# Fuzzy Graceful Labeling on the Double Fan Graphs and the Double Wheel Graphs

**R. Shanmugapriya, P.K. Hemalatha, M. Suba**

***Abstract:** A graph G admits a fuzzy graceful labeling and if all the vertex labelings are distinct then we can say G is a fuzzy vertex graceful graph. Here, we discuss the fuzzy vertex graceful labeling on certain classes of double fan graphs and double wheel graphs.*

***Keywords:** Fuzzy labeling, Graceful labeling, Fuzzy double fan graphs, Fuzzy double wheel graphs.*

## I. INTRODUCTION

In 20th century, Zadeh was introduced fuzzy in 1965. The fuzzy sets can be applied in the field of cluster analysis, neutral networks etc. Fuzzy relations on fuzzy sets which had better feature in making the fuzzy graph model were developed by Zadeh. Fuzzy basic ideas were introduced by Kauffmann in 1973. The Rosenfeld discussed the concepts of fuzzy graph and explained the fuzzy relations between fuzzy sets and the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya gave some remarks on fuzzy graphs.

Assigning the values to the vertices and edges of a graph is called labeling. A fuzzy graceful labelling graph $G=(\rho,\mu)$ if $\rho: V \to [0,1]$ and $\mu: V\text{x}V \to [0,1]$ is bijective such that $\mu(u,v) \leq \rho(u) \wedge \rho(v)$ for all $u,v \epsilon V$ where $\rho$ *and* $\mu$ are distinct. In graph theory, graph labeling are used in many more applications. Fuzzy vertex graceful labeling on fan graph and wheel graph were discussed by Jebesty and Vimala.

## II. DOUBLE FAN GRAPHS AND DOUBLE WHEEL GRAPHS

A Fan graph $F_n$ can be constructed from a wheel graph by deleting one edge on the n-cycle. A joining of two graphs $\overline{K_n} + P_m$ is called a fan graph $F_{n,m}$ where $\overline{K_n}$ has empty set of n vertices is and $P_m$ is the m vertices of a path graph . The case m=1corresponds to the usual fan graph, while m=2 corresponds to the double fan graph. A double fan graph $F_{n,m}$ with fuzzy labeling is called a fuzzy double fan graph is shown in figure 1.

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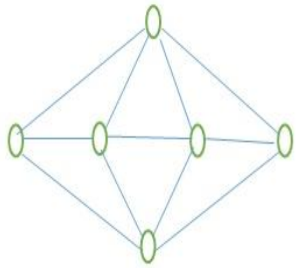

**Fig. 1 Double fan graph $F_{2,4}$**

A wheel graph of order n is obtained by joining a new vertex called 'Hub' to each vertex of a cycle graph of order n-1 and is denoted by $W_{1,n}$ $(n \geq 3)$.In a fuzzy wheel graph if all vertex values are distinct then it is said to be fuzzy vertex graceful labeling wheel graph.

A fuzzy wheel graph consists of two vertex sets V and U with |V| = 1and |U| > 1, such that $\mu(v, u_j) > 0, 1 \leq j \leq n-1$ and $\mu(u_j, u_{j+1}) > 0$ where $1 \leq j \leq n-2$.A double wheel graph $\mathbb{D}\omega_m$ of order n consist of $2C_n$+$K_1$. A double wheel graph with fuzzy labeling is called a fuzzy double wheel graph is show in figure 2.

Petrie and Smith applied various methods in constraint programming to graceful graphs and provided solution for the double wheel graceful graph problems. They showed no labeling $\mathbb{D}\omega_3$ is graceful. A fuzzy double wheel graph with labeling is called a fuzzy Double wheel graph. Fuzzy vertex graceful labelling on double fan graph and double wheel graph were discussed in [2].

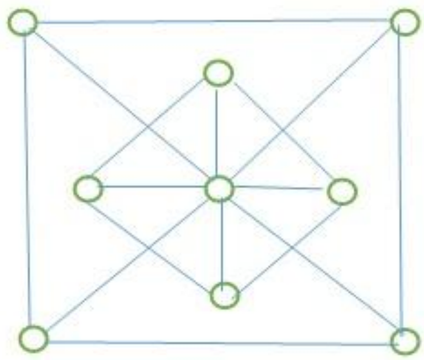

**Fig. 2 Double wheel graph $\mathbb{D}\omega_4$**

### Theorem 1

For the positive integer m, every fuzzy double fan graph $F_{2,m}$ is fuzzy vertex graceful fan graph.

