section VI. A conclusion of the paper is given in Section VII.

Notations: The superscript  $(\cdot)^H$  indicates conjugate transpose; the squared Frobenius norm of H represented by  $\|H\|_F^2$  and  $I_N$  is the  $N \times N$  identity matrix.

## II. MIMO SYSTEM MODEL

In a MIMO system data are transmitted with  $N_t$  transmitting antenna arrays. The receiver is constructed with  $N_r$  antenna arrays.

A spatial multiplexing MIMO system transmits different data symbols from each transmitter. The signals from each transmitter combine over the air and are received by multiple receiver antennas.

The MIMO model can be described as

$$y = Hx + n \quad (1)$$

Where H is the channel matrix, x is the transmitted signal, y is the received signal and n is the noise.

## III. THE CAPACITY OF A MIMO SYSTEM

MIMO contains  $N_t$  transmitter antennas and  $N_r$  receiver antennas. H contains  $N_r \times N_t$  channel coefficients.

If the channel is unknown to the transmitter than the signals are assumed to be independent and equal power is transmitted from transmitter antennas. The capacity of the MIMO for the unknown CSI at the transmitter is given by

$$C = \log_2 \det \left( I_{N_r} + \frac{E_S}{N_t N_o} H H^H \right) \quad (2)$$

Given that  $H H^H = Q \lambda Q^H$ , then the capacity of MIMO can be

$$C = \log_2 \det \left( I_{N_r} + \frac{E_S}{N_t N_o} Q \lambda Q^H \right) \quad (3)$$

Using the identity of matrix

$$C = \sum_{i=1}^r \log_2 \left( 1 + \frac{E_S \lambda_i}{N_t N_o} \right) \quad (4)$$

The rank of the channel matrix is  $r$  and  $\lambda_i (i=1, 2, \dots, r)$  are the positive eigenvalues of  $HH^H$ .

In the absence of channel knowledge equal transmit energy allocated to each spatial data pipe.

Given a fixed total power  $\|H\|_F^2 = \sum_{i=1}^r \lambda_i = \xi$

Where,  $\|H\|_F^2$  is the total power gain of the channels.

Consider a full rank MIMO channel with  $N_t = N_r = N$  so that

$r=N$ . If  $\lambda_i = \lambda_j = \frac{\xi}{N} (i, j=1, \dots, N)$  and  $H$  is the orthogonal

matrix such that  $HH^H = H^H H = \frac{\xi}{N} I_N$  then resulting capacity will be

$$C = N \log_2 \left( 1 + \frac{E_S}{N_t N_o} \frac{\xi}{N} \right) \quad (5)$$

If elements of  $H$  satisfy such that  $\|H_{ij}\|^2 = 1 \Rightarrow \|H\|_F^2 = \xi = N^2$  then

$$C_{max} = N \log_2 \left( 1 + \frac{E_S}{N_o} \right) \quad (6)$$

Which is equal to the  $N$  times the capacity of SISO channel for unknown CSI.

For CSI known, power allocation strategy extracts the information regarding conditions of channels which are good and which are not, and accordingly power is allocated to the channels. In this way, the overall capacity of the system is maximized so that the spectral efficiency is improved. This strategy is also referred to as water filling algorithm in MIMO communications.

After applying SVD and further processing, the conventional MIMO system becomes similar to a collection of non-interfering SISO systems. SVD is helpful not only in decomposing the interfering paths into parallel non-interfering channels but also it helps to identify the quality of those parallel channels. Based on the quality of channels, the power is allocated such that maximum data rate can be achieved. The diagonal elements  $\sum$  are the singular values of the channel matrix  $H$  which are the square roots of the eigenvalues. A number of non-zero diagonal elements of  $\sum$  cannot exceed the rank of the channel matrix,  $H$ , as the number of non-zero eigenvalues is the rank of the matrix.

