$\alpha_{(\gamma,\gamma')(\beta,\beta')}$-Open, Closed Mappings in Topological Spaces

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Abstract: In this paper the concept of $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -open, closed mappings have been introduced and some of its properties have been studied.

Keywords: $\alpha_{(\gamma,\gamma')}$ -open set, $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -open mapping, $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -closed mapping, $\alpha_{(\beta,\beta')(\gamma,\gamma')}$ -continuous mapping.

I. INTRODUCTION

O.Njastad [7], Kasahara [4,5], Ogata [8,9] and Kalaivani, Sai Sundara Krishnan[1,2,3] discussed about the α -open sets, operation on topological spaces, $\tau_{\alpha-\gamma}$ and α -(γ,γ') -open sets. Maki & Noiri [6], Umehara, Maki& Noiri[10] and Umehara[11] analyzed the concept of Bioperations in topological spaces.

In this article the $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -open mappings has been introduced and its properties are analyzed. $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -closed mappings, has been introduced and properties are discussed.

Notations: $\alpha_{(\gamma,\gamma')}$ -open set $\rightarrow \alpha$ -(γ , γ')-open set , $\tau_{\alpha_{(\gamma,\gamma')}} \rightarrow \tau_{\alpha-(\gamma,\gamma')}$ $\rightarrow \alpha_{(\gamma,\gamma')}$ -open sets, $X_{TS} \rightarrow (X,\tau)$, $Y_{TS} \rightarrow (Y,\tau)$,OS $\rightarrow$open set ,CS $\rightarrow$closed set, OSs$\rightarrow$open sets, CSs$\rightarrow$closed sets, TS $\rightarrow$ topological space, CM $\rightarrow$continuous mapping, OPM$\rightarrow$open mapping, CLM $\rightarrow$closed mapping, C$\rightarrow$continuous, M$\rightarrow$mapping, NEIGH$\rightarrow$ neighbourhood, INV- IMA inverse image, iff $\rightarrow$if and only if, ima$\rightarrow$ image, impth $\rightarrow$implies that, theex $\rightarrow$there exists, sucth-such that, OP INJ $\rightarrow$open injection, OP SUR $\rightarrow$open surjection ,INJ M$\rightarrow$ injection mapping, SUR M$\rightarrow$ surjection Mapping, subs$\rightarrow$ sub set.

II. PRELIMINARIES

Theorem 2.1. Let $\{A_\alpha : \alpha \in J\}$ be the family of $\alpha_{(\gamma,\gamma')}$ -OSs in X_{TS} .Then $\bigcup_{\alpha\in J} A_\alpha$ is also an $\alpha_{(\gamma,\gamma')}$ -OS in X_{TS}

.**Definition 2.1.** A M f_M is called an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CM iff if for every $\alpha_{(\beta,\beta')}$ -OS, E of Y_{TS} , $f^{-1}{}_M$ (E) -the INV- IMA of E, is an $\alpha_{(\gamma,\gamma')}$ -OS in X_{TS} .

Definition 2.2. A M $f_M : X_{TS} \rightarrow Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CM iff for each point e in X_{TS} and each $\alpha_{(\beta,\beta')}$ -NEIGH D of f_M (e), there is an $\alpha_{(\gamma,\gamma')}$ -NEIGH E of b sucth f_M $(E) \subseteq D$.

Theorem 2.2. Let f_M be a M. Then the statements mentioned below are equivalent:

(i) $f_M : X_{TS} \rightarrow Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CM;

(ii) f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -cl(E)) $\subseteq$ $\sigma_{\alpha_{(\beta,\beta')}}$ -cl(f_M (E)),for every subset E of X_{TS} ;

(iii) For every $\alpha_{(\beta,\beta')}$ -CS, F of Y_{TS} , $f^{-1}{}_M$ (F) is an $\alpha_{(\gamma,\gamma')}$ -CS in X_{TS} .

III. $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -OPEN MAPPINGS

Definition 3.1. A M $f_M : X_{TS} \rightarrow Y_{TS}$ is assumed to be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -OPM iff if for each $\alpha_{(\gamma,\gamma')}$ -OS, $H \in \tau_{\alpha_{(\gamma,\gamma')}}$, the ima f_M (H) $\in \sigma_{\alpha_{(\beta,\beta')}}$

Example 3.1. Let $X_{TS} = \{u,v,w\}$, $\tau = \{\phi, X_{TS}, \{u\}, \{w\}, \{u,v\}, \{u,w\}\}$ $Y_{TS} = \{u,v,w\}$, $\sigma = \{\phi, Y_{TS}, \{u\}, \{v\}, \{u,v\}, \{u,w\}\}$.

