Standard Few Results on Graphs Difference Cordial Labeling and Its Applications

C. Sujatha, A. Manickam

***Abstract:** The admittance of difference cordial labelling by a graph is called a difference cordial graph. In this research paper, the difference cordial labelling behaviour of $P_n^2 \cdot K_1$, $P_n^2 \cdot 2K_1$ $P_n^2 \cdot K_2$, corona of the book B_p with K_1, K_2 and $2K_1$. is examined. In this research article application part, finally we conclude that this paper will be useful in the field of coding theory, astronomy, circuit design and communication networks etc. An over view and new ideas have been proposed here.*

***Keywords:** Book B_p, Cordial Labelling, Graph P_n^2.*

I. INTRODUCTION

In this article, only the undirected and simple graph was taken. For (p,q) ,let $G=(V,E)$ be a graph. The number of edges of *G* is known as the size of *G*and the number of vertices of *G* is called the order of *G*. S. Sathish Nayanan,R. Pronraj, R. Kala have presented difference cordial labelling [4]. In [4] difference cordial tagging behaviour of many graphs like cycle, path, complete graph, bistar, wheel, complete bipartite graph, some more typical graphs were examined. In [2, 5], difference cordiality of $G \odot P_n$, $G \cdot mK_1 (m=1,2,3)$ where *G* is either tree or unicucle, crown $C_n \odot K_1$, comb $P_n \odot K_1, P_n \odot C_m, C_n \odot C_m$, $W_n \cdot K_2, W_n \cdot 2K_1, L_n \cdot K_1$, $L_n \odot 2K_1, L_n \odot K_2$ have been examined. We notice the difference cordial labelling behaviour of few more corona graphs like $P_n^2 \odot K_1, P_n^2 \odot 2K_1, P_n^2 \odot 2K_1$, corona of the book B_p with K_1, K_2 and $2K_1$. Let us consider x as any real number. Then x stands for the smallest integer equal or greater than x and the illustration x stands for the largest integer less than or equal to x. Definitions and terms that are not defined here are used in Harary'ssense [3].

II. DIFFERENCE CORDIAL LABELLING

Definition 2.1

Let's assume $\emptyset$ be a map from $V(G)$ to $(1,2,\ldots,p)$. Let *G* be a (p,q) graph. For every edge uv, allot the label $|f(u)-f(v)|$.f is called difference cordial labelling if $|e_\phi(0)-e_\phi(1)| \le 1$ and $\emptyset$ is 1-1,where $e_\phi(0)$ and $e_\phi(1)$ signify the number of edges not labelled with 1 and labelled with 1 respectively. Difference cordial graph is a graph with a variance cordial labelling. The following descriptions are used for the succeeding units: The book B_p is the graph $S_p \times P_2$ where through $p+1$ vertices, S_p is the star.The graph P_n^2 is attained from the path P_n by totalling edges that joins all vertices u and v with $d(u,v)=2$.

Theorem 2.2

Let *G* be a (p,q) graph. If *G* is any one of the succeeding, then $G \odot pK_n (p=1,2)$ is difference cordial.

a. Let C_n be the cycle $x_1x_2\ldots x_nx_1$. Let's assume that*G*is a graph with $V(G)=V(C_n)\cup\{z_i : 1\le i\le q\}$ $E(G)=E(C_n)\cup\{x_iz_i, x_{i+1(\mathrm{mod}\, n)}\ z_i : 1\le i\le q\}$

b. The book B_p.

Proof:

Case (i): $p=1$.

Sub case (a): Let C_n be the cycle $x_1x_2\ldots x_nx_1$.

Let's assume*G*as a graph with $V(G)=V(C_n)\cup\{z_i : 1\le i\le q\}$ and $E(G)=E(C_n)\cup\{x_iz_i, x_{i+1(\mathrm{mod}\ n)}\ z_i : 1\le i\le q\}$

Let $V(G\odot K_1)=V(G)\{d_i,u_i : 1\le i\le q\}$ and $E(G\odot K_1)=E(G)\{x_id_i, z_iu_i : 1\le i\le q\}$.

Describe a map $\emptyset$ with domain $V(G\odot K_1)$ and co domain $\{1,2,3,\ldots 4q\}$ as below:

$\phi(x_i)=4i-1\ 1\le i\le q$, $\phi(d_i)=4i\ 1\le i\le q$,

$\phi(z_i)=4i+2\ 1\le i\le \left\lceil \frac{q-1}{2} \right\rceil$

Revised Manuscript Received on July 10, 2019.

C.Sujatha, Principal, Professor of Mathematics, Marudu Pandiyar College, Vallam Post, Thanjavur–613 403 (Tamil Nadu), India.