Received signal at the receiver without preprocessing at the transmitter will be  $r = Hx + n$ . With preprocessing at transmitter transmitted signal will be  $HVx$  instead of  $Hx$  and Received signal, in this case, will be  $r = HVx + n$ .

Where  $H = U \sum V^H$

$$r = U \sum V^H V x + n$$

$$r = U \sum x + n \quad (7)$$

At receiver with processing  $y = U^H r$

$$y = U^H (U \sum x + n) \quad (8)$$

$$y = \sum x + U^H n \quad (9)$$

$$y_i = \sqrt{\lambda_i} x_i + [U^H n]_i \quad (10)$$

MIMO channels have been virtually decomposed as parallel SISO channels through the process of transformations at transmitter and receiver.

The capacity of the MIMO model is algebraic sum of the capacities of all channels

$$C = \sum_{i=1}^r \log_2 (1 + Y_i) \quad (11)$$

Water filling algorithm can be applied with the following expression.

$$y_i = \sqrt{\frac{E_S}{N_t}} \sqrt{\lambda_i} x_i + [U^H n]_i \quad (12)$$

$$C = \sum_{i=1}^r \log_2 \left( 1 + \frac{E_S \lambda_i}{N_t N_o} Y_i \right) \quad (13)$$

Maximum capacity can be find using, [7][8]

$$C = \max_{\sum_{i=1}^r \gamma_i = N_t} \sum_{i=1}^r \log_2 \left( 1 + \frac{1}{N_t} \frac{1}{N_o} Y_i \right) \quad (14)$$

The channel which contains higher SNR will be given more power, and more power increases the sum of data rates in all subchannels.  $\lambda_i$  indicates the strength of the signal of MIMO subchannel. If  $\lambda_i$  is higher more power allocated to the same subchannel.  $Y_i$  is optimum power allocated to given subchannel.

The power given to each subchannel is calculated using, [5][16].

$$Power \text{ allocated} = \frac{P_t + \sum_{i=1}^r \frac{1}{H_i}}{\sum \text{channels}} - \frac{1}{H_i} \quad (15)$$

Where  $P_t$  is the power of the MIMO model and channel matrix is represented by  $H$ .

By knowing channel statistics, the extra power is allocated at the transmitter side hence capacity can be increased according to the water filling algorithm.

### IV. MASSIVE MIMO

Massive MIMO model is described where BS is equipped with arrays of  $M$  antennas that receive data from  $N$  single antenna users where  $M \gg N$ . [17],[19],[20].

In the uplink the BS receives

$$y = \sqrt{p} G x + n \quad (16)$$

Where  $x = [x_1, x_2, \dots, x_N]^T$  is the vector of information symbols,  $x_N$  is transmitted by the single antenna  $N^{th}$  terminal. The  $G$  is the channel matrix between  $M$  antennas at the BS and  $N$  users are  $G \in \mathbb{C}^{M \times N}$ . The  $p$  is the normalized SNR of each user and  $n$  is the vector of additive white Gaussian noise with zero mean and variance 1.

The sum channel capacity of the channel model described above is, [6][13][14][15]

$$C_{sum} = \log_2 (\det(I_N + p G^H G)) \quad (17)$$

### V. LINEAR DETECTORS

Using Maximum Likelihood Detector (MLD) optimal performance can be achieved, but it will require complex signal processing methods at the BS. Linear processing techniques like ZF and MRC will give a suboptimal performance, and it will require less complex signal processing. Linear receivers perform well when the BS is equipped with a large number of antennas compared to the serving users. [6],[18],[21]

MRC and ZF detectors have been considered here. In this paper, we have simulated the SER for ZF and MRC only. Suboptimal low complexity Massive MIMO detection at BS combining is done using  $r=Ay$ .

For MRC:  $A=G^H$

(18)

For Zero Forcing:  $A=(G^H G)^{-1} G^H$

(19)

From (16) and (18), the received vector for MRC is given by

$$r=G^H(\sqrt{p}Gx+n) \tag{20}$$

It is assumed that users  $x_1, x_2, x_3, \dots, x_N$  are independent.

