

# Optimal Sequence in General Flow Shop with Job Block Criteria using Fuzzy Index Technique

Arti Tyagi, Ashima Kalra, Cheshta Kashyap

**Abstract:** Here, we are finding a novel methodology to solve a problem of scheduling of general flow shop where proceeding time of job is indeterminate. The parameters required to solve such problem was considered to be in triangular fuzzy number. The concept of job block concept has been introduced to understand relative interference of one job with other. The novelty of this method lies in the section that it will not convert the fuzzy processing time into classical numbers to figure out the near optimal sequence of jobs. The method has been made clearer by giving a numerical example to demonstrate the purposed technique.

**Index Terms:** Triangular fuzzy number (TFN), Fuzzy processing time, Flow shop, Fuzzy ranking, Location index etc.

## I. INTRODUCTION

Scheduling is very crucial in a production method or in any multifarious system. It is required in variety of domains such as institutes, hospitals, airlines, trains, company etc. The scheduling techniques can also act as a decision maker in various cases such as; the duration for which operators remain idle to utilize that time for some other task; in industry or in business can be utilized in a cost-effective way. In flow shop scheduling, our objective is to form a series of jobs i.e. to process machines in a pre-defined order or to find some well-defined criteria; such as minimization of make span or total time elapsed, production cost, rental cost of machines, mean flow time, idle time or satisfaction level of element maker etc.

A Heuristic procedure was first introduced by [1] Johnson for flow shop scheduling in two and restrictive 3 stage problems to reduce the total elapsed time for completion of all the jobs. [2] Gupta J.N.D. and Dudek had done an experimental study inclusive of measurement of its performance in flow shop scheduling. The study endorses the utilization of blend of measures like make span and total flow time in order to minimize the cost of scheduling. [3] Maggu and Dass gave the idea of equivalent job for a job block giving priority of one job over another in production scheduling. [4] Singh T.P. added various practical concepts in scheduling theory making problems to be more significant and wider. But all these researchers have assumed the processing time and set up time of jobs, deterministic in nature.

In literature there were varieties of fuzzy methods for the problem of flow shop scheduling. [5] MaccMahon and Lee were first to introduce the method of job sequencing with fuzzy

processing time. [6] Ishibuchi et al and [7] Hong et al. offered a new triangular fuzzy Johnson algorithm. [8] S. Reza Hejari et al then proposed an enhanced version of McMahon & Lee algorithm. Most of the authors have applied [9] Yager's formulae to defuzzify the processing time in their study. In this paper we have apply the fuzziness indexing technique in a general m stage flow shop scheduling with triangular fuzzy parameters. The results appear fairly good. The idea of corresponding job for a job block has also been included. By applying a new type of fuzzy arithmetic and a fuzzy ranking method using location index, the results have been derived through fuzziness index function technique given by [10] Ming Ma et al . In this technique, there is no need to convert the fuzzy processing time into classical numbers, differing with the work of Yager's (1981) and other authors.

[11], [12], [13], [14], [15], [16] T.P. Singh et al explored a lot of scheduling models under fuzzy environments and extended the scheduling research in fuzzy system with applications. Recently [17] Arti and Namita applied the fuzzy index function techniques in n jobs 3 stage scheduling problem. The present research is further an addition to this work.

**The purposed study is made clear through numerical example.**

## II. PRELIMINARIES:

In this section we give some notions and results which were used in further study

### A. FUZZY SET:-

A fuzzy set  $\tilde{A}$  is defined by  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$ . In the pair  $\{(x, \mu_{\tilde{A}}(x))\}$ , the element  $x$  belongs to the classical set  $A$ , whereas the other element  $\mu_{\tilde{A}}(x)$ , belongs to the interval  $[0, 1]$ , termed as membership function or grade of membership.

Zadeh L.A. [1965] introduces fuzzy set as an addition to the classical notation of set.

### B. FUZZY NUMBER:

A fuzzy set  $\tilde{A}$  defined with the help of set of real numbers  $R$  is termed as fuzzy number if its membership functions

$\tilde{A}: R \rightarrow [0, 1]$  fulfill the following study.