The operation γ,γ' on τ is given as A^γ = A U $\{u\}$ and $A^{\gamma'} = \begin{cases} A \text{ if } A = \{u\} \\ A\text{U}\{w\} \text{ if } A \neq \{u\} \end{cases}$ for every $A \in \tau$.

The operation β,β' on σ is given as $A^\beta = \begin{cases} A \text{ if } v \notin A \\ cl(A) \text{ if } v \in A \end{cases}$ and $A^{\beta'} = \begin{cases} A \text{ if } v \in A \\ cl(A) \text{ if } v \notin A \end{cases}$ for every $A \in \sigma$.

The M f_M is given as f_M (u) = v, f_M (v) = w and f_M (w) = u. Then the ima of every $\alpha_{(\gamma,\gamma')}$ -OS is an $\alpha_{(\beta,\beta')}$ - OS under the M f_M . Hence f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - OPM.

Theorem 3.1. If $f_M : X_{TS} \rightarrow Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -OPM and $g_M : (Y_{TS},\sigma) \rightarrow (Z_{TS},\delta)$ is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ -OPM, then $g_M \circ f_M : (X_{TS},\tau) \rightarrow (Z_{TS},\delta)$ is an $\alpha_{(\gamma,\gamma')(\delta,\delta')}$ - OPM.

Proof. The proof follows from the Definition 3.1.

Theorem 3.2. A M $f_M : X_{TS} \rightarrow Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -OPM iff for each b $\in X_{TS}$, and for every $E \in \tau_{\alpha_{(\gamma,\gamma')}}$ such that b $\in$ E , theex a

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$D \in \sigma_{\alpha_{(\beta,\beta')}}$ sucth f_M (b) $\in D$ and $D \subseteq f_M$ (E).

Proof. Let E be an $\alpha_{(\gamma,\gamma')}$- OS of b $\in X_{TS}$. Then f_M (b) $\in f_M$ (E). Therefore f_M (E) is an $\alpha_{(\beta,\beta')}$-NEIGH of f_M (b) in Y_{TS} . Then by Theorem 2.2 theex an $\alpha_{(\gamma,\gamma')}$-open NEIGH, $D \in \sigma_{\alpha_{(\beta,\beta')}}$ sucth f_M (b) $\in D \subseteq f_M$ (E).

Conversely, let $E \in \tau_{\alpha_{(\gamma,\gamma')}}$ such that b $\in E$. Then , theex a $D \in \sigma_{\alpha_{(\beta,\beta')}}$ sucth f_M (b) $\in D \subseteq f_M$ (E) . Therefore f_M (E) is an $\alpha_{(\beta,\beta')}$- NEIGH of f_M (b) in Y_{TS} and this impth f_M (E) = $\mathrm{U}_{f(x)\in f(A)} D$. Then by Theorem 2.1 f_M (E) is an $\alpha_{(\beta,\beta')}$-OS in Y_{TS} . Hence f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM.

Theorem 3.3. A M $f_M : X_{TS} \to Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM iff if for each b $\in X_{TS}$, and for every $\alpha_{(\gamma,\gamma')}$- NEIGH U of b $\in X_{TS}$,theex an $\alpha_{(\beta,\beta')}$-NEIGH V of f_M (b) sucth $V \subseteq f_M$ (U).

Proof. Let U be an $\alpha_{(\gamma,\gamma')}$- NEIGH of b $\in X$. Then by Definition 2.1 theex an $\alpha_{(\gamma,\gamma')}$-OS, W sucth b $\in W \subseteq U$. This impth f_M (b) $\in f_M$ (W) $\subseteq f_M$ (U). Since f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM, f_M (W) is an $\alpha_{(\beta,\beta')}$- OS. Hence V = f_M (W) is an $\alpha_{(\beta,\beta')}$-NEIGH of f_M (b) and $V \subseteq f_M$ (U).

Conversely, let $U \in \tau_{\alpha_{(\gamma,\gamma')}}$ and b $\in U$. Then U is an $\alpha_{(\gamma,\gamma')}$- NEIGH of b and hence, theex an $\alpha_{(\beta,\beta')}$- NEIGH V of f_M (b) sucth f_M (b) $\in V \subseteq f_M$ (U). That is, f_M (U) is an $\alpha_{(\beta,\beta')}$- NEIGH of f_M (b). Thus f_M (U) is an $\alpha_{(\beta,\beta')}$- NEIGH of each of its points. Therefore f_M (U) is an $\alpha_{(\beta,\beta')}$-OS. Hence f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$- OPM.