SINR for nth received element for MRC is given by

$$SINR_{MRC,n} = \frac{Signal\ Power}{Interference\ Power + Noise\ Power}$$

$$SINR_{MRC,n} = \frac{p \|g_n\|^4}{p \sum_{i \neq n}^N |g_n^H g_i|^2 + \sigma_n^2 \|g_n\|^2} \tag{21}$$

From (16) and (19), the received vector for ZF linear detector is

$$r = (G^H G)^{-1} G^H(\sqrt{p}Gx+n) \tag{22}$$

$$= \sqrt{p}x + (G^H G)^{-1} G^H n \tag{23}$$

It will able to suppress Multiuser interference. SINR for ZF is given by

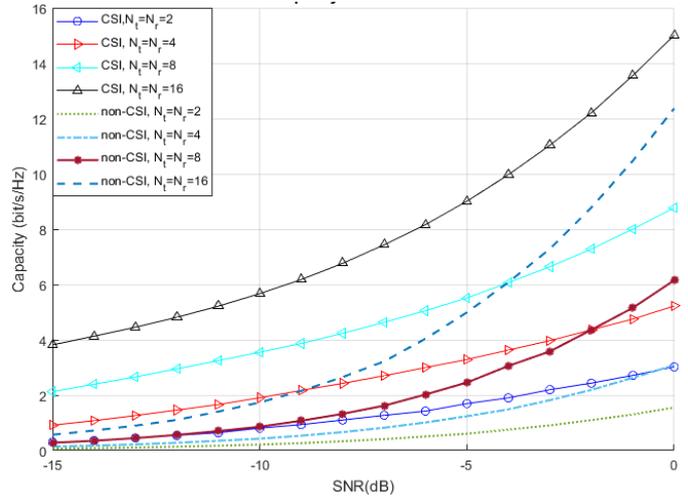
$$SINR_{ZF,n} = \frac{p}{[(G^H G)^{-1}]_{nn}} \tag{24}$$

**VI. SIMULATION RESULTS**

The results of the simulations are presented in this section. A comparison has been carried out with different size of arrays at transmitter and receiver side. At the end of this section comparison and analysis has been carried out for the linear detectors, ZF and MRC.

From Fig. 1 analysis has been carried out in Table- 1 for different size of antennas at transmitter and receiver side. It shows there is an improvement in the capacity if the size of the antenna has been increased. Due to water filling applied for known CSI it will give more capacity compared to unknown CSI for the same number of antenna combinations at transmitter and receiver side.

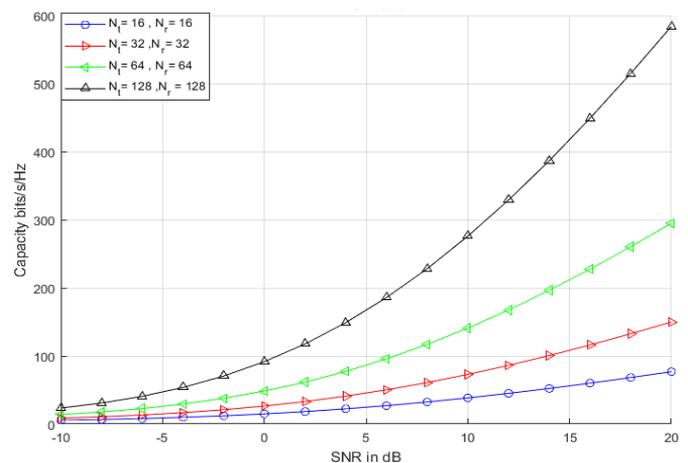
From Fig. 2 analysis has been done in Table 2 for different number of combination of antennas at transmitter and receiver side for known channel state information. There is a huge improvement in the capacity in terms of Bits/s/Hz if the size of antennas has been increased from the 16x16 to 128x128. At 20 dB SNR, 16x16 will give 75 Bits/s/Hz while 128x128 antennas will give 600 Bits/s/Hz.