Dr. A. Manickam, Associate Professor of Mathematics, Vel Tech Rangarajan Dr.Sagunthala R&D Institute of Science and Technology, (Deemed to be University), No.42.Avadi-Veltech road, Avadi, Chennai-600 062.Tamilnadu, India. Email: dramanickam@veltech.edu.in, manickammaths2011@gmail.com

$$\phi\left(z\left[\frac{q-1}{2}\right]+i\right)=4\left[\frac{q-1}{2}\right]+4i+1 \quad 1\le i\le\left[\frac{q-1}{2}\right]$$

$$\phi(u_i)=4i+1 \quad 1\le i\le\left[\frac{q-1}{2}\right]$$

$$\phi\left(u\left[\frac{q-1}{2}\right]+i\right)=4\left[\frac{q-1}{2}\right]+4i+2 \quad 1\le i\le\left[\frac{q-1}{2}\right].$$

$\phi(z_n)=2$ and $\phi(u_n)=1$.

The following table (i) displays that Ø is a difference cordial labeling.

Table (i)

Nature of n	$e_\phi(0)$	$e_\phi(1)$
$q\equiv 0 (\text{mod } 2)$	$\frac{5q}{2}$	$\frac{5q}{2}$
$q\equiv 1 (\text{mod } 2)$	$\frac{5q-1}{2}$	$\frac{5q+1}{2}$

Sub case (b): The book B_p.

The book B_p is the graph $S_p \times P_2$ where S_p is a star with $p+1$ vertices.

Let $V(B_p)=\{x,y,x_i,y_i:1\le i\le p\}$ and

$E(B_p)=\{xy,xx_i,yy_i,x_iy_i:1\le i\le p\}$.

Let $V(B_p\cdot K_1)=V(B_p)\cup\{r,s,z_iu_i:1\le i\le p\}$ and

$E(B_p\cdot K_1)=E(B_p)\cup\{xr,ys,x_iz_i,y_iu_i:1\le i\le p\}$.

The size and order of $B_p\cdot K_1$ are $5p+3$ and $4p+4$ respectively. $\phi:V(B_p\cdot K_1)\to\{1,2...4p+4\}$

$$\phi(x)=2,\phi(y)=3,\phi(r)=1,\phi(s)=4,$$

$$\phi(y_i)=4i+3, 1\le i\le p,$$

$$\phi(u_i)=4i+4, 1\le i\le p,$$

$$\phi(x_i)=4i-1 \quad 1\le i\le q,\ \phi(y_i)=4i \quad 1\le i\le q,$$

$$\phi(x_i)=4i+2 \quad 1\le i\le\left[\frac{p-2}{2}\right] \quad \phi(z_i)=4i$$

$$1\le i\le\left[\frac{p-2}{2}\right]$$

$$\phi\left(x\left[\frac{p-2}{2}\right]+i\right)=4\left[\frac{p-2}{2}\right]+4i+1 \quad 1\le i\le\left[\frac{p+2}{2}\right]$$

$$\phi\left(z\left[\frac{p-2}{2}\right]+i\right)=4\left[\frac{p-2}{2}\right]+4i+2 \quad 1\le i\le\left[\frac{p+2}{2}\right].$$

The next table (ii) displays that Ø is a difference cordial labelling of $B_p\cdot K_1$.

Table (ii)

Nature of q	$e_\phi(0)$	$e_\phi(1)$
$p\equiv 0 (\text{mod } 2)$	$\frac{5p+2}{2}$	$\frac{5p+4}{2}$
$p\equiv 1 (\text{mod } 2)$	$\frac{5p+3}{2}$	$\frac{5p+3}{2}$

In figure (i), the difference cordial labelling of $B_6 \odot K_1$ is provided.

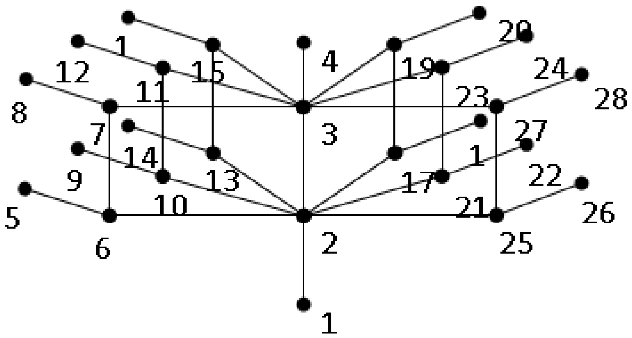

Figure (i)

Theorem 2.3

Let G be a (p,q) graph. Then $G\odot K_2$ is difference cordial, if G is any one of the below.

