Linearization of Statcom Model

C E Geethadevi, R Devanathan

Abstract*: Flexible AC Transmission System (FACTS) devices can generate and absorb reactive power from power systems. Static Compensator (Statcom) is a FACTS controller used in power systems to enhance power transmission capability and to introduce damping during power system transients. The design of Statcom has been a subject of research. Different techniques have been used including exact linearization which results in zero dynamics that needs to be stabilized. In this paper, considering operation about an operating point, Statcom equations are reduced essentially to a quadratic form. The resulting equation is quadratic linearized. Issues of zero dynamics do not appear in the proposed method. A numerical example illustrates the theory.

***Index Terms*: Statcom, FACTS controller, linearizatin, quadratic linearization.**

I. INTRODUCTION

Power system stability is required for efficient power transfer capability. Conventional methods like shunt and series compensators have been used to achieve power system stability. Modern systems use flexible ac transmission systems (FACTS). Flexible ac transmission systems are power control devices using thyristors. They are characterized by higher speed of control response and controlled power flow. Static synchronous compensator (Statcom) is a FACTS device with an ability to generate and absorb electric power. Statcom output can be controlled independent of system voltage. Dynamic equations describing Statcom model is nonlinear involving products of input and state variables. Many linearization techniques have been applied in the literature to linearize the Statcom model and build the controller based on the linearized model. A few limitations have been observed in these techniques. This paper proposes a new method which attempts to eliminate some of the limitations of existing techniques. An equivalent circuit of Statcom is given in figure 1.

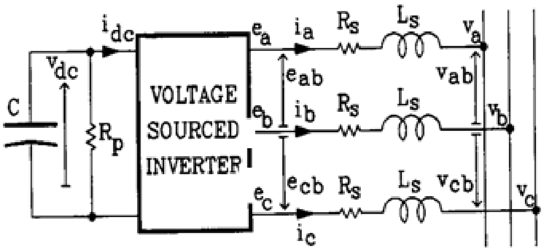

Fig.1 Equivalent Circuit of Statcom

In figure 1, R_s is the series resistance on the AC side which represents the inverter and transformer conduction losses. L_s is the leakage inductance of the transformer. R_p represents the switching losses in the inverter and C is the capacitor on the dc side. e_a, e_b, e_c are Statcom output voltages. v_a, v_b, v_c are line voltages.

The dynamic equations representing Statcom are given in (1) -(3) as

$$\frac{di_q}{dt} = \frac{-R_S}{L_S} i_q - \omega i_d + \frac{<u> \sin\alpha}{L_S} V_{dc} \qquad (1)$$

$$\frac{di_d}{dt} = \omega i_q - \frac{R_s}{L_s} i_d - \frac{<u> \cos\alpha}{L_s} V_{dc} + E/L_s \qquad (2)$$

$$\frac{dV_{dc}}{dt} = -\frac{3}{4} < u > \left(\frac{\cos\alpha}{C} i_d - \frac{\sin\alpha}{C} i_q\right) - \frac{V_{dc}}{RpC} \qquad (3)$$

In (1)-(3), i_q is quadrature axis current, i_d is direct axis current, V_{dc} is Statcom output voltage, $<u>$ represents generalized averaged value of the intermittent switching of the inverter and α is the firing angle in the switching circuit of the inverter which is the key control input. E is the bus voltage of Statcom at the point of connection to the line. E is assumed to be kept constant through a separate control loop and hence it will not be considered in the linearization analysis that follows.

II. EXISTING DESIGN METHODS OF STATCOM

C. Schaduer & H. Mehta [1] proposed a method in which α is varied together with a factor which relates two parameters, viz., the DC side voltage and the amplitude of AC voltage. A decoupled control of e_q and e_d is proposed in [1]. A. Petitclair and S. Bacha [2] proposed a nonlinear control technique using exact feedback linearization taking i_q as output. Nonlinear derivatives of output have been taken successively until input appears. Introduction of feedback linearization of Statcom introduced zero dynamics which cannot be observed and controlled. H. Chen and Zhou [3] modeled a Statcom connected to external transmission lines. Using external line reactance, one more equation in addition to those in (1)-(3) is formed. In [3] nonlinearity is taken as a bounded uncertainty and robust control is applied. The method does not address the nonlinearity directly. P.W. Lehn & M.R. Iravani [4] proposed a technique in which first the system is linearized about an operating point and a linear control law using multi-variable approach is applied. Zhichang Yuan *et .al* [5] introduced the modulation index m as a new control input and exact linearization is applied. In [5] undesired internal dynamics of direct axis current is eliminated but zero dynamics still exists which needs to be stabilized. Y.H. Liu and N.R. Watson [6] modeled not only the Statcom but the multi-source converter. State