**Fig. 1. Channel Capacity comparison of Non-CSI and known CSI for  $N_t=N_r=2, N_t=N_r=4, N_t=N_r=8, N_t=N_r=16$**

**Table- 1 Capacity comparison**

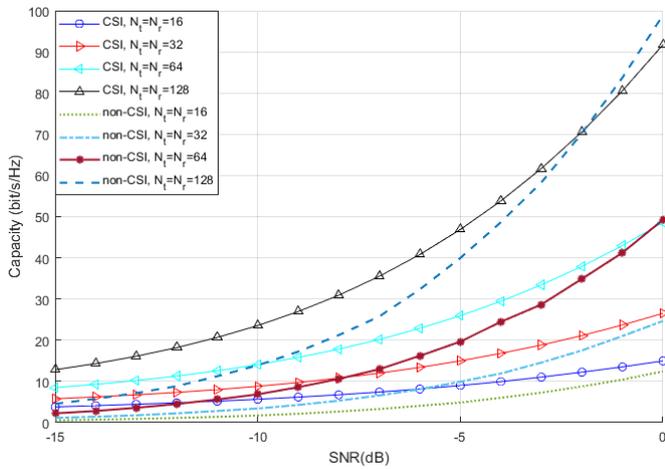
SNR in dB	Antenna size at Tx and Rx.	Capacity Bits/s/Hz
0	Non-CSI 2x2	≈2
0	CSI Known 2x2	≈3
0	Non-CSI 16x16	≈12
0	CSI 16x16	≈15



**Fig.2. Channel Capacity comparison of known CSI for  $N_t=N_r=16, N_t=N_r=32, N_t=N_r=64, N_t=N_r=128$**

**Table-2 Capacity comparison for different size of Antennas**

SNR in dB	Antenna size at Tx and Rx.	Capacity Bits/s/Hz
20	CSI 16x16	≈75
20	CSI 32x32	≈150
20	CSI 64x64	≈300
20	CSI 128x128	≈600

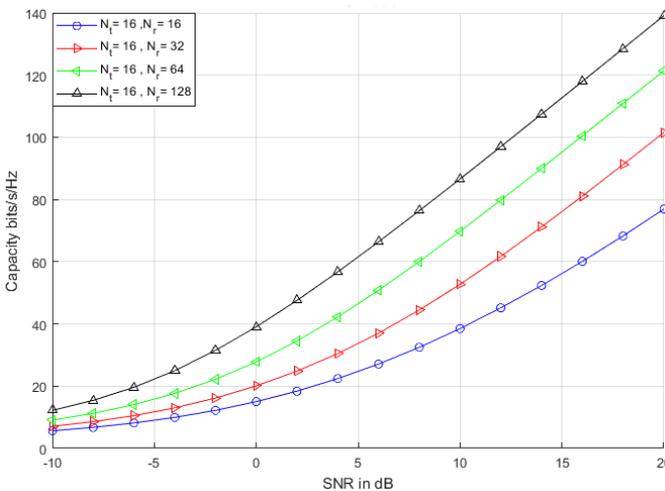


**Fig. 3. Channel Capacity comparison of Non-CSI and known CSI for  $N_t=N_r=16$ ,  $N_t=N_r=32$ ,  $N_t=N_r=64$ ,  $N_t=N_r=128$ .**

From Fig. 3 it can be observed that as the antenna size increases for known CSI and non-CSI there is an improvement in the capacity in Bits/s/Hz. Further, it has been observed that at very low SNR known CSI will give better result compared to unknown CSI as it can be increased with the help of water filling model. At higher SNR known CSI and non-CSI will provide similar results for a higher number of antennas at transmitter and receiver side.

It has been observed from Fig. 4 that for a fixed number of transmitter antennas and if the size of the receiver antenna has been increased at receiver side then capacity improvement will be there.