(a) Let C_q be the cycle $x_1,x_2,....x_qx_1$.
Let G be a graph with $V(G)=V(C_q)\cup\{z_i:1\le i\le q\}$ and $E(G)=E(C_q)\cup\{x_iz_i,x_{i+1(\text{mod } q)}\ z_i:1\le i\le q\}$.

(b) The book B_p.

(c) The graph P_q^2.

Proof:

Case (a)

Let C_q be the cycle $x_1x_2...x_qx_1$.

Let G be a graph with $V(G)=V(C_q)\cup\{z_i:1\le i\le q\}$ and $E(G)=E(C_q)\cup\{x_iz_i,x_{i+1(\text{mod } n)}\ z_i:1\le i\le q\}$.

Let $V(G\odot K_2)=V(G)\cup\{d_i,d_i',u_i,u_i':1\le i\le q\}$ and

$E(G\odot K_2)=E(G)\cup\{x_id_i,x_id_i',d_id_i',z_iu_i,z_iu_i',u_iu_i':1\le i\le q\}$.

The size and order of $G\odot K_2$ are $9q$ and $6q$ respectively.

Explain an injective map ϕ from the vertices $\phi:V(G)\to G\odot K_2$ to the set $\{1,2,3,...6q\}$ by

$\phi(g_i)=6i-3$, $1\le i\le\left[\frac{q}{2}\right]$

$\phi(d_i)=6i-4$, $1\le i\le\left[\frac{q}{2}\right]$

$\phi(d_i')=6i-5, 1\le i\le\left[\frac{q}{2}\right]$

$\phi(z_i)=6i-2$, $1\le i\le q$, $\phi(u_i)=6i$, $1\le i\le q$, $\phi(u_i')=6i-1, 1\le i\le q$

$\phi(g_{\left[\frac{q}{2}\right]+i})=6\left[\frac{q}{2}\right]+6i-5$, $1\le i\le\left[\frac{q}{2}\right]$,

$\phi(g'_{\left[\frac{q}{2}\right]+i})=6\left[\frac{q}{2}\right]+6i-3, 1\le i\le\left[\frac{q}{2}\right]$

$\phi(d'_{\left[\frac{q}{2}\right]+i})=6\left[\frac{q}{2}\right]+6i-4, 1\le i\le\left[\frac{q}{2}\right]$

The belowtable (iii) displays that ϕ is a difference cordial labeling of $G \circledast K_2$.

Table (iii)

Nature of n	$e_\phi(0)$	$e_\phi(1)$
$q\equiv 0 \pmod 2$	$\frac{9q}{2}$	$\frac{9q}{2}$
$q\equiv 1 \pmod 2$	$\frac{9q-1}{2}$	$\frac{9q+1}{2}$

Case (b): The book B_p.

Let

$V(B_p\cdot K_2)=V(B_p)\cup\{y_1,y_2,z_1,z_2,w_i,w_i',x_i,x_i':1\le 1\le p\}$

and $E(B_p\cdot K_1)=$

$E(B_p)\cup\{uy_1,uy_2,y_1y_2,vz_1,vz_2,z_1z_2,u_iw_i,u_iw_i',w_iw_i',v_ix_i,v_ix_i,x_ix_i':1\le i\le p\}$.

The size and order of $B_p\cdot 2K_1$ are $9p+7$ and $6p+6$ respectively. State an injective map $\emptyset$ from the vertices of $B_p\cdot 2K_1$ to the set $\{1,2,3,\ldots 6p+6)$ as below:

$\phi(u)=3,\ \phi(v)=4,\ \phi(y_1)=2,\ \phi(y_2)=1,$

$\phi(z_1)=6,\ \phi(z_2)=5,$

$\phi(u_1)=6i+3$

, $1\le i\le\left[\frac{p}{2}\right]-1$ $\phi(w_1)=6i+2$, $1\le i\le\left[\frac{p}{2}\right]-1$

$\phi(v_1)=6i+4, 1\le i\le\left[\frac{p}{2}\right]-1$

$\phi(x_1)=6i+6$, $1\le i\le\left[\frac{p}{2}\right]-1$ $\phi(w_1')=6i+1$

$1\le i\le\left[\frac{p}{2}\right]-1$ $e_\phi(1)=2p+3, 1\le i\le\left[\frac{p}{2}\right]-1$

$\phi\left(u_{\left[\frac{p}{2}\right]-1+i}\right)=6\left[\frac{p}{2}\right]+6i-5$

