α(γ,γ')(β,β') - Open, Closed Mappings in Topological Spaces

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Abstract: In this paper the concept of α(γ,γ')(β,β') -open, closed mappings have been introduced and some of its properties have been studied.

Keywords: α(γ,γ') -open set, α(γ,γ')(β,β') -open mapping, α(γ,γ')(β,β') -closed mapping, α(β,β')(γ,γ') -continuous mapping.

I. INTRODUCTION


In this article the α(γ,γ')(β,β') -open mappings has been introduced and its properties are analyzed. α(γ,γ')(β,β') -closed mappings, has been introduced and properties are discussed.

Notations: α(γ,γ') -open set → α -(γ,γ') -open set , τα(γ,γ') → τα -(γ,γ') → α(γ,γ') -open sets, XTS → (X,τ), YTS → (Y,τ), OS → open set , CS → closed set, OSs → open sets, CSs → closed sets, TS → topological space, CM → continuous mapping, OPM → open mapping, CL → closed mapping, C → continuous, M → mapping, NEIGH→ neighbourhood, INV-IMA inverse image, iff → if and only if, ima → image, impt → implies that, theex → there exists, such→such that, OP INJ → open injection, OP SUR → open surjection, INJ M → injection mapping, SUR M → surjection Mapping, subs→ sub set.

II. PRELIMINARIES

Theorem 2.1. Let { Aα :α∈J } be the family of α(γ,γ') -OS in XTS .Then ∪
α∈J Aα is also an α(γ,γ') -OS in XTS.

Definition 2.1. A M fM is called an α(γ,γ')(β,β') -CM iff for every α(β,β') -OS, E of YTS , fM -1(E)-the INV-IMA of E, is an α(γ,γ') -OS in XTS.

Definition 2.2. A M fM :XTS →YTS is an α(γ,γ')(β,β') -CM iff for each point e in XTS and each α(β,β') -NEIGH D of fM (e), there is an α(γ,γ') -NEIGH E of b such fM (E) ⊆ D.

Theorem 2.2. Let fM be a M. Then the statements mentioned below are equivalent:
(i) fM :XTS →YTS is an α(γ,γ')(β,β') -CM;
(ii) fM (τα(γ,γ') -cl(E)) ⊆ σα(β,β'), cl(fM(E)), for every subset E of XTS;
(iii) For every α(β,β') -CS, F of YTS , fM -1(F) is an α(γ,γ') -CS in XTS.

III. α(γ,γ')(β,β') -OPEN MAPPINGS

Definition 3.1. A M fM :XTS →YTS is assumed to be an α(γ,γ')(β,β') -OPM iff if for each α(γ,γ') -OS, H ∈ τα(γ,γ'), the ima fM (H) ∈ σα(β,β').

Example 3.1. Let XTS = {u, v, w} , YTS = {u, v, w} .

The operation β, β' on σ is given as Aβ = A U {u} and Aβ' = A U {u} for every A ∈ τ .

Definition 3.2. A M fM :XTS →YTS is an α(γ,γ')(β,β') -OPM and gM : YTS →ZTS is an α(β,β')(δ,δ') -OPM, then gM ∘ fM : XTS →ZTS is an α(γ,γ')(δ,δ') -OPM.

Proof. The proof follows from the Definition 3.1.

Theorem 3.2. A M fM :XTS →YTS is an α(γ,γ')(β,β') -OPM iff for each b ∈ XTS , and for every E ∈ τα(γ,γ'), such that b ∈ E , then a
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**Proof.** Let \(E\) be an \(\alpha_{(\gamma)}\) - OSM of \(b \in X_{TS}\). Then \(f_{M}(b) \subseteq f_{M}(E)\). Therefore \(f_{M}(E)\) is an \(\alpha_{(\nu)}\)-NEIGH of \(f_{M}(b)\) in \(Y_{TS}\). Then by Theorem 2.2 theore an \(\alpha_{(\gamma)}\)-open NEIGH, \(D \in \sigma_{\alpha_{(\gamma)}}\), such \(f_{M}(b) \subseteq f_{M}(E)\).