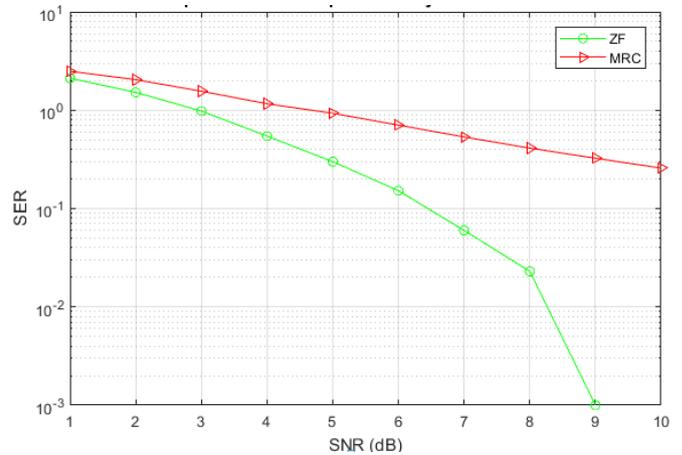
Linear receivers perform well when the BS contains a large number of antenna elements at the BS and is serving tens of mobile terminals (MTs) which are the case of Massive MIMO. The system parameters that were used in simulations of linear detection, ZF and MRC for Massive MIMO are shown in Table 3. For the comparison of linear detectors like ZF and MRC, hundreds of base station antennas and small number of single antenna user terminals have been taken for simulation.



**Fig. 4. Channel Capacity comparison with known CSI for  $N_t=16$  and  $N_r=16, 32, 64$  and  $128$**

**Table-3 Simulation Parameters For Linear Detectors In Massive MIMO**

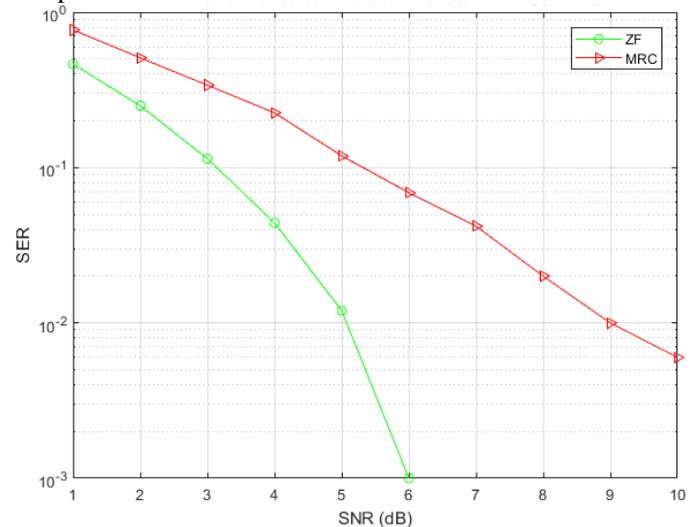
Description	Parameters
No. of Receive Antennas at BS =M	128,256,512
No. of Users =N	16
SNR in dB	1:10
Linear Detectors	MRC and ZF



**Fig. 5.: SER Performance Comparison for  $M=128$  Receive Antennas at Base station and  $N=16$  Users for ZF and MRC**

As shown in result of Fig. 5, Massive MIMO using the ZF and MRC techniques was compared, and clearly, there is an improvement in terms of SER when using ZF compared to MRC. It has been observed that the MRC performs worst in terms of error rate performance.

As per simulation result of Fig. 5 and Fig. 6 analysis has been carried out in table 4, and it shows that there is an improvement in the performance of SER when the number of antennas has been increased at BS from 128 to 256. Also, it has been shown that ZF will give good performance compared to MRC.