### Proof

Consider the double fan graph $F_{2,m}$ the two vertices are $F_{v_1}$,$F_{v_2}$ and m contains $F_{v1'}, F_{v2'}, F_{v3'}, F_{v4'}, F_{v5'}, F_{v6'}$. A fuzzy double fan graph is vertex gracef



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$E\left(F_{v2},F_{(m+1)'}\right)-E\left(F_{v_2},F_{m'}\right)=E(F_{(m-1)'},F_{m'})$, where $m=1,2,3\ldots$ and
ul, then
$E\left(F_{v_1},F_{(m+1)'}\right)-E\left(F_{v_1},F_{m'}\right)=E(F_{m'},F_{(m+1)'})$, where $m=1,2,3,\ldots$
$V(F_{m'})=V\left(F_{v_1}\right)-E(F_{v_1},F_{m'})$or
$V(F_{m'})=V(F_{v2})-E(F_{v2},F_{m'})$

(i) If the value of the vertex $F_{v_1}$, $m=\frac{q-1}{1000}$, where q=$v_1,v_2,\ldots..v_m$.
Here, $E\left(F_{v_1},F_{v1'}\right)=0.0001$,
$E\left(F_{v_1},F_{v2'}\right)=0.0003=E\left(F_{v_1},F_{v2'}\right)+E\left(F_{v_1},F_{v1'}\right)$,
$E\left(F_{v_1},F_{v3'}\right)=0.0006=E\left(F_{v_1},F_{v1'}\right)+E(F_{v1'},F_{v3'})$,
$E\left(F_{v_1},F_{v4'}\right)=0.001=E\left(F_{v_1},F_{v3'}\right)+E(F_{v3'},F_{v4'})$,
$E\left(F_{v_1},F_{v5'}\right)=0.0015=E\left(F_{v_1},F_{v4'}\right)+E(F_{v4'},F_{v5'})$,
$E\left(F_{v_1},F_{v6'}\right)=0.0021=E\left(F_{v_1},F_{v5'}\right)+E(F_{v5'},F_{v6'})$.
In general,
$E\left(F_{v_1},F_{(m+1)'}\right)-E\left(F_{v_1},F_{m'}\right)=(0.0001)(m+1)$, where $m=1,2,\ldots$-----(a)
Similarly we can find,
$E(F_{v1'},F_{v2'})=(0.0001)2=0.0002$,
$E(F_{v2'},F_{v3'})=(0.0001)3=0.0003$,
$E(F_{v3'},F_{v4'})=(0.0001)4=0.0004$,
$E(F_{v4'},F_{v5'})=(0.0001)5=0.0005$,
$E(F_{v5'},F_{v6'})=(0.0001)6=0.0006$.
In general,
$$E\left(F_{m'},F_{(m+1)'}\right)=(0.0001)(m+1),$$
where $m=1,2,3,\ldots$-----(b)
Using above equations (a) and (b) we get
$E\left(F_{v_1},F_{(m+1)'}\right)-E\left(F_{v_1},F_{m'}\right)=E\left(F_{m'},F_{(m+1)'}\right)$, where $m=1,2,\ldots$
$V(F_{v1'})=0.0049=V\left(F_{v_1}\right)-E\left(F_{v_1},F_{v1'}\right)$,
$V(F_{v2'})=0.0047=V\left(F_{v_1}\right)-E\left(F_{v_1},F_{v2'}\right)$,
$V(F_{v3'})=0.0044=V\left(F_{v_1}\right)-E\left(F_{v_1},F_{v3'}\right)$,
$V(F_{v4'})=0.004=V\left(F_{v_1}\right)-E\left(F_{v_1},F_{v4'}\right)$,
$V(F_{v5'})=0.0035=V\left(F_{v_1}\right)-E\left(F_{v_1},F_{v5'}\right)$,
$V(F_{v6'})=0.0029=V\left(F_{v_1}\right)-E\left(F_{v_1},F_{v6'}\right)$.
In general,$V(F_m)=V\left(F_{v_1}\right)-E\left(F_{v_1},F_m\right)$.