Theorem 3.4. A M $f_M : X_{TS} \to Y_{TS}$ is an $\alpha_{(\gamma,\gamma')}$-OPM iff f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int (P)) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (P)) , for all $P \subseteq X_{TS}$.

Proof. Let b $\in \tau_{\alpha_{(\gamma,\gamma')}}$ -int (P) . Then theex $U \in \tau_{\alpha_{(\gamma,\gamma')}}$ sucth b $\in U \subseteq P$. So f_M (b) $\subseteq f_M$ (U) $\subseteq f_M$ (P). Since f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - OPM, f_M (U) is an $\alpha_{(\beta,\beta')}$-OS in Y_{TS} . Hence f_M (b) $\in \sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (P)). Thus f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int (P)) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (P)).

Conversely, let $U \in \tau_{\alpha_{(\gamma,\gamma')}}$ and hence f_M (U) = f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int (U)) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int ((f_M (U)) $\subseteq f_M$ (U) or f_M (U) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (U)) $\subseteq f_M$ (U) . This impth f_M (U) is an $\alpha_{(\beta,\beta')}$-OS. Thus f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM.

Theorem 3.5. A M $f_M : X_{TS} \to Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM iff $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)) $\subseteq f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int (Q)), for all $Q \subseteq Y_{TS}$.

Proof. Let Q be any subset of Y_{TS} . Clearly $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)) is an $\alpha_{(\gamma,\gamma')}$-OS in X_{TS} . Also f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)) $\subseteq f_M$ ($f^{-1}{}_M$ (Q)) $\subseteq Q$. Since f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - OPM and by Theorem 3.4 , f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int (Q). Hence $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)) $\subseteq f^{-1}{}_M$ (f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)))). This impth $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (Q)) $\subseteq f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int (Q)) for all $Q \subseteq Y_{TS}$.

Conversely , let $P \subseteq X_{TS}$, we obtain $\tau_{\alpha_{(\gamma,\gamma')}}$ -int (P) $\subseteq \tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (f (P)) $\subseteq f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int(f_M (P))). This impth that f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int (P)) $\subseteq f_M$ ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (f_M (P)))) $\subseteq f_M$ ($f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (P)))) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (P)). Consequently f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -int (P)) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$ -int (f_M (P)), for all $P \subseteq X_{TS}$. By Theorem 3.4 f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - OPM.

Theorem 3.6. A M $f_M : X_{TS} \to Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM if f $f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -cl (D)) $\subseteq \tau_{\alpha_{(\gamma,\gamma')}}$ -cl ($f^{-1}{}_M$ (D)), for all $D \subseteq Y_{TS}$.

Proof. Let D be any subset of Y_{TS} . By Theorem 3.5 $\tau_{\alpha_{(\gamma,\gamma')}}$ -int($f^{-1}{}_M$ ($Y_{TS} - D$)) $\subseteq f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int ($Y_{TS} - D$)). Then $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($X_{TS} - f^{-1}{}_M$ (D)) $\subseteq f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int ($Y_{TS} - D$)). As $\sigma_{\alpha_{(\beta,\beta')}}$ -int (D) = $Y_{TS} - \sigma_{\alpha_{(\beta,\beta')}}$ - cl ($Y_{TS} - D$), therefore $X_{TS} - \tau_{\alpha_{(\gamma,\gamma')}}$ - cl ($f^{-1}{}_M$ (D)) $\subseteq f^{-1}{}_M$ ($Y_{TS} - \sigma_{\alpha_{(\beta,\beta')}}$ -cl (D)) or $X_{TS} - \tau_{\alpha_{(\gamma,\gamma')}}$ - cl ($f^{-1}{}_M$ (D)) $\subseteq X_{TS} - f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ - cl (D)). Hence $f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -cl (D)) $\subseteq \tau_{\alpha_{(\gamma,\gamma')}}$ -cl ($f^{-1}{}_M$ (D)).