(a)  $\tilde{A}$  is convex,  $\tilde{A}\{\lambda x_1 + (1-t)x_2\} = \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}$ , for all  $x_1, x_2 \in R$  and  $\lambda \in [0, 1]$

(b)  $\tilde{A}$  is normal i.e. there exists an  $x \in R$  such that  $\tilde{A}(x) = 1$

(c)  $\tilde{A}$  is piecewise continuous.

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Arti Tyagi Chandigarh Group of colleges, Landran, Panjab, India  
Ashima Kalra, PhD Research scholar, IK Gujral PTU, Jalandhar, India  
Cheshta Kashyap, Maharishi Markandeshwar University, Sadopur, Ambala, Haryana.

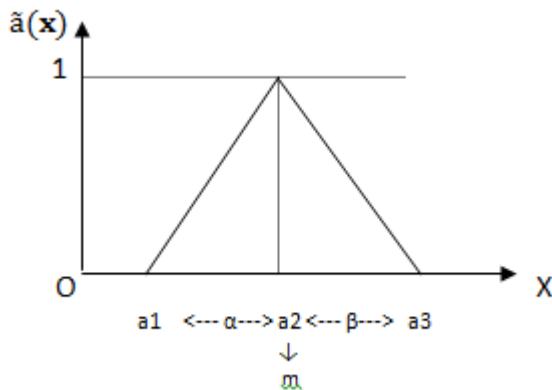
**C. TRIANGULAR FUZZY NUMBER (TFN):**

Any fuzzy number  $\tilde{a}$  defined on  $R$ , called as a triangular fuzzy number (TFN) or linear fuzzy number only if its membership functions is given by  $\tilde{a}: R \rightarrow [0,1]$  has the following features:

$$\tilde{a}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{elsewhere} \end{cases}$$

The parameters  $(a_1, a_2, a_3)$  with  $(a_1 < a_2 < a_3)$  determine  $x$  coordinates of three corners of underlying triangular MF.

(a) Fuzzy numbers is denoted by  $\tilde{a} = (a_1, a_2, a_3)$ . We use  $F(R)$  to represent the set of all triangular fuzzy numbers.



**Fig 1: Triangular fuzzy number  $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$ .**

If  $m = a_2$  symbolizes the modal value or midpoint,  $\alpha = (a_2 - a_1)$  denotes the left spread and  $\beta = (a_3 - a_2)$  characterizes the right spread of the triangular fuzzy number  $\tilde{a} = (a_1, a_2, a_3)$ , then the triangular fuzzy number  $\tilde{a}$  can be given with the triple  $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$ .

(b) A triangular fuzzy number  $\tilde{a} \in F(R)$  can also be denoted as a pair;  $\tilde{a} = (\underline{a}, \bar{a})$  of functions and  $\underline{a}(r)$  and  $\bar{a}(r)$  for  $0 \leq r \leq 1$  which fulfills the following conditions:

- (i)  $\underline{a}(r)$  is a bounded monotonic increasing left continuous function.
- (ii)  $\bar{a}(r)$  is a bounded monotonic decreasing left continuous function.
- (iii)  $\underline{a}(r) \leq \bar{a}(r), 0 \leq r \leq 1$

**D. LOCATION INDEX:**

For any random triangular fuzzy number  $\tilde{a} = (\underline{a}, \bar{a})$ , the number  $a_0 = \left(\frac{\underline{a}(1) + \bar{a}(1)}{2}\right)$  is called as location index number of  $\tilde{a}$ . The two non-decreasing left continuous functions  $a_* = (a_0 - \underline{a}), a^* = (\bar{a} - a_0)$  were said to be left fuzziness index function as well as right fuzziness index function respectively. Therefore every triangular fuzzy number  $\tilde{a} = (a_1, a_2, a_3)$  can also be symbolized by  $\tilde{a} = (a_0, a_*, a^*)$

**E. RANKING OF TRIANGULAR FUZZY NUMBERS**

Many different methodologies were given in the literature for the ranking of fuzzy numbers. [18] Abbas bandy and Hajjari offered a novel method of ranking which is centered on the left and the right ranges at some  $\alpha$  –levels of fuzzy numbers.