$1\le i\le\left[\frac{p}{2}\right]+1$ $\phi\left(w_{\left[\frac{p}{2}\right]-1+i}\right)=6\left[\frac{p}{2}\right]+6i-4$

$1\le i\le\left[\frac{p}{2}\right]+1$

$\phi\left(v_{\left[\frac{p}{2}\right]-1+i}\right)=6\left[\frac{p}{2}\right]+6i-2$

$1\le i\le\left[\frac{p}{2}\right]+1$ $\phi\left(x_{\left[\frac{p}{2}\right]-1+i}\right)=6\left[\frac{p}{2}\right]+6i-1$

$1\le i\le\left[\frac{p}{2}\right]+1$

$\phi\left(w'_{\left[\frac{p}{2}\right]-1+i}\right)=6\left[\frac{p}{2}\right]+6i-3$

$1\le i\le\left[\frac{p}{2}\right]+1$ $\phi\left(x'_{\left[\frac{p}{2}\right]-1+i}\right)=6\left[\frac{p}{2}\right]+6i$

$1\le i\le\left[\frac{p}{2}\right]+1$

The below table (iv) displays that $\emptyset$is a difference cordial labeling of $B_p\cdot K_2$.

Table (iv)

Nature of n	$d_f(0)$	$d_f(1)$
$n\equiv 0 \pmod 2$	$\frac{9m+6}{2}$	$\frac{9m+8}{2}$
$n\equiv 1 \pmod 2$	$\frac{9m+7}{2}$	$\frac{9m+7}{2}$

Case (c): The graph P_q^2

Let $V(P_q^2 \not\leqslant K_2) = V(P_q^2) \cup \{d_i, z_i : 1 \le 1 \le q\}$ and

$$E(P_q^2 \not\leqslant K_2) = E(P_q^2) \cup \{x_i d_i, x_i z_i, d_i z_i : 1 \le i \le q\}.$$

The number of edges and vertices in $P_q^2 \not\leqslant K_2$ are $5q-3$ and $3q$ respectively. Describe a map $\phi : V(P_n^2 \not\leqslant K_2) \to \{1,2,...3q\}$ as below.

$$\phi(g_{2i-1}) = 6i-3 \quad , \quad 1 \le i \le \left[\frac{q-2}{2}\right]$$

$$\phi(z_{2i-1}) = 6i-5 \quad 1 \le i \le \left[\frac{q-2}{2}\right]$$

$$\phi(d_{2i-1}) = 6i-4 \quad 1 \le i \le \left[\frac{q-2}{2}\right]$$

$$\phi(g_{2i}) = 6i-2 \quad 1 \le i \le \left[\frac{q-2}{2}\right]$$

$$\phi(z_{2i}) = 6i-1, 1 \le i \le \left[\frac{q-2}{2}\right] \quad \phi(d_{2i}) = 6i$$

$$1 \le i \le \left[\frac{q-2}{2}\right]$$

$$\phi\left(g_{2\left[\frac{n}{2}\right]-1}\right) = 6\left[\frac{q}{2}\right] - 5,$$

$$\phi\left(z_{2\left[\frac{n}{2}\right]-1}\right) = 6\left[\frac{q}{2}\right] - 3,$$

$$\phi\left(w_{2\left[\frac{n}{2}\right]}\right) = 6\left[\frac{q}{2}\right], \ \phi\left(v_{2\left[\frac{n}{2}\right]}\right) = 6\left[\frac{q}{2}\right] - 1 \text{ and}$$

when q is odd, $\phi(g_n) = 3q-2$ and $\phi(z_n) = 3q$. The below table (v) displays that $\emptyset$ is a difference cordial labeling of $P_q^2 \cdot K_2$.

Table (v)

Nature of n	$e_\phi(0)$	$e_\phi(1)$
$q \equiv 0 (\text{mod } 2)$	$\frac{5q-4}{2}$	$\frac{5q-2}{2}$
$q \equiv 1 (\text{mod } 2)$	$\frac{5q-3}{2}$	$\frac{5q-3}{2}$

In figure (ii), the difference cordial labelling of $P_q^2 \cdot K_2$ is provided.

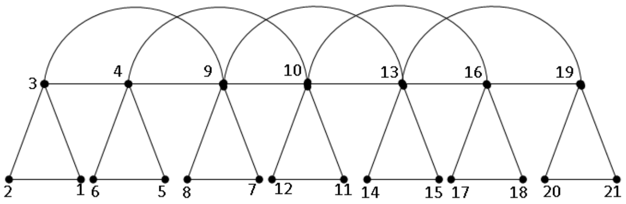

Figure (ii)

Theorem 2.3

$(K_2 + pK_1) \cdot 2K_1$ is difference cordial.