(ii) If the value of the vertex $F_{v_2}$,$m=\frac{q+1}{1000}$, where $q=v_1,v_2,\ldots,v_m$
From the above cases we can find,
$E\left(F_{v_2},F_{v6'}\right)=0.0041$,
$E\left(F_{v_2},F_{v5'}\right)=0.0035$,
$E\left(F_{v_2},F_{v4'}\right)=0.003$,
$E\left(F_{v_2},F_{v3'}\right)=0.0026$,
$E\left(F_{v_2},F_{v2'}\right)=0.0023$,
$E\left(F_{v_2},F_{v1'}\right)=0.0021$.
In general,
$E\left(F_{v_2},F_{(m-1)'}\right)$-$E\left(F_{v_2},F_{m'}\right)=(0.0001)m$, where$m=1,2,3,\ldots$-----(a)
$E(F_{v1'},F_{v2'})=0.0002$,
$E(F_{v2'},F_{v3'})=0.0003$,
$E(F_{v3'},F_{v4'})=0.0004$,and so on.
$E\left(F_{(m-1)'},F_{m'}\right)=(0.0001)m$, where$m=1,2,3,\ldots$-----(b)
Using above equations, (b) in (a) we get
$E\left(F_{v_2},F_{m'}\right)-E\left(F_{v_2},F_{(m-1)'}\right)=E\left(F_{(m-1)'},F_{m'}\right)$
$V(F_{m'})=V\left(F_{v_2}\right)-V(F_{v_2},F_{m'})$
Note: If $m=\frac{q-1}{100}$, where q=$v_1,v_2,\ldots..,v_m$
Here,$E\left(F_{v_1},F_{v1'}\right)=0.001$,
$E\left(F_{v_1},F_{v2'}\right)=0.003=E\left(F_{v_1},F_{v1'}\right)+E(F_{v1'},F_{v2'})$,
$E\left(F_{v_1},F_{v3'}\right)=0.006=E\left(F_{v_1},F_{v2'}\right)+E(F_{v2'},F_{v3'})$,
$E\left(F_{v_1},F_{v4'}\right)=0.01=E\left(F_{v_1},F_{v3'}\right)+E(F_{v3'},F_{v4'})$, and so.
In general, $E\left(F_{v_1},F_{(m+1)'}\right)=E\left(F_{v_1},F_{m'}\right)+(m+10.001$, $m$=1,2,3,…
Similarly,
$E(F_{v1'},F_{v2'})=0.002$,
$E(F_{v2'},F_{v3'})=0.003$,
$E(F_{v3'},F_{v4'})=0.004$, and so on.
Therefore, $E\left(F_{m'},F_{(m+1)'}\right)=0.001(m+1)$, where $m=1,2,3,\ldots$
$E\left(F_{v_1},F_{(m+1)'}\right)-E\left(F_{v_1},F_{m'}\right)=E\left(F_{m'},F_{(m+1)'}\right)$, where $m=1,2,3,\ldots$
Since all the membership value of edges $E\left(F_{v_1},F_j\right)>0$ and $E\left(F_{v1'},F_j\right)>0$, $1\le j\le m$
and $E\left(F_jF_{j+1}\right)$>0 , $1\le j\le m-1$and $E(u,v)\le V(u)\wedge V(v)$.

Therefore all membership values of edges and vertices are distinct. The fuzzy vertex graceful labelling on double fan graph have been proved. Fuzzy vertex double fan graph $F_{2,6}$ is shown in figure 3.

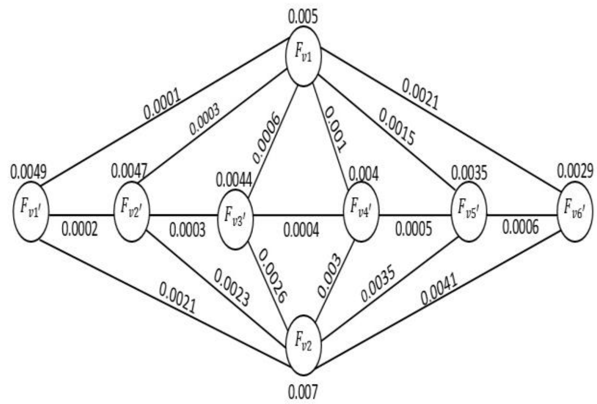


**Fig. 3 $\mathbb{D}_{2,6}$A fuzzy graceful labelling graph**

**Theorem 2**

For the positive integerm, the double wheel graph $\mathbb{D}\omega_m$admits fuzzy vertex graceful wheel graph.

**Proof**

In the double wheel graph, V is a central vertex, $u_j$ denotes the vertices in the inner and outer cycle.

Here $\rho: V \to [0,1]$ and $\rho: u_j \to [0,1]$ defined as follows

Inner cycle:

$\rho(u_j) = \rho(v) - E(v, u_j)$, where $E(v, u_j)$=(0.0001)j

$E(u_j, u_{j+1}) = E(v, u_j) - E(v, u_{j+1})$, $1 \le j \le m-1$.

Therefore,

$E(u_{m-1}, u_m)$= $E(v, u_{m-1}) - E(v, u_m)$

Outer cycle:

$\rho(u_{j'}) = \rho(v) - E(v, u_{j'})$, where $E(v, u_{j'})$=0.0001(j′ + m, 1≤j′≤m and

while$E(u_{m'}, u_{1'})$= $E(v, u_{m'}) - E(v, u_{1'}) - (0.0001)(m-2)$.