Revised Manuscript Received on July 05, 2019

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equations are expanded using the energy stored in the inductors and capacitors. But in the method of [6], the transformation used cannot be studied easily because it does not follow established nonlinear theory.

III. LINEARIZATON OF STATCOM MODEL

In this paper, an approximate linearization of Statcom is proposed. A third order approximation of the terms $V_{dc}\cos\alpha$ and $V_{dc}\sin\alpha$ in (1)-(3) using Taylor's series expansion about an operating point is implemented. Formulating (1)-(3) using deviation about an operating point of state variables, a form involving quadratic nonlinearities is obtained. By applying the method given by Krener and Kang [7],[8], approximate linearization is carried out on the resulting equation to remove the quadratic terms. This leaves only third and higher order nonlinearity in the system, which can be considered negligible. An advantage of this method is that it avoids internal dynamics and allows larger variations of input α , the firing angle.

Statcom equations (1)-(3) can be represented as

$$\dot{x}_1 = -a_1 x_1 - \omega x_2 + k(sin\alpha)\, x_3 \tag{4}$$

$$\dot{x}_2 = \omega x_1 - a_1 - + k(cos\alpha)\, x_3 \tag{5}$$

$$\dot{x}_3 = -\frac{3}{4} a_2 (cos\alpha) x_2 + \frac{3}{4} a_2 (sin\alpha) x_1 - a_3 x_3 \tag{6}$$

where

$$x_1 = i_q, x_2 = i_d, x_3 = V_{dc}$$

and

$$a_1 = \frac{R_s}{L_s}$$

$$a_2 = \frac{<u>}{C}$$

$$a_3 = \frac{1}{RC}$$

$$k = \frac{<u>}{L_s}$$

Considering operation around an operating point we can write

$$x_i = x_{i0} + \xi_i \text{ , i=1,2,3} \tag{7}$$

where $x_{i0,}$ i=1,2,3 represent the steady state operating points corresponding to the state variables.

$$\dot{x}_i = \dot{\xi}_i \text{ . i=1,2,3}$$

Using (7), (4)-(6) reduce to (8)-(10) as below

$$\dot{\xi}_1 = -a_1 x_{10} - a_1\xi_1 - \omega x_{20} - \omega\xi_2 + k(\sin\alpha)x_{30} + k(\sin\alpha)\xi_3 \tag{8}$$

$$\dot{\xi}_2 = \omega x_{10} + \omega\xi_1 - a_1 x_{20} - a_1\xi_2 - k(\cos\alpha)x_{30} - k(\cos\alpha)\xi_3 \tag{9}$$

$$\dot{\xi}_3 = -\frac{3}{4}a_2(\cos\alpha)x_{20} - \frac{3}{4}a_2(\cos\alpha)\xi_2 + \frac{3}{4}a_2(\sin\alpha)x_{10} + \frac{3}{4}a_2(\sin\alpha)\xi_1 - a_3 x_{30} - a_3\xi_3 \tag{10}$$

Expanding sin α and cosα by Taylor's series about the operating point $\alpha_{0,}$ we obtain

$$\sin\alpha|_{\alpha_0} = \sin\alpha_0 + (\cos\alpha_0)(\alpha-\alpha_0) - (\sin\alpha_0)(\alpha-\alpha_0)^2/2 - (\cos\alpha_0)(\alpha-\alpha_0)^3/6 + \ldots \tag{11}$$

$$\cos\alpha|_{\alpha_0} = \cos\alpha_0 - (\sin\alpha_0)(\alpha-\alpha_0) - (\cos\alpha_0) * (\alpha-\alpha_0)^2/2 + (\sin\alpha_0)(\alpha-\alpha_0)^3/6 + \ldots \tag{12}$$

Taking

$\mu_1 = \alpha - \alpha_0$

$\mu_2 = (\alpha - \alpha_0)^2$

$\mu_3 = (\alpha - \alpha_0)^3$

we write

$\sin\alpha = \sin\alpha_0 + \mu_1\cos\alpha_0 - (\sin\alpha_0/2)\,\mu_2 - (\cos\alpha_0/6)\mu_3$

$\cos\alpha = \cos\alpha_0 - \mu_1\sin\alpha_0 - (\cos\alpha_0/2)\,\mu_2 + (\sin\alpha_0/6)\mu_3$

with the higher order terms considered negligible.