Proof:

Let $V(K_2 + pK_1) = \{x, y, z_i : 1 \le i \le p\}$ and $E(K_2 + pK_1) = \{xy, xz_i, yz_i : 1 \le i \le p\}$ and

$V((K_2 + pK_1) \cdot 2K_1) = V(K_2 + pK_1)$ $\cup \{x_1, x_2, y_1, y_2, u_i, u_i : 1 \le i \le p\}$ and

$E((K_2 + pK_1) \cdot 2K_1) = E(K_2 + pK_1)$ $\cup \{xx_1, xx_2, yy_1, yy_2, z_i u_i, z_i z_i : 1 \le i \le p\}$.

Evidently, the number of vertices and edges in $(K_2 + pK_1) \cdot 2K_1$ are $3p+6$ and $4p+5$ respectively.

Define an injective map from the vertices of $(K_2 + pK_1) \cdot 2K_1$ to the set $\{1,2,...3p+6\}$ by

$\phi(x) = 3p+2, \phi(x_1) = 3p+1, \quad \phi(x_2) = 3p+3,$

$\phi(v) = 3p+4, \quad \phi(y_1) = 3p+5, \quad \phi(y_2) = 3p+6,$

$\phi(z_i) = 3i-1, \ 1 \le i \le p \quad \phi(u_i) = 3i-2, \ 1 \le i \le p$

$\phi(u_i') = 3i, \ 1 \le i \le p$

Since, $e_\phi(1) = 2p+3$ and $e_\phi(0) = 2p+2$ $\emptyset$ is a difference cordial labeling of $(K_2 + pK_1) \cdot 2K_1$.

3. Applications of Graph Labelling In Networks of Communication

The Graph Theory field plays an important role in several fields. One of the significant parts in graph theory is Graph Labelling used in numerous applications like coding theory, astronomy, radar, X-ray crystallography, circuit design, and communication network addressing data base procedure. This articleprovides an impression of labelling of graphs in heterogeneous fields to some degree but mostly focuses on the communication networks. Communication network has two kinds 'Wireless communication' and 'Wired communication'. Day by day wireless networks have been established to comfort message between any two systems, outcomes more effective communication. This article also discovered the role of labelling in increasing the value of this channel task progression in communication networks. Several study papers based on graph labelling have been identified and observed its practice towards communication networks. This article discourses how the idea of graph labelling can be useful to network addressing, network security, social networks and channel assignment process. An impression and new thoughts have been projected here [1, 6, 7].

III. CONCLUSION

We examine the difference cordial labelling behaviour of $P_n^2 \not\subset K_1$, $P_n^2 \not\subset 2K_1$ $P_n^2 \not\subset K_2$, corona of the book B_m with K_1, K_2 and $2K_1$, lastly we complete that this article will be very beneficial in the engineering field and its applications has been discussed clearly and comprehended in this research paper.

CONFLICT OF INTEREST

There is no conflict of interest to affirm this publication.

ACKNOWLEDGEMENT

We would like to express our sincere thanks to the Editor-in-Chief for this valued recommendation towards improvisation of the paper.

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AUTHORS PROFILE

Dr. C.Sujatha a well known academician has taken the charge as principal of Marudupandiyar College, Thanjavur, Since December 2018. She had obtained her doctoral degree from Bharathidasan University Tiruchirappalli, which is ranked among the top world universities. She is a Mathematician by profession and specialized in Graph theory and applications. She has more than 15 years of teaching and 7 years of research experience. She has published many research articles in reputed in International Journals and conferences and also organized many Conference, Seminar and Workshops. She has the vision of fostering academic learning and research among students and research scholars for the development of innovative concepts and products to the betterment of society. She firmly believes that quality education can only lead to innovations and new product developments. She her wide and rich administrative research and international exposure to achieve the vision and Build Institutions.

Dr. A. Manickam, Associate Professor in the Department of Mathematics has 10 years of teaching and research experience, including 8 years of research experience in the field if Mathematical Modeling and Graph theory and Applications. He has received both his B.Sc., Degree in Mathematics, M.Sc., Degree from Bharathidasan University, B.Ed., Degree from Tamil Nadu Teachers Education University and M.Phil., Ph.D from the Bharathidasan University. He has published over 35 research publications in referred International Journals and in proceedings of international conferences. He is a recognized supervisor of Bharathidasan University. His current research work includes, Mathematical modeling deal with Differential Equations. He has organized 05 International conference, seminar and workshops. He has acting as a reviewer for 06 International peer reviewed Journals. (Synergistically activities).