Mean Square Cordial Labeling of Some Snake Graphs

S Dhanalakshmi, S Thirunavukkarasu , N Parvathi

***Abstract*: In this paper we investigate that some snake related graph admits mean square cordial labeling. In particular , mean square cordial labeling of a Triangular snake Tk, Subdivision of a triangular snake S(Tk), Quadrilateral snake QSk, Subdivision of a quadrilateral snake S(QSk) are discussed.**

Index Terms: Mean square cordial labeling, Triangular snake, Subdivision of a triangular snake, Quadrilateral snake, Subdivision of a Quadrilateral snake.

I. INTRODUCTION

Numerous applications of graph labeling in real life induce the motivation among the researchers. Many research activities are tested from different types of graph labeling [1] which lead to many publications in this topic and the number of papers persists to be on the crease. In this work, there is a contribution of some interesting results on mean square cordial labeling. Snake related graphs are taken here for the discussion because these graphs are very common as a network design at the access layer of computer networks. Also snake related graphs are good models of network transits using probe tools like trace route. We follow Harary[2] for basic terminology and notations . Cahit[3] initiated the concept of cordial labeling .Mean cordial labeling was first studied by and Ponraj and et al[3].A.N. Murugan introduced the labeling technique of mean square cordial and they have investigated the same for some special graphs in [5,6,7] Dhanalakshmi et al explored some ideas on mean square cordial labeling of some acyclic graphs and its rough approximations[8] and projected the same labeling technique for some cyclic graphs[9]. In this paper we investigate that some snake related graph admits mean square cordial labeling. In particular , mean square cordial labeling of a Triangular snake Tk, Subdivision of a triangular snake S(Tk), Quadrilateral snake QSk, Subdivision of a quadrilateral snake S(QSk) are analysed.

II. PRELIMINARIES

Definition 1: Let G = (V,E) be a graph with p vertices and q edges. “A Mean Square Cordial labeling of a Graph G(V,E) with p vertices and q edges is a bijection from V to

{0, 1} such that each edge uv is assigned the label where (ceil(x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled 0 and the number of edges labeled with 1 differ by at most 1”.

Definition 2: “A Triangular Snake **T**K is obtained from a path u1,u2,…….uk by joining ui and ui+1 to a new vertex vi for $1 \leq i \leq k-1$. That is The triangular snake Tk is obtained from the path Pk by replacing each edge of the path by a triangle C3.”

Definition 3: “The Quadrilateral snake Q(Sk) is obtained from a path u1,u2,…….uk by joining ui and ui+1 for $1 \leq i \leq k-1$,to two new vertices vi and wi and then joining vi and wi.That is the path Pn by replacing each edge of the path by a cycle C4.”

Definition 4: “The pentagonal snake P(Sk) is obtained from a path u1,u2,…….uk by joining ui and ui+1 for $1 \leq i \leq k-1$,to two new vertices vi , wi , xi and then joining vi , xi and xi ,wi.That is the path Pn by replacing each edge of the path by a cycle C5.””

Definition 5: “Let G be a graph. The subdivision graph S(G) is obtained from G by subdividing each edge of G with a vertex.”

III. MAIN RESULTS

Theorem 1: Triangular snake T_k admits mean square cordial labeling , $\forall k \geq 2$ Proof: Let P_k be the path $u_1,u_2,\ldots,u_k$. Let $V(T_k) = V(P_k) \cup \{v_i$:i varies from 1 to k-1}. $E(T_k)=\{[(u_iu_{i+1})$:i varies from 1 to k-1] $\cup$ [$(u_iv_i$: i varies from 1 to k-1 } $\cup$ [$(v_iu_{i+1}$: i varies from 1 to k-1 }Here $|V| = 2k-1$ *and* $|E| = 3k-3$

Define f maps $V(T_k)$ to { 0,1}

Case (i) : For even k

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$$f(u_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } k/2 \\ 1, i \text{ varies from } (k+2)/2 \text{ to } k \end{cases}$$

$$f(v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } k/2 \\ 1, i \text{ varies from } (k+2)/2 \text{ to } k-1 \end{cases}$$

Hence the edge labeling is

$$f(u_i u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-2)/2 \\ 1, i \text{ varies from } k/2 \text{ to } k-1 \end{cases}$$

$$f(u_i v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } k/2 \\ 1, i \text{ varies from } (k+2)/2 \text{ to } k-1 \end{cases}$$

$$f(u_{i+1} v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-2)/2 \\ 1, i \text{ varies from } k/2 \text{ to } k-1 \end{cases}$$