Conversely , let $D \subseteq Y_{TS}$ and hence, $f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ - cl ($Y_{TS} - D$)) $\subseteq \tau_{\alpha_{(\gamma,\gamma')}}$ -cl ($f^{-1}{}_M$ ($Y_{TS} - D$)). Then $X_{TS} - \tau_{\alpha_{(\gamma,\gamma')}}$ -cl ($f^{-1}{}_M$ ($Y_{TS} - D$)) $\subseteq X_{TS} - f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ - cl ($Y_{TS} - D$)). Hence $X_{TS} - \tau_{\alpha_{(\gamma,\gamma')}}$ -cl ($X_{TS} - f^{-1}{}_M$ (D)) $\subseteq f^{-1}{}_M$ ($Y_{TS} - \sigma_{\alpha_{(\beta,\beta')}}$ - cl ($Y_{TS} - D$)). This gives that $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($f^{-1}{}_M$ (D)) $\subseteq f^{-1}{}_M$ ($\sigma_{\alpha_{(\beta,\beta')}}$ -int (D)). Using Theorem 3.5, it follows that f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-OPM.

Theorem 3.7. Let $f_M:(X_{TS},\tau)\to(Y_{TS},\sigma)$ and $g_M:(Y_{TS},\sigma)\to(Z_{TS},\delta)$ be two Ms such that $g_M \circ f_M:(X_{TS},\tau)\to(Z_{TS},\delta)$ is an $\alpha_{(\gamma,\gamma')(\delta,\delta')}$ - CM. Then
(i) If g_M is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ -OP INJ then f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CM.
(ii) If f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -OP SUR then g_M is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ - CM.

Proof. (i) Let $U \in \sigma_{\alpha_{(\beta,\beta')}}$ .Since g_M is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ -OPM, then g_M $(U)\in \zeta_{\alpha-\delta}$. Since g_M is INJ and $g_M \circ f_M$ is an $\alpha_{(\gamma,\gamma')(\delta,\delta')}$ -CM, $(g_M \circ f_M)^{-1}$ (g_M (U)) = ($f_M^{-1} \circ g_M^{-1}$) (g_M (U)) = f^{-1}_M (g^{-1}_M (g_M (U)) = f^{-1}_M (U) is an $\alpha_{(\gamma,\gamma')}$ - OPM in X_{TS}. This proves that f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CM.

(ii) Let $V \in \varsigma_{\alpha_{(\delta,\delta')}}$. Since $g_M \circ f_M$ is an $\alpha_{(\gamma,\gamma')(\delta,\delta')}$ - CM, then $(g_M \circ f_M)^{-1}$ $(V)\in \tau_{\alpha_{(\gamma,\gamma')}}$ .Also f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - OPM, so f_M $((g_M \circ f_M)^{-1}$ $(V))$ is an $\alpha_{(\beta,\beta')}$ - OS in Y_{TS}. Since f_M is SUR, $(f_M \circ (g_M \circ f_M)^{-1})$ (V) = $(f_M \circ (f_M^{-1} \circ g_M^{-1}))$ (V) = $((f_M \circ f_M^{-1}) \circ g_M^{-1})$ (V) = g^{-1}_M (V). It follows that g^{-1}_M $(V) \in \sigma_{\alpha_{(\beta,\beta')}}$. This proves that g_M is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ -CM.

IV. $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CLOSED MAPPINGS

Definition 4.1. A M $f_M:X_{TS}\to Y_{TS}$ is defined to be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CLM, iff the ima f_M (D) is an $\alpha_{(\beta,\beta')}$ -CS for each $\alpha_{(\gamma,\gamma')}$ - C subs D of X_{TS}.

Example 4.1. Let $X_{TS}=\{g,h,i\}$, $\tau=\{\phi,X_{TS},\{g\},\{i\},\{g,h\},\{g,i\}\}$ $Y_{TS}=\{g,h,i\}$, $\sigma=\{\phi,Y_{TS},\{g\},\{h\},\{g,h\}\}$.

The operation γ,γ' on τ is defined as A^{γ} = cl(A) and $A^{\gamma'}$ =int(cl(A)) for every $A\in\tau$.

The operation β,β' on σ is defined as $A^{\beta}=\begin{cases} A \text{ if } h\notin A \\ cl(A) \text{ if } h\in A\end{cases}$ and $A^{\beta'}=\begin{cases} A \text{ if } h\in A \\ cl(A) \text{ if } h\notin A\end{cases}$ for every $A\in\sigma$.

The M f_M is given as f_M (a) = a, f_M (b) = b and f_M (c) = b. Then the ima of every $\alpha_{(\gamma,\gamma')}$ - CS is an $\alpha_{(\beta,\beta')}$ - CS under f_M . Hence f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CLM.