Application of deviation about an operating point of α into (8)- (10) and noting that the steady state operating point terms add up to zero, we get

$$\dot{\xi} = A\,\xi + B\mu + g^{(1)}(\xi)_\mu + O^{(3)} \tag{13}$$

where

$$A = \begin{bmatrix} -a_1 & -\omega & ksin\alpha_0 \\ \omega & -a_1 & -kcos\alpha_0 \\ \frac{3}{4}a_2(sin\alpha_0) & -\frac{3}{4}a_2(cos\alpha_0) & -a_3 \end{bmatrix}$$

$$B = \begin{bmatrix} k(\cos\alpha_0)x_{30} & -\frac{k}{2}(\sin\alpha_0)x_{30} & -\frac{k}{6}(\cos\alpha_0)x_{30} \\ k(\sin\alpha_0)x_{30} & \frac{k}{2}(\cos\alpha_0)x_{30} & -\frac{k}{6}(\sin\alpha_0)x_{30} \\ \left\{\frac{3}{4}a_2(\sin\alpha_0)x_{20} \right. & \left\{\frac{3}{4}a_2\left(\frac{\cos\alpha_0}{2}\right)x_{20}\right. & \left\{-\frac{3}{4}a_2\left(\frac{\sin\alpha_0}{6}\right)x_{20}\right. \\ \left. +\frac{3}{4}a_2(\cos\alpha_0)x_{10}\right\} & \left. -\frac{3}{4}a_2\left(\frac{\sin\alpha_0}{2}\right)x_{10}\right\} & \left. -\frac{3}{4}a_2\left(\frac{\cos\alpha_0}{6}\right)x_{10}\right\} \end{bmatrix}$$

$$g^{(1)}(\xi) = \begin{bmatrix} k(\cos\alpha_0)\xi_3 & -\frac{k}{2}(\sin\alpha_0)\xi_3 & -\frac{k}{6}(\cos\alpha_0)\xi_3 \\ k(\sin\alpha_0)\xi_3 & \frac{k}{2}(\cos\alpha_0)\xi_3 & -\frac{k}{6}(\sin\alpha_0)\xi_3 \\ \left\{\frac{3}{4}a_2(\sin\alpha_0)\xi_2 \right. & \left\{\frac{3}{4}a_2\left(\frac{\cos\alpha_0}{2}\right)\xi_2\right. & \left\{-\frac{3}{4}a_2\left(\frac{\sin\alpha_0}{6}\right)\xi_2\right. \\ \left. +\frac{3}{4}a_2(\cos\alpha_0)\xi_1\right\} & \left. -\frac{3}{4}a_2\left(\frac{\sin\alpha_0}{2}\right)\xi_1\right\} & \left. -\frac{3}{4}a_2\left(\frac{\cos\alpha_0}{6}\right)\xi_1\right\} \end{bmatrix}$$

and $O^{(3)}$ corresponds to third and higher order terms which can be considered negligible.

Taking input transformation as

$$\mu = [I + \beta^{(1)}(\xi)]\,\eta \tag{14}$$

where $\beta^{(1)}(\xi)$ is a 3x3 matrix with elements of first order in ξ and η is the new input and applying (14) to (13), we get

$$\dot{\xi} = A\,\xi + B\eta + B\,\beta^{(1)}(\xi)\eta + g^{(1)}(\xi) + O^{(3)}(\xi) \tag{15}$$

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$$\beta^{(1)}(\xi) = -(B^{-1})(g^1)(\xi) \qquad (16)$$

where B^{-1} is assumed to exist. Substituting (16) into (15), we can get

$$\dot{\xi} = A\,\xi + B\eta + O^{(3)} \qquad (17)$$

Notice that there is no quadratic term in (17) and hence quadratic linearization of the Statcom model is completed.