For an arbitrary triangular fuzzy number  $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$  with parametric form  $\tilde{a} = (\underline{a}(r), \bar{a}(r))$ , we state the value of the triangular fuzzy number  $\tilde{a}$  by

$$\text{Mag}(\tilde{a}) = \frac{1}{2} \left( \int_0^1 (\underline{a} + \bar{a} + a_0) f(r) dr \right) = \frac{1}{2} \left( \int_0^1 (a^* + 4a_0 - a_*) f(r) dr \right)$$

Where the function  $f(r)$  is a non-negative and growing function in interval  $[0,1]$  with  $f(0)=0, f(1)=1$  and  $\int_0^1 f(r) dr = \frac{1}{2}$ . The function  $f(r)$  can also be deliberated as a weighting function. In real world problems,  $f(r)$  can be selected by the decision maker according to the situation. In present research, for our suitability we refer  $f(r) = r$ .

So the magnitude of a triangular fuzzy number  $\tilde{a}$  is:

$$\text{Mag}(\tilde{a}) = \left( \frac{a^* + 4a_0 - a_*}{4} \right)$$

The magnitude of a triangular fuzzy number  $\tilde{a}$  provides the data about every membership degree, and significance of this magnitude is pictorial and normal.  $\text{Mag}(\tilde{a})$  is used to rank fuzzy numbers, Larger the  $\text{Mag}(\tilde{a})$  larger is the fuzzy number. For any two triangular fuzzy numbers  $\tilde{a} = (a_0, a_*, a^*)$  and  $\tilde{b} = (b_0, b_*, b^*)$  in  $F(R)$ , we can define the ranking of  $\tilde{a}$  and  $\tilde{b}$  on the basis of the  $\text{Mag}(\tilde{a})$  and  $\text{Mag}(\tilde{b})$  on  $R$  as follows:

- (i)  $\tilde{a} \geq \tilde{b}$  if and only if  $\text{Mag}(\tilde{a}) \geq \text{Mag}(\tilde{b})$
- (ii)  $\tilde{a} \leq \tilde{b}$  if and only if  $\text{Mag}(\tilde{a}) \leq \text{Mag}(\tilde{b})$
- (iii)  $\tilde{a} \approx \tilde{b}$  if and only if  $\text{Mag}(\tilde{a}) = \text{Mag}(\tilde{b})$

**F. SYMMETRIC TFN:**

A triangular fuzzy number  $\tilde{a} = (a_0, a_*, a^*)$  is said to be symmetric if  $a_* = a^*$ .

**(a) POSITIVE FUZZY NUMBER:**

A triangular fuzzy number  $\tilde{a} = (a_0, a_*, a^*)$  is termed as non-negative in the only case if  $\text{Mag}(\tilde{a}) \geq 0$  and is denoted by  $\tilde{a} \geq \tilde{0}$ . Further if  $\text{Mag}(\tilde{a}) > 0$ , then  $\tilde{a} = (a_0, a_*, a^*)$  is called as a positive fuzzy number and is denoted by  $\tilde{a} > \tilde{0}$

**G. EQUIVALENT FUZZY NUMBERS:**

Two triangular fuzzy numbers  $\tilde{a} = (a_0, a_*, a^*)$  and  $\tilde{b} = (b_0, b_*, b^*)$  in  $F(R)$  are termed as comparable if and only if

$$\text{Mag}(\tilde{a}) = \text{Mag}(\tilde{b})$$

That is  $\tilde{a} \approx \tilde{b}$  if and only if  $\text{Mag}(\tilde{a}) = \text{Mag}(\tilde{b})$ . Two triangular fuzzy numbers  $\tilde{a} = (a_0, a_*, a^*)$  and  $\tilde{b} = (b_0, b_*, b^*)$  in  $F(R)$  are said to be equal if and only if  $a_0 = b_0, a_* = b_*, a^* = b^*$ . That is  $\tilde{a} \approx \tilde{b}$  if and only if  $a_0 = b_0, a_* = b_*, a^* = b^*$ .

**H. ARITHMETIC OPERATIONS ON TRIANGULAR FUZZY NUMBERS**

Ming Ma et al. (1999) have introduced a new fuzzy arithmetic on the basis of both location index function and fuzziness index function.