Theorem 4.1. Let $f_M:X_{TS}\to Y_{TS}$ be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CLM, then the statements mentioned below hold good.
(i) if $g_M:(Y_{TS},\sigma)\to(Z_{TS},\delta)$ is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ - CLM, then $g_M \circ f_M:(X_{TS},\tau)\to(Z_{TS},\delta)$ is an $\alpha_{(\gamma,\gamma')(\delta,\delta')}$ - CLM.
(ii) $\sigma_{\alpha_{(\beta,\beta')}}$ -cl (f_M (P))$\subseteq$ f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -cl (P)), for every subset P of X_{TS} .
(iii) $\sigma_{(\beta,\beta')}$ -cl ($\sigma_{(\beta,\beta')}$ -int ($\sigma_{(\beta,\beta')}$ -cl (f_M (P)))) $\subseteq$ f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -cl (P)), for every subset P of X_{TS} .
(iv) for each subset Q of Y_{TS} and each $\alpha_{(\gamma,\gamma')}$ - OS, P in X_{TS} containing f^{-1}_M (B), theex an $\alpha_{(\beta,\beta')}$ -OS, W in Y_{TS} containing Q sucth f^{-1}_M $(W)\subseteq P$.

Proof. The proofs are analogous to the proof of the Theorems 3.1, 3.2, 3.4, 3.5 and 3.6.

Theorem 4.2. Let $f_M:X_{TS}\to Y_{TS}$ be a bijective M. Then the below mentioned statements are equivalent:
(i) f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CLM.
(ii) f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - OPM.
(iii) f^{-1}_M is an $\alpha_{(\beta,\beta')(\gamma,\gamma')}$ -CM.

Proof. (i)$\Longrightarrow$ (ii) The proof follows from the Definitions 3.1 and 4.1.

(ii) $\Longrightarrow$ (iii) Let P be an $\alpha_{(\gamma,\gamma')}$ - CS in X_{TS} . Then $\tau_{\alpha_{(\gamma,\gamma')}}$ -cl (P) = P . By (ii) and by Theorem 3.5 , f^{-1}_M ($\sigma_{\alpha_{(\beta,\beta')}}$ -cl (f_M (P))) $\subseteq$ $\tau_{\alpha_{(\gamma,\gamma')}}$ -cl (f^{-1}_M (f_M (P))) impth $\sigma_{\alpha_{(\beta,\beta')}}$ -cl (f_M (P)) $\subseteq$ f_M ($\tau_{\alpha_{(\gamma,\gamma')}}$ -cl (P)). Thus $\sigma_{\alpha_{(\beta,\beta')}}$ -cl ($(f_M^{-1})^{-1}$ (P)) $\subseteq$ $(f_M^{-1})^{-1}$ (P), for every subs P of X_{TS} , it follows that f^{-1}_M is an $\alpha_{(\beta,\beta')(\gamma,\gamma')}$ - CM.

(iii)$\Longrightarrow$ (i) Let P be an $\alpha_{(\gamma,\gamma')}$ -CS in X_{TS} . Then $X_{TS}-P$ is an $\alpha_{(\gamma,\gamma')}$ - OS in X_{TS} . Since f^{-1}_M is an $\alpha_{(\beta,\beta')(\gamma,\gamma')}$ -CM, $(f_M^{-1})^{-1}$ ($X_{TS}-P$) is an $\alpha_{(\beta,\beta')}$ - OS in Y_{TS} . But $(f_M^{-1})^{-1}$ ($X_{TS}-P$) = f_M ($X_{TS}-P$) = $Y_{TS}-f_M$ (P). Thus f_M (P) is an $\alpha_{(\beta,\beta')}$ -CS in Y_{TS} . This proves that f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CLM.

Definition 4.2. Let $id_M:\tau\to P(X_{TS})$ be the identity operation. A M $f_M:(X_{TS},\tau)\to(Y_{TS},\sigma)$ is said to be an $\alpha_{id(\beta,\beta')}$ - CLM if for any α -CS, F of X_{TS} , f_M (F) is an $\alpha_{(\beta,\beta')}$ -CS in Y_{TS} .

Theorem 4.3. If f_M is a bijective M and $f_M^{-1}:(Y_{TS},\sigma)\to(X_{TS},\tau)$ is an $\alpha_{(\beta,\beta')id}$ -CM, then f_M is an $\alpha_{id(\beta,\beta')}$ -CM.

Proof. The proof follows from the Definitions 4.1, 4.2.

Theorem 4.4. Suppose that f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CM. Then
(i) If P is an $\alpha_{(\gamma,\gamma')}$ g -CS in X_{TS} , then the ima f_M (P) is an $\alpha_{(\beta,\beta')}$ g - CS.