The above labeling pattern satisfied the cardinality of vertices and edges which are mentioned in the following table. In table, cardinality of vertices and edges are represented in terms of "t" throughout this section.

t	0	1
$\lvert v_f(t)\rvert$	k	$k-1$
$\lvert e_f(t)\rvert$	$\frac{3k}{2}-2$	$\frac{3k}{2}-1$

Case (ii) : For odd k

$$f(u_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2 \text{ to } k \end{cases}$$

$$f(v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

Hence the edge labeling is

$$f(u_i u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2-1 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(u_i v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(v_i u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2-1, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

0The above labeling pattern satisfied the cardinality of vertices and edges which are mentioned in the following table.

t	0	1
$\lvert v_f(t)\rvert$	k	$k-1$
$\lvert e_f(t)\rvert$	$\frac{3k-3}{2}$	$\frac{3k-3}{2}$

Hence the theorem is proved.

Example:1

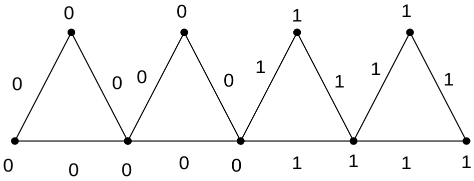

Figure (1) : Mean square cordial labeling of a triangular snakes T_5

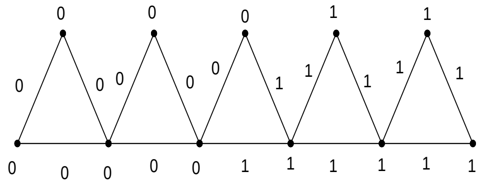

Figure (2) : Mean square cordial labeling of a triangular snakes T_6

Theorem 2 :The subdivision of a triangular snake $S(T_k)$ admits mean square cordial labeling, $\forall k \geq 3$ *and* k *is odd* .

Proof: Let P_k be the path $u_1, u_2, \ldots, u_k$. Let $V(T_k) = V(P_k) \cup \{v_i$: i varies from 1 to k-1$\}$ and $V(S(T_k)) = V(T_k) \cup \{ x_i, y_i, w_i$: i varies from 1 to k-1$\}$.Then $E(S(T_k))=\{[(u_i x_i) \cup (x_i v_i) \cup (y_i v_i) \cup (y_i u_{i+1}) \cup (u_i w_i) \cup (w_i u_{i+1})$:i varies from 1 to k-1 $\}$

Here $|V| = 5k-4$ *and* $|E| = 6k-6$

Define f maps $V(S(T_k))$ to $\{0,1\}$

$$f(u_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2 \text{ to } k \end{cases}$$

$$f(v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from} (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(x_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from} (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(y_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from} (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from} (k+1)/2 \text{ to } k-1 \end{cases}$$

Hence the edge labeling is

$$f(u_i x_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(x_i v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(v_i y_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(u_i w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(u_i w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(u_{i+1} w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(u_{i+1} y_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

The above labeling pattern satisfied the cardinality of vertices and edges which are mentioned in the following table.

t	0	1
$\lvert v_f(t) \rvert$	$\frac{5k-3}{2}$	$\frac{5k-5}{2}$
$\lvert e_f(t) \rvert$	$3k-3$	$3k-3$

Hence the theorem is proved.

Example: 2

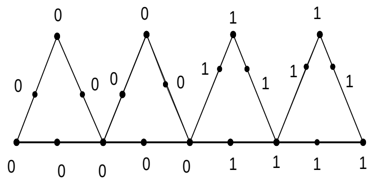

Figure (3) : Mean square cordial labeling of subdivision of a triangular snakes $S(T_5)$

Remark 1: Subdivision triangular snake $S(T_k)$ doesn't satify the above labeling for even value of k since the edge difference labeled with 0 and 1 is two, contradicts the edge condition of mean square cordial labeling.

Theorem 3: Quadrilateral snake QS_k admits mean square cordial labeling , $\forall k \geq 3, k \text{ is odd}$

Proof: Let P_k be the path $u_1, u_2, \ldots, u_k$. Let $V(QS_k) = V(P_k) \cup \{v_i, w_i$:i varies from 1 to k-1}. $E(QS_k)=\{[(u_i u_{i+1})$:i varies from 1 to k-1] $\cup$ [$(u_i v_i)$: i varies from 1 to k-1 } $\cup$ [$(v_i w_i)$:i varies from 1 to k-1] $\cup$ [$(w_i u_{i+1})$: i varies from 1 to k-1 }

Here $|V| = 3k-2 \text{ and } |E| = 4k-4$

Define f maps $V(QS_k)$ to { 0,1}

$$f(u_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2 \text{ to } k \end{cases}$$

$$f(v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2 \\ 1, i \text{ varies from} (k+1)/2 \text{ to } k-1 \end{cases}$$

Hence the edge labeling is

$$f(u_i u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(u_i v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(v_i w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{to} (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to} k-1, \end{cases}$$

$$f(u_{i+1} w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{to } (k-1)/2 \\ 1, i \text{ varies from} (k+1)/2 \text{to } k-1, \end{cases}$$

The above labeling pattern satisfied the cardinality of vertices and edges which are mentioned in the following table.