The location index number is considered in the ordinary



arithmetic, while the fuzziness index function were supposed to follow the lattice rule which is least upper bound in the lattice L. that is for  $a, b \in L$  we define  $a \vee b = \max\{a, b\}$  and  $a \wedge b = \min\{a, b\}$ .

For any triangular fuzzy numbers  $\tilde{a} = (a_0, a_*, a^*)$  and  $\tilde{b} = (b_0, b_*, b^*)$  and  $*$  = {+, -, x, ÷}, the arithmetic operations on the triangular fuzzy numbers are defined by  $\tilde{a} * \tilde{b} = (a_0 * b_0, a_* \vee b_*, a^* \vee b^*)$ .

In specific case; for any two triangular fuzzy numbers  $\tilde{a} = (a_0, a_*, a^*)$  and  $\tilde{b} = (b_0, b_*, b^*)$ , we define

- (i) Addition:  $\tilde{a} + \tilde{b} = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$
- (ii) Subtraction:  $\tilde{a} - \tilde{b} = (a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$
- (iii) Multiplication:  $\tilde{a} \times \tilde{b} = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$
- (iv) Division:  $\tilde{a} \div \tilde{b} = (a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

### III. METHODOLOGY

This section Deals with some notations, assumptions, Method and Procedure to solve a problem.

#### A. ASSUMPTIONS:

- (a) No job anticipation is permitted.
- (b) The machine can only execute one job at a time.
- (c) All jobs are offered at the commencement of the scheduling time horizon.
- (d) Each job has m operations.
- (e) Each job must be accomplished once it is initiated.

#### B. EQUIVALENT JOB THEOREM FOR JOB BLOCK:

We consider the job block  $(\alpha k, \alpha m)$  having processing time  $(A_{\alpha k}, A_{\alpha m})$  is corresponding to single job  $\beta$  (called equivalent job). The treating time of job  $\beta$  on machines are given by Maggu and Das (1977) in equivalent job block theorem as,

$$A_\beta = A_{\alpha k} + A_{\alpha m} - \min(B_{\alpha k}, A_{\alpha m})$$

$$B_\beta = B_{\alpha k} + B_{\alpha m} - \min(B_{\alpha k}, A_{\alpha m})$$

Job block  $(\alpha k, \alpha m)$  implies that job  $\alpha k$  is preferred over  $\alpha m$  and in the order  $\alpha k, \alpha m$ .

#### C. NOTATIONS

- $S_k$  : Sequence obtained on application of Johnson's procedure,  $k=1,2,3,\dots,m$ .
- $M_j$  : Machine  $j$ ,  $j=1,2,3,\dots,m$ .
- $\tilde{M}$  : Minimum makes span
- $\tilde{A}_{ij}$  : Fuzzy processing time of  $i^{\text{th}}$  job on machine  $M_j$ ,  $i=1, 2, 3,\dots,n; j=1,2, 3,\dots,m$ .
- $\tilde{t}_{ij}(S_k)$  : Completion time of  $i^{\text{th}}$  job of sequence  $S_k$  on machine  $M_j$ .
- $\tilde{I}_{ij}(S_k)$  : Idle time of machine  $M_j$  for job  $i$  in the sequence  $(S_k)$

- $\tilde{U}_j(S_k)$  : Utilization time for which machine  $M_j$  is required
- $\beta$  → Equivalent job for job block
- $\tilde{CT}(S_i)$  : Total completion time of the jobs for sequence  $S_i$

### D. PROBLEM FORMULATION

Suppose some job  $i$  ( $i=1,2,3,\dots,n$ ) is to be done on  $j$  machines  $j=(1,2,\dots,m)$  with the condition  $(\alpha k, \alpha m)$  as a job block. Let  $\tilde{A}_{ij}$  be the processing time of  $i^{\text{th}}$  job on  $j^{\text{th}}$  machine defined by triangular fuzzy numbers. Our objective is to figure out the sequence  $\{S_k\}$  of the jobs which reduce the make duration of machines i.e. Our target is to reduce the net pass by time on machines when minimizing their utilization time is subject to constraints of equivalent job for job block .

### E. ALGORITHM

Algorithm for heuristic approach was build up to minimize the utilization time and hence scheduling along with processing time in fuzzy environment.