IV. EXPERIMENTAL RESULTS & DISCUSSION

In this section, we provide an illustrative example to the quadratic linearization presented earlier.

Assuming the values

$\omega = 314$

$R_s = 0.0033\ \Omega$

$L_s = 0.05$ H

$R_p = 26.17\ \Omega$;

C= 0.2933 F

$< u >= \frac{4}{\pi}$ =1.2738

k=25.477

Assuming the steady sate input as α_0 = -0.045 radians, we can determine the following:

$sin\alpha_0 = -0.04498$

$cos\alpha_0 = 0.998$

(4) to (6) can be written as

$\dot{x}_1 = -0.066x_1 - 314x_2 + 25.4(\sin\alpha)\,x_3$

$\dot{x}_2 = 314x_1 - 0.066x_2 - 25.4\,(\cos\alpha)\,x_3$

$\dot{x}_3 = 3.254(\sin\alpha)\,x_1 - 3.25\,(\cos\alpha)\,x_2 - 0.1302\,x_3$

Assuming per unit values for the steady state values of the state variables x_{10}, x_{20}, x_{30} to be unity, Statcom model (13) can be calculated as

$$A=\begin{bmatrix} -0.066 & -314 & -1.142492 \\ 314 & -0.066 & -25.4267 \\ -0.1465 & -3.2509 & -0.1302 \end{bmatrix}$$

$$B=\begin{bmatrix} 25.4267 & 0.5728 & -4.2378 \\ -1.1456 & 12.7130 & 0.1909 \\ 3.3973 & 1.6986 & 0.5173 \end{bmatrix}$$

$$g^{(1)}(\xi)=\begin{bmatrix} 25.426\xi_3 & 0.5729\xi_3 & -4.2376\xi_3 \\ -1.1459\xi_3 & 12.7130\xi_3 & 0.1909\xi_3 \\ (3.2508\xi_1-0.1465\xi_2) & (1.6254\xi_2+0.073\xi_1) & (0.0244\xi_2-0.5418\xi_1) \end{bmatrix}$$

Computing

$$B^{-1}=\begin{bmatrix} 0.0178 & -0.0214 & 0.1538 \\ 0.0035 & 0.0785 & 0.0000 \\ -0.1286 & -0.1175 & 0.9229 \end{bmatrix}$$

we apply input transformation as in (14) where

$\beta^{(1)}(\xi) = -(B^{-1})g^{(1)}(\xi)$

That is,

$\beta^{(1)}(\xi) = [b_{ij}(\xi)]$ i, j=1,2,3

where the individual elements are given as

$b_{11} = -0.4999\xi_1 + 0.0225\xi_2 - 0.477\xi_3$

$b_{12} = -0.112\xi_1 - 0.2499\xi_2 + 0.2617\xi_3$

$b_{13} = 0.0833\xi_1 - 0.0375\xi_2 + 0.0794\xi_3$

$b_{21} = -0.001\xi_3$

$b_{22} = -0.999\xi_3$

$b_{23} = 0.0002\xi_3$

$b_{31} = -3.006\xi_1 + 0.1352\xi_2 + 3.404\xi_3$

$b_{32} = -0.0675\xi_1 - 1.4999\xi_2 + 1.567\xi_3$

$b_{33} = 0.500\xi_1 - 0.225\xi_2 - 0.5226\xi_3$

It is verified that

$B\beta^{(1)}(\xi) + g^{(1)}(\xi) \cong 0$

subject to high order decimal precision.

Therefore (13) reduces to

$\dot{\xi} = A\,\xi + B\eta + O^{(3)}$

with the quadratic term eliminated.

V. CONCLUSION

Considering the limitation of existing work on the design of Statcom an approximate linearization of Statcom is proposed in this paper. We formulate the Statcom equations in terms of deviation from operating points resulting in quadratic nonlinearity. Approximate linearization technique due to Kang and Krener is used to remove the dominant quadratic terms leaving only third and higher order terms which can be considered negligible. The main advantages of the proposed method are that it avoids zero dynamics, allows larger variations of control input and is simpler to implement.

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