	0	1
$\lvert v_f(t)\rvert$	$\frac{3k-1}{2}$	$\frac{3k-3}{2}$
$\lvert e_f(t)\rvert$	$\frac{4k-4}{2}$	$\frac{4k-4}{2}$

Hence the theorem is proved.

Example:3

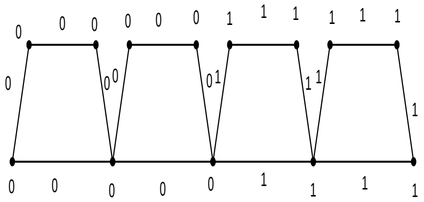

Figure (4): Mean square cordial labeling of a quadrilateral snake QS_5

Remark 2: Quadrilateral snake QS_k doesn't satify the above labeling for even value of k since the edge difference labeled with 0 and 1 is two, contradicts the edge condition of mean square cordial labeling.

Theorem: 4 Subdivision of a quadrilateral snake $S(QS_k)$ admits mean square cordial labeling ,

$\forall k \geq 3$ *and* k *is odd* .

Proof: Let P_k be the path $u_1,u_2,\ldots,u_k$. Let $V(QS_k)$ = $V(P_k) \cup \{v_i, w_i$:i varies from 1 to k-1} and $V(S(QS_k))$ = $V(QS_k) \cup \{ x_i,y_i,z_i,p_i$:i varies from 1 to k-1} Then $E(S(QS_k))$ ={[(u_ix_i): i varies from 1 to k-1] $\cup$ [(v_ix_i): i varies from 1 to k-1] $\cup$ [(v_iy_i): i varies from 1 to k-1 } $\cup$ [$(y_iw_i$:i varies from 1 to k-1] $\cup$ [$(z_iw_i$:i varies from 1 to k-1] $\cup$ [$(z_iu_{i+1}$: i varies from 1 to k-1 $\cup$ [$(p_iu_{i+1}$: i varies from 1 to k-1 $\cup$ [$(p_iu_i$: i varies from 1 to k-1 }

Here $|V| = 7k-6$ *and* $|E| = 7k-3$

Define f maps $V(S(QS_k))$ to { 0,1}

$$f(u_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2 \text{ to } k \end{cases}$$

$$f(v_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(x_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(y_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(z_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(p_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

Hence the edge labeling is

$$f(u_ix_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(x_iv_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(v_iy_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(y_iw_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(w_iz_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(z_iu_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(u_{i+1}p_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(p_iu_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

The above labeling pattern satisfied the cardinality of vertices and edges which are mentioned in the following table.

t	0	1
$\lvert v_f(t)\rvert$	$\frac{7k-5}{2}$	$\frac{7k-7}{2}$
$\lvert e_f(t)\rvert$	$\frac{7k-3}{2}$	$\frac{7k-3}{2}$

Hence the theorem is proved.

Example: 4

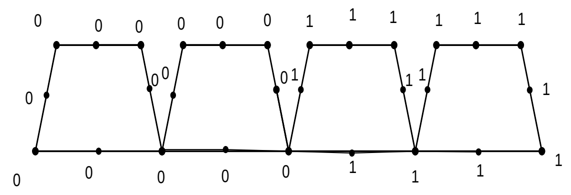

Figure (5): Mean square cordial labeling of $S(QS_5)$

Remark 3: Subdivision of quadrilateral snake $S(QS_k)$ doesn't satify the mean square cordial labeling for even value of k since the edge difference labeled with 0 and 1 is two, contradicts the edge condition of mean square cordial labeling.

IV. CONCLUSION

In this section mean square cordial labeling is investigated for some snake related graphs. It can be further investigated by the researcher for some more snake related graphs like alternate triangular snake graphs, double triangular snake graphs, alternate quadrilateral snake graphs, double quadrilateral graphs etc. Graph operations like union, intersection, corona of two graphs etc., can also be discussed for mean square cordial labeling in future.

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