(ii) If Q is an $\alpha_{(\beta,\beta')}$ g - CS of Y_{TS}, then the set f^{-1}_M (Q) is an $\alpha_{(\gamma,\gamma')}$ g -CS.

Proof. (i) Let V be any $\alpha_{(\beta,\beta')}$ -OS in Y_{TS} sucth f_M $(P) \subseteq V$. By using Theorem 2.2, f^{-1}_M (V) is an $\alpha_{(\gamma,\gamma')}$ -OS containing P. By assumption we have $\tau_{\alpha_{(\gamma,\gamma')}}$ - cl $(A) \subseteq f^{-1}_M$ (V), so f_M $(\tau_{\alpha_{(\gamma,\gamma')}}$ -cl $(P))) \subseteq V$. Since f_M is $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - C, f_M $(\tau_{\alpha_{(\gamma,\gamma')}}$ - cl $(P))$ is an $\alpha_{(\beta,\beta')}$ -CS containing f_M (P), impth $\sigma_{\alpha_{(\beta,\beta')}}$ -cl $(f_M$ $(P)) \subseteq$ $\sigma_{\alpha_{(\beta,\beta')}}$ -cl$(f_M$ $(\tau_{\alpha_{(\gamma,\gamma')}}$ - cl$(P))) = f_M$ $(\tau_{\alpha_{(\gamma,\gamma')}}$ - cl$(A)) \subseteq$ V. Hence f_M (P) is an $\alpha_{(\beta,\beta')}$ g -CS.

(ii) Let U be an $\alpha_{(\gamma,\gamma')}$ - OS of X_{TS} such that f^{-1}_M $(Q) \subseteq U$ for any subs B in Y_{TS}. Put F = $\tau_{\alpha_{(\gamma,\gamma')}}$ -cl $(f^{-1}_M$ $(Q))$ I $(X_{TS} - U)$. It follows from the Remark 3.14 (ii) and Theorem 3.21[3],that F is an $\alpha_{(\gamma,\gamma')}$ -CS in X_{TS}. Since f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CLM, f_M (F) is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CS in Y_{TS}. By Theorem 5.5 [3] and Theorem 2.2 (ii) and from the following inclusion, f_M $(F) \subseteq$ $\sigma_{\alpha_{(\beta,\beta')}}$ -cl $(Q) - Q$, it is obtained that f_M $(F) = \phi$, and hence $F = \phi$. This impth $\tau_{\alpha_{(\gamma,\gamma')}}$ -cl $(f^{-1}_M$ $(Q)) \subseteq U$. Therefore f^{-1}_M (Q) is an $\alpha_{(\gamma,\gamma')}$ g -CS.

Theorem 4.5. Let $f_M : X_{TS} \to Y_{TS}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -C and $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - CLM. Then

(i) If f_M is an INJ M and Y_{TS} is an $\alpha_{(\beta,\beta')}$ - $T_{\frac{1}{2}}$ then X_{TS} is an $\alpha_{(\gamma,\gamma')}$ - $T_{\frac{1}{2}}$ space.

(ii) If f_M is a SUR M and X_{TS} is an $\alpha_{(\gamma,\gamma')}$ - $T_{\frac{1}{2}}$ then Y_{TS} is an $\alpha_{(\beta,\beta')}$ - $T_{\frac{1}{2}}$ space.

Proof. (i) Let P be an $\alpha_{(\gamma,\gamma')}$ g -CS in X_{TS}. Then by Theorem 4.4 (i) f_M (P) is an $\alpha_{(\beta,\beta')}$ g - CS. Therefore by assumption P is an $\alpha_{(\gamma,\gamma')}$ -CS in X_{TS}. Therefore X_{TS} is an $\alpha_{(\gamma,\gamma')}$ - $T_{\frac{1}{2}}$ space.

(ii) Let Q be an $\alpha_{(\beta,\beta')}$ g -CS in Y_{TS}. Then it follows from the Theorem 4.4 (ii) and the assumption that f^{-1}_M (Q) is an $\alpha_{(\gamma,\gamma')}$ -CS. Hence f_M is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -CLM, implies that f_M $(f^{-1}_M$ $(Q))$ = Q is an $\alpha_{(\beta,\beta')}$ -CS in Y_{TS}. Therefore Y_{TS} is an $\alpha_{(\beta,\beta')}$ - $T_{\frac{1}{2}}$ space.

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