Let there are  $m$  machines  $M_1, M_2, \dots, M_m$ . This problem can be converted to a two machine problem, if one of the following conditions is fulfilled.

Let  $\tilde{A}_{i1}, \tilde{A}_{i2}, \dots, \tilde{A}_{im}$  be the processing times on machines  $M_1, M_2, \dots, M_m$  respectively.

$$\min_i \tilde{A}_{i1} \geq \max_i \tilde{A}_{ij}, j = 2, 3, \dots, m - 1$$

(or)

$$\min_i \tilde{A}_{im} \geq \max_i \tilde{A}_{ij}, j = 2, 3, \dots, m - 1$$

Then the problem can be converted to a two machine problem. Introduce two fictitious machines G and H such that

$$\tilde{G}_i = \tilde{A}_{i1} + \tilde{A}_{i2} + \tilde{A}_{i3} + \dots + \tilde{A}_{i(m-1)}, i = 1, 2, 3, \dots, n$$

$$\tilde{H}_i = \tilde{A}_{i2} + \tilde{A}_{i3} + \tilde{A}_{i4} + \dots + \tilde{A}_{im}, i = 1, 2, 3, \dots, n$$

Where  $\tilde{G}_i$  and  $\tilde{H}_i$  are the processing times for job  $i$  on machines G and H respectively.

Obtain the sequence  $S_k$  (say) by applying [1] Johnson's (1954) algorithm on machines G & H and apply equivalent job block theorem.



IV. NUMERICAL ILLUSTRATION

Imagine 5 jobs, 3 machines flow shop problem whose processing time was given by triangular fuzzy numbers as given on table 1. Equivalent job  $\beta$  for job block  $\beta = (1, 5)$  i.e. jobs 1 and 5 are held as a job block. Our aim is to find an optimum schedule to diminish the net elapsed time of the machines.

Table 1: Fuzzy Processing Times  $\tilde{a} = (a_1, a_2, a_3)$

Job	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>
1	(14, 16, 18)	(10, 12, 14)	(7, 8, 9)	(4, 5, 6)
2	(14, 15, 16)	(11, 12, 13)	(8, 9, 10)	(5, 6, 7)
3	(17, 18, 19)	(12, 13, 14)	(9, 10, 11)	(6, 7, 8)
4	(16, 17, 18)	(11, 13, 15)	(10, 11, 12)	(7, 8, 9)
5	(13, 14, 15)	(10, 11, 12)	(7, 9, 11)	(4, 6, 8)

A. SOLUTION:-

In the given scheduling problems, all the decision parameters are represented using triangular fuzzy numbers in the form of  $\tilde{a} = (a_1, a_2, a_3)$ . As per definition 2.4, these numbers can also be represented in convenient form

$\tilde{a} = (a_1, a_2, a_3) = (a_0, a^*, a^*)$  where  $a^* = (a_0 - \alpha)$   $\rightarrow$  left fuzziness index function of  $\tilde{a}$  respectively. Now for machine M<sub>1</sub> and job 1, the processing time can be calculated as,

$$\frac{x - a_0}{a^* - a_0} = \alpha$$

$$\Rightarrow x - a_0 = \alpha(a^* - a_0) \Rightarrow x = \alpha(a^* - a_0) + a_0 \rightarrow \bar{a}(r)$$

$$\text{Also } \frac{a^* - x}{a^* - a_0} = \alpha$$

$$\Rightarrow a^* - x = \alpha(a^* - a_0)$$

$$\Rightarrow x = a^* - \alpha(a^* - a_0) \rightarrow \bar{a}(r)$$

Take e.g. processing time (12, 13, 14) we have

$$\frac{x - 12}{13 - 12} = \alpha \Rightarrow x = \alpha + 12 \rightarrow \underline{a} = 13 \text{ for } \alpha = 1$$

$$\text{Or } \frac{14 - x}{14 - 13} = \alpha \Rightarrow x = 14 - \alpha \rightarrow \bar{a} = 13 \text{ for } \alpha = 1$$

$$\therefore \tilde{a} = (12, 13, 14) \approx (13, 1, 1)$$

The given data of table 1 in form of  $\tilde{a} = (a_0, a^*, a^*)$  with the help of index function can be presented in table 2 as:

Table 2: Fuzzy Processing time  $\tilde{a} = (a_0, a^*, a^*)$

Jobs	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>
1	<16, 2, 2>	<12, 2, 2>	<8, 1, 1>	<5, 1, 1>
2	<17, 1, 1>	<12, 1, 1>	<9, 1, 1>	<6, 1, 1>
3	<18, 1, 1>	<13, 1, 1>	<10, 1, 1>	<7, 1, 1>
4	<15, 1, 1>	<13, 2, 2>	<11, 1, 1>	<8, 1, 1>
5	<14, 1, 1>	<11, 1, 1>	<9, 2, 2>	<6, 2, 2>

Minimum Processing time on M<sub>1</sub> = <14, 1, 1>  
 Maximum Processing time on M<sub>2</sub> = <13, 2, 2>  
 Minimum Processing time on M<sub>3</sub> = <8, 1, 1>  
 Clear, Min Processing of M<sub>1</sub>  $\geq$  Maximum Processing time of M<sub>2</sub>

As from section II (E),  $\text{mag}(\tilde{a}) = \frac{a + 4a_0 - a_4}{4}$

$\text{mag}(\tilde{a})$  on M<sub>1</sub> =  $\frac{1 + 4 \times 13 - 1}{4} = 14$

$\text{mag}(\tilde{a})$  on M<sub>2</sub> =  $\frac{2 + 4 \times 12 - 2}{4} = 13$

Convert the problem in two fictitious machines G and H such that

$$\tilde{G}_i = \tilde{M}_{i1} + \tilde{M}_{i2} + \tilde{M}_{i3} + \tilde{M}_{i4}$$

$$\tilde{H}_i = \tilde{M}_{i2} + \tilde{M}_{i3} + \tilde{M}_{i4}$$

and represent the data in table 3 as follows:

Table 3

Jobs	G	$\tilde{H}$
1	<36, 2, 2>	<25, 2, 2>
2	<38, 1, 1>	<27, 1, 1>
3	<41, 1, 1>	<30, 1, 1>
4	<39, 2, 2>	<32, 1, 1>
5	<34, 2, 2>	<26, 2, 2>

Given equivalent job  $\beta = (1, 5)$  for job block on applying equivalent job block theorem, we get

$$G_\beta = <51, 2, 2>$$

$$H_\beta = <23, 2, 2>$$

the processing time on G and H in given in table 4.

Table 4: Processing time on G and H

Jobs	G	$\tilde{H}$
$\beta$	<50, 2, 2>	<32, 2, 2>
2	<38, 1, 1>	<27, 1, 1>
3	<41, 1, 1>	<30, 1, 1>
5	<34, 2, 2>	<26, 1, 1>

Apply Johnson's technique, the sequence we find

$$S_1 = \boxed{\beta \quad 3 \quad 2 \quad 5} \quad \text{or} \quad \boxed{1, 4, 3, 2, 5}$$

Other possible sequence

$$S_2 = \boxed{4, 1, 3, 2, 5}$$



**B. VALIDITY of model and to find total elapsed time:**

For first sequence  $S_1$ , we have,

**Table 5:**

Jobs	M <sub>1</sub> In - out	M <sub>2</sub> In - out	M <sub>3</sub> In - out	M <sub>4</sub> In - out
1	(0,0,0) - (16,2,2)	(16,2,2) - (29,2,2)	(29,2,2) - (37,2,2)	(38,2,2) - (45,2,2)
4	(16,2,2) - (31,2,2)	(31,2,2) - (45,2,2)	(45,2,2) - (56,2,2)	(56,2,2) - (64,2,2)
3	(31,2,2) - (49,2,2)	(49,2,2) - (62,2,2)	(62,2,2) - (72,2,2)	(72,2,2) - (79,2,2)
2	(49,2,2) - (66,2,2)	(66,2,2) - (78,2,2)	(78,2,2) - (90,2,2)	(90,2,2) - (96,2,2)
5	(66,2,2) - (80,2,2)	(80,2,2) - (91,2,2)	(91,2,2) - (100,2,2)	(100,2,2) - (106,2,2)

For sequence  $S_2$ , we have

**Table 6:**

Jobs	M <sub>1</sub> In - out	M <sub>2</sub> In - out	M <sub>3</sub> In - out	M <sub>4</sub> In - out
4	(0,0,0) - (15,1,1)	(15,1,1) - (29,2,2)	(29,2,2) - (40,2,2)	(40,2,2) - (48,2,2)
1	(15,1,1) - (31,2,2)	(31,2,2) - (43,2,2)	(43,2,2) - (51,2,2)	(51,2,2) - (56,2,2)
3	(31,2,2) - (49,2,2)	(49,2,2) - (63,2,2)	(63,2,2) - (73,2,2)	(73,2,2) - (80,2,2)
2	(49,2,2) - (66,2,2)	(66,2,2) - (78,2,2)	(78,2,2) - (87,2,2)	(87,2,2) - (93,2,2)
5	(66,2,2) - (80,2,2)	(80,2,2) - (91,2,2)	(91,2,2) - (100,2,2)	(100,2,2) - (106,2,2)

Hence, the optimal sequence of the said problem  $S_1 = (1, 4, 3, 2, 5)$

The minimum total elapsed time = (106, 2, 2)  
∴ Idle time on Machine 1 = (106, 2, 2) - (80, 2, 2) = (26, 2, 2)

Idle time on Machine 2 = (15,1,1) + (2, 2, 2)+(6, 2, 2)+(3, 2, 2) + (2, 2, 2) = (28, 2, 2)

Idle time on machine 3 = (29, 2, 2) + (3, 2, 2) + (8, 2, 2) + (5, 2, 3) + (4, 2, 2) = (49, 2, 2)

Idle time on machine 4 = (40, 2, 2) + (3, 2, 2) + (17, 2, 2) + (7, 2, 2) + (7, 2, 2) = (74, 2, 2)

Utilization time of Machine 1 = (80, 2, 2) hours  
Utilization time of Machine 2 = (91, 2, 2) - (28, 2, 2) = (53, 2, 2) hours

Utilization time of Machine 3 = (100, 2, 2) - (49, 2, 2) = (51, 2, 2) hours

Utilization time of Machine 4 = (106, 2, 2) - (74, 2, 2) = (32, 2, 2) hours

**V. CONCLUSION**

Fuzzy scheduling is of excessive use for the effective application in real time events. In this paper we have investigated the 3 stages specially structured flow shop scheduling with uncertain parameters. A new technique using fuzziness index function has been proposed to minimize the total pass by time where processing times of machines are fuzzy in nature. The validity of the said model is being examined with numerical illustration and through in-out table. The study can be extended for high inventory cost between

more than three machines and more constraints can be added to wider the problem.

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**AUTHORS PROFILE**



scheduling, machine learning.

**Dr. Arti Tyagi**, Ph.D. in Mathematics from MM University, Mullana in 2016 and M.Phil in 2009. She has published 12 papers in reputed journals. She has guided one M.phil. Student in 2017. Her research activities include Queuing model networks, fuzzy systems,



**Ashima Kalra**, Gold Medalist in

ed By:  
Dile Eye Intelligence Engineering  
& Sciences Publication



## Optimal Sequence in General flow shop with Job block criteria using Fuzzy index Technique

B Tech in Electronics from Kurukshetra University, Kurukshetra in 2003 .Received M.Tech degree from Punjab Technical University, Kapurthla (Punjab)in 2008 and pursuing PhD from Punjab Technical University , Kapurthla (Punjab )in the field of soft Computing. She has published more than 25 papers in reputed journals and 3 book chapters in Springer series. Her research activities include designing model identification using neural networks, fuzzy systems, supervised learning .machine learning. Lifetime IEEE member and has been serving as a fellow member of IEEE Delhi section, India. Organized 3 IEEE international conferences as a chairperson.



**Dr. Cheshta kashyap**, Ph.D in Finance and Human Resource, M. Phil in Management in 2008, M.Com in 2010, M.B.A in 2004. She has Published 6 papers in reputed journals. She presented paper at IIT Rurkee in 2014 .Her research activities inculcate Stock Market, Pay for Performance, Finance.