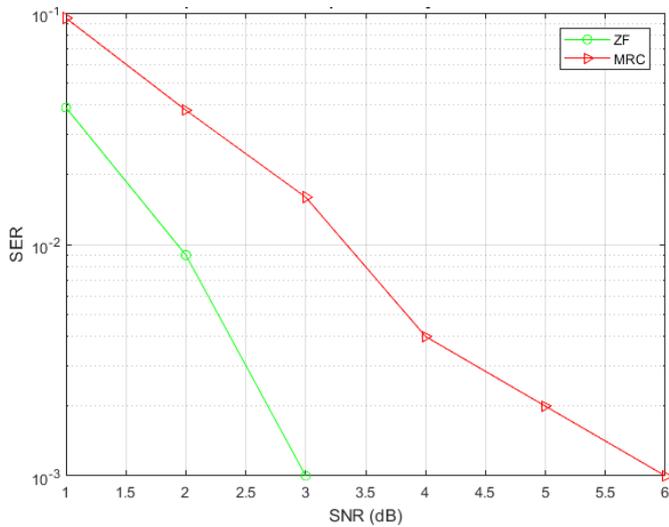


**Fig. 6.: SER Performance Comparison for  $M=256$  Receive Antennas at Base station and  $N=16$  Users for ZF and MRC**

**Table 4 Simulation results of MRC and ZF Linear Detectors**

SNR (dB)	M=128 Antennas at BS and N=16		M=256 Antennas at BS and N=16	
	SER of MRC	SER of ZF	SER of MRC	SER of ZF
1	2.492	2.118	0.769	0.464
2	2.047	1.521	0.509	0.25
3	1.561	0.981	0.339	0.114
4	1.164	0.546	0.225	0.044
5	0.931	0.3	0.119	0.012
6	0.706	0.152	0.069	0.001

From Table 4 it has been observed that at 6 dB SNR for M=128 antennas and N=16 users MRC will give 0.706 SER while for same SNR for M=256 antennas and N=16 users SER will be 0.069. At 6 dB SNR for M=128 and N=16 users will provide a 0.152 SER while for M=256 and N=16 users SER will be only 0.001 in case of ZF linear detection.



**Fig. 7.: SER Performance Comparison for M=512 Receive Antennas at Base station and N=16 Users for ZF and MRC**

**Table 5 MRC and ZF Linear Detectors comparison for M=512 Antennas at BS and N=16 users**

SNR(dB)	SER of MRC	SER of ZF
1	0.095	0.039
2	0.038	0.009
3	0.016	0.001

From Fig.7 and analysis as per table 5, for higher number of antennas like 512 antennas at BS for a small number of users then there is an improvement in the SER. It has been analyzed that at 1 dB SNR MRC will give a 0.095 SER while ZF will give a 0.039 SER.

**VII. CONCLUSION**

In this paper capacity comparison has been carried out for known CSI and unknown CSI in MIMO wireless communication model. We have compared capacity from a small number of antennas to hundreds of antennas at transmitter and receiver side. It was found that using higher number of antennas capacity improvement is possible for

known CSI and unknown CSI. Using water filling model we can increase the capacity at low SNR for known CSI. It was also observed that at higher SNR, known CSI and unknown CSI will give similar results for larger number of antennas.

In Massive MIMO systems MRC detector will give worst result compared to ZF. It was also concluded that increasing the antennas at the BS for small number of serving users then performance improvement is possible in terms of SER.

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## AUTHORS PROFILE



Haresh L. Judal is currently working as an assistant professor at Government Engineering College Patan, Gujarat, India. He has completed his B.E. (Electronics and Communication Engineering) from the North Gujarat University. He has completed his M.E. from L. D. College of Engineering Gujarat University Ahmedabad. Currently he is working in the field of MIMO and OFDM.



Kishor G. Maradia has completed his B.E. (Electronics and Communication Engineering) from the Bhavnagar University in 1992 and M.E. (Electronics and Communication) with specialization in the field of communication systems from L. D. College of Engineering, Gujarat in year 2003. He has completed his Ph.D. from M. S. University Baroda. He is a life member of IETE, CSI, LAA and other professional societies. He is guiding research students in the field of MIMO, OFDM, Optical communication and wireless communication.