(i) If the value of the vertex$\frac{m+1}{1000}$

Here, when $\rho: V \to [0,1]$ and $\rho: u_j \to [0,1]$ as follows

Inner cycle:

$\rho(u_1) = \rho(v) - 0.0001$=0.0069,

$\rho(u_2) = \rho(v) - 0.0002$=0.0068,

$\rho(u_3) = \rho(v) - 0.0003$ =0.0067,

$\rho(u_4) = \rho(v) - 0.0004$ =0.0066,

$\rho(u_5) = \rho(v) - 0.0005$ =0.0065,

$\rho(u_6) = \rho(v) - 0.0006$ =0.0064.

In general,

$\rho(u_j) = \rho(v) - \mu(v, u_j)$, where $E(v, u_j) = (0.0001)j$,$1 \le j \le m$

also,

$E(u_1, u_2) = E(v, u_1) - E(v, u_2) = 0.0001$

$E(u_2, u_3) = E(v, u_2) - E(v, u_3) = 0.0001$ and so on.

In general,

$E(u_{m-1}, u_m) = E(v, u_{m-1}) - E(v, u_m)$, while

$E(u_m, u_1) = E(v, u_m) - E(v, u_1) - (0.0001)(m-2)$

Outer cycle:

$\rho(u_{j'}) = \rho(v) - (0.0001)(j' + m)$

$\rho(u_{j'}) = \rho(v) - E(v, u_{j'})$, where $E(v, u_{j'}) = 0.0001(j' + m)$

$\rho(u_{1'}) = \rho(v) - 0.0007$=0.0063,

$\rho(u_{2'}) = \rho(v) - 0.0008$=0.0062,

$\rho(u_{3'}) = \rho(v) - 0.0009$=0.0061,

$\rho(u_{4'}) = \rho(v) - 0.0010$=0.0060,

$\rho(u_{5'}) = \rho(v) - 0.0011$ =0.0059,

$\rho(u_{6'}) = \rho(v) - 0.0012$ =0.0058.

(ii) When $\rho(v)$ starts from $\frac{m-1}{1000}$

Here, when $\rho: V \to [0,1]$ and $\rho: u_j \to [0,1]$ as follows

Inner cycle:

From the above case, we can find the following values

$\rho(u_1) = 0.0049$,

$\rho(u_2) = 0.0048$,

$\rho(u_3) = 0.0047$,

$\rho(u_4) = 0.0046$,

$\rho(u_5) = 0.0045$,

$\rho(u_6) = 0.0044$.

In general,

$\rho(u_j) = \rho(v) - \mu(v, u_j)$, where $E(v, u_j) = (0.0001)j$, $1 \le j \le m$

In general, |

$E(u_{m-1}, u_m) = E(v, u_{m-1}) - E(v, u_m)$

While

$E(u_m, u_1) = E(v, u_m) - E(v, u_1) - (0.0001)(m-2)$

Outer cycle:

$\rho(u_{j'}) = \rho(v) - (0.0001)(j' + m)$

$\rho(u_{j'}) = \rho(v) - E(v, u_{j'})$, where $E(v, u_{j'}) = 0.0001(j' + m)$

Therefore, for any values of $\rho: V \to [0,1]$, the labeling of all vertices in the inner and the outer cycle $u_j$ and $u_{j'}$are distinct,$1 \le j = j' \le m$. By using the above condition the double wheel graph $\mathbb{D}\omega_m$is a fuzzy vertex graceful labeling.Fuzzy vertex double wheel graph $\mathbb{D}\omega_6$ is shown in figure 4.

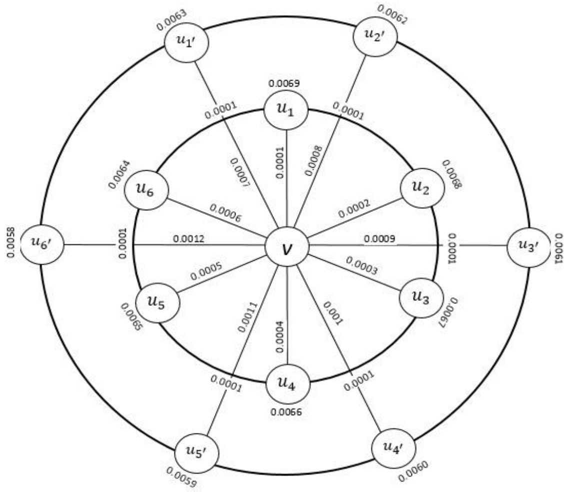


**Fig. 4 Fuzzy vertex graceful labelling on double wheel graph $\mathbb{D}\omega_6$**

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