Conversely, let \(E \in \tau_{\alpha_{(\gamma)}}\) such that \(b \in E\). Then, thea a \(D \in \sigma_{\alpha_{(\gamma)}}\) such \(f_{M}(b) \subseteq f_{M}(E)\). Therefore \(f_{M}(E)\) is an \(\alpha_{(\nu)}\)-NEIGH of \(f_{M}(b)\) in \(Y_{TS}\) and this imph \(f_{M}(E) = U_{f_{M}(b)}(A)\). Then by Theorem 2.1 \(f_{M}(E)\) is an \(\alpha_{(\nu)}\)-OS in \(Y_{TS}\). Hence \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM.

**Theorem 3.3.** A M \(f_{M}: X_{TS} \rightarrow Y_{TS}\) is an \(\alpha_{(\gamma)}\)-OPM iff if for each \(b \in X_{TS}\), and for every \(\alpha_{(\gamma)}\)-NEIGH \(U\) of \(b \in X_{TS}\), thea an \(\alpha_{(\nu)}\)-NEIGH \(V\) of \(f_{M}(b)\) such \(V \subseteq f_{M}(U)\).

**Proof.** Let \(U\) be an \(\alpha_{(\gamma)}\)-NEIGH of \(b \in X\). Then by Definition 2.1 theex an \(\alpha_{(\gamma)}\)-OS, \(W\) such \(b \in W \subseteq U\).

This imph \(f_{M}(b) \subseteq f_{M}(W) \subseteq f_{M}(U)\). Since \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM, \(f_{M}(W)\) is an \(\alpha_{(\nu)}\)-OS. Hence \(V = f_{M}(W)\) is an \(\alpha_{(\nu)}\)-NEIGH of \(f_{M}(b)\) and \(V \subseteq f_{M}(U)\).

Conversely, let \(U \subseteq \tau_{\alpha_{(\gamma)}}\) and \(b \in U\). Then \(U\) is an \(\alpha_{(\gamma)}\)-NEIGH of \(b\) and hence, theex an \(\alpha_{(\nu)}\)-NEIGH \(V\) of \(f_{M}(b)\) such \(f_{M}(b) \subseteq V \subseteq f_{M}(U)\). That is, \(f_{M}(U)\) is an \(\alpha_{(\nu)}\)-NEIGH of \(f_{M}(b)\). Thus \(f_{M}(U)\) is an \(\alpha_{(\nu)}\)-NEIGH of each of its points. Therefore \(f_{M}(U)\) is an \(\alpha_{(\nu)}\)-OS. Hence \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM.

**Theorem 3.4.** A M \(f_{M}: X_{TS} \rightarrow Y_{TS}\) is an \(\alpha_{(\gamma)}\)-OPM iff \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM.

**Proof.** Let \(b \in \tau_{\alpha_{(\gamma)}}\) such \(b \in U \subseteq P\). So \(f_{M}(b) \subseteq f_{M}(U) \subseteq f_{M}(P)\). Since \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM, \(f_{M}(U)\) is an \(\alpha_{(\nu)}\)-OS in \(Y_{TS}\). Hence \(f_{M}(b) \in \sigma_{\alpha_{(\nu)}}\) - int \((f_{M}(P)) \subseteq \sigma_{\alpha_{(\nu)}}\) - int \((f_{M}(P))\), for all \(P \subseteq X_{TS}\).

**Theorem 3.5.** A M \(f_{M}: X_{TS} \rightarrow Y_{TS}\) is an \(\alpha_{(\gamma)}\)-OPM iff \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM.

**Proof.** Let \(Q\) be any subset of \(Y_{TS}\). Clearly \(\tau_{\alpha_{(\gamma)}}\) - int \((f_{M}(Q))\) is an \(\alpha_{(\gamma)}\)-OS in \(X_{TS}\). Also \(f_{M}\) is \(\alpha_{(\gamma)}\)-OPM and by Theorem 3.4, \(f_{M}\) is \(\alpha_{(\gamma)}\)-OPM.

This imph \(\tau_{\alpha_{(\gamma)}}\) - int \((f_{M}(Q)) \subseteq \sigma_{\alpha_{(\nu)}}\) - int \((f_{M}(Q))\). Hence \(\tau_{\alpha_{(\gamma)}}\) - int \((f_{M}(Q)) \subseteq f_{M}\) \(\tau_{\alpha_{(\gamma)}}\) - int \((f_{M}(Q))\). This imph \(\tau_{\alpha_{(\gamma)}}\) - int \((f_{M}(Q)) \subseteq f_{M}\) \(\tau_{\alpha_{(\gamma)}}\) - int \((f_{M}(Q))\), for all \(Q \subseteq Y_{TS}\).

Conversely, let \(D \subseteq Y_{TS}\) and hence, \(f_{M}\) is an \(\alpha_{(\gamma)}\)-OPM.
Theorem 3.7. Let \( f_M : (X_{TS}, \tau) \rightarrow (Y_{TS}, \sigma) \) and \( g_M : (Y_{TS}, \sigma) \rightarrow (Z_{TS}, \delta) \) be two Ms such that \( g_M \circ f_M : (X_{TS}, \tau) \rightarrow (Z_{TS}, \delta) \) is an \( \alpha_{(\gamma', \delta, \delta, \delta)}^\beta \) - CLM. Then

(i) If \( g_M \) is an \( \alpha_{(\beta, \beta, \delta, \delta)} \) - OP INJ then \( f_M \) is an \( \alpha_{(\gamma', \gamma', \beta, \beta)} \) - CM.

(ii) If \( f_M \) is an \( \alpha_{(\gamma', \gamma', \beta, \beta)} \) - OP SUR then \( g_M \) is an \( \alpha_{(\beta, \beta, \delta, \delta)} \) - CM.

Proof. (i) Let \( U \in \sigma_{(\alpha, \beta, \beta)} \). Since \( g_M \) is an \( \alpha_{(\beta, \beta, \delta, \delta)} \) - OPM, then \( g_M (U) \in \zeta_{(\alpha, \beta, \beta)} \). Since \( g_M \) is INJ and \( g_M \circ f_M \) is an \( \alpha_{(\gamma', \delta, \delta, \delta)} \) - CM, \( (g_M \circ f_M)^{-1} (g_M (U)) \) is an \( \alpha_{(\gamma', \delta, \delta, \delta)} \) - CM.

\( \Rightarrow (Mf, Mf) \rightarrow (\alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - CM.

\( \Rightarrow (Mf, Mf) \rightarrow (\alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - OPM. So \( f_M \) is an \( \alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - OPM and \( g_M \circ f_M \) is an \( \alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - OP INJ. Thus \( g_M \circ f_M \) is an \( \alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - CM.

\( \Rightarrow (Mf, Mf) \rightarrow (\alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - CM.

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\( \Rightarrow (Mf, Mf) \rightarrow (\alpha_{(\gamma', \gamma', \beta, \beta), \beta}) \) - CM.
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(ii) If \( Q \) is an \( \alpha(\beta, \beta') \) g-CS of \( Y_{TS} \), then the set \( f^{-1}_M(Q) \) is an \( \alpha_{(\gamma, \gamma')} \) g-CS.

**Proof.** (i) Let \( V \) be any \( \alpha(\beta, \beta') \)-OS in \( Y_{TS} \) such that \( f_M(P) \subseteq V \). By using Theorem 2.2, \( f^{-1}_M(V) \) is an \( \alpha_{(\gamma, \gamma')} \)-OS containing \( P \). By assumption we have \( \tau_{a(\beta, \beta')} = \text{cl}(A) \subseteq f^{-1}_M(V) \), so \( f_M(\tau_{a(\beta, \beta')} - \text{cl}(P)) \subseteq V \). Since \( f_M \) is an \( \alpha_{(\gamma, \gamma')} \)-CS containing \( f_M(P) \), then \( f_M(\tau_{a(\beta, \beta')} - \text{cl}(P)) = f_M(\tau_{a(\beta, \beta')} - \text{cl}(A)) \subseteq V \). Hence \( f_M(P) \) is an \( \alpha(\beta, \beta') \) g-CS.

(ii) Let \( U \) be an \( \alpha_{(\gamma, \gamma')} \)-OS of \( X_{TS} \) such that \( f^{-1}_M(Q) \subseteq U \) for any sub \( B \) in \( Y_{TS} \). Put \( F = \tau_{a(\beta, \beta')} - \text{cl}(f^{-1}_M(Q)) \cap (X_{TS} - U) \). It follows from the Remark 3.14 (ii) and Theorem 3.21 [3] that \( F \) is an \( \alpha_{(\gamma, \gamma')} \)-CS in \( X_{TS} \). Since \( f_M \) is an \( \alpha_{(\gamma, \gamma')} \)-CLM, \( f_M(F) \) is an \( \alpha_{(\gamma, \gamma')} \)-CS in \( Y_{TS} \). By Theorem 5.5 [3] and Theorem 2.2 (ii) and from the following inclusion, \( f_M(F) \subseteq \sigma_{a(\beta, \beta')} - \text{cl}(Q) - Q \), it is obtained that \( f_M(F) = \phi \), and hence \( F = \phi \). This implies \( \tau_{a(\beta, \beta')} - \text{cl}(f^{-1}_M(Q)) \subseteq U \).

Therefore \( f^{-1}_M(Q) \) is an \( \alpha_{(\gamma, \gamma')} \) g-CS.

**Theorem 4.5.** Let \( f_M : X_{TS} \rightarrow Y_{TS} \) is an \( \alpha_{(\gamma, \gamma')} \)-CS and \( \alpha_{(\gamma, \gamma')} \)-CLM. Then

(i) If \( f_M \) is an INJ M and \( Y_{TS} \) is an \( \alpha_{(\beta, \beta')} - T_\alpha \) then \( X_{TS} \) is an \( \alpha_{(\gamma, \gamma')} - T_\alpha \) space.

(ii) If \( f_M \) is a SUR M and \( X_{TS} \) is an \( \alpha_{(\gamma, \gamma')} - T_\alpha \) then \( Y_{TS} \) is an \( \alpha_{(\beta, \beta')} - T_\alpha \) space.

**Proof.** (i) Let \( P \) be an \( \alpha_{(\beta, \beta')} \) g-CS in \( X_{TS} \). Then by Theorem 4.4 (i) \( f_M(P) \) is an \( \alpha_{(\beta, \beta')} \) g-CS. Therefore by assumption \( P \) is an \( \alpha_{(\gamma, \gamma')} \)-CS in \( X_{TS} \). Therefore \( X_{TS} \) is an \( \alpha_{(\gamma, \gamma')} - T_\alpha \) space.

(ii) Let \( Q \) be an \( \alpha_{(\beta, \beta')} \) g-CS in \( Y_{TS} \). Then it follows from the Theorem 4.4 (ii) and the assumption that \( f^{-1}_M(Q) \) is an \( \alpha_{(\gamma, \gamma')} \)-CS. Hence \( f_M \) is an \( \alpha_{(\gamma, \gamma')} \)-CLM, implies that \( f_M(f^{-1}_M(Q)) = Q \) is an \( \alpha_{(\beta, \beta')} \) g-CS in \( Y_{TS} \). Therefore \( Y_{TS} \) is an \( \alpha_{(\beta, \beta')} - T_\alpha